KIT DEVELOPED FOR DOING EXPERIMENTS IN PHYSICS

(Experiments suitable at B.Sc, M.Sc and Post-M.Sc. Physics levels)

R. Srinivasan
and
K.R. Priolkar

INSTRUCTION MANUAL

SPONSORED BY

INDIAN ACADEMY OF SCIENCES, BANGALORE
INDIAN NATIONAL SCIENCE ACADEMY, DELHI
THE NATIONAL ACADEMY OF SCIENCES INDIA, ALLAHABAD

August 1, 2013

CONTENTS

PREFACE

The laboratory programs in the BSc (Physics) and MSc (Physics) courses in various universities in the country are in a very poor state. Over the years instruction in colleges, both at the undergraduate and postgraduate levels, has neglected experimental physics at the cost of theory. Neither is the theory being taught well. The Indian Academy of Sciences, Bangalore, entrusted the responsibility of developing simple, but effective, experiments of low cost to me with the aim of improving the laboratory programs in colleges and universities. In this task I sought and obtained the unstinted co-operation of Dr. K.R.Priolkar and Prof. Sadique of Goa University and Dr. Efrem DeSa of Carmel College, Margao. We were clear in our mind that the experiments must verify physical laws and illustrate physical principles, besides yielding values of physical quantities with reasonable accuracy (within 10%). We felt that, at the level of B.Sc and First year M.Sc the readings should be taken manually by a student so that he/she understands how the measurements are done. For example the student should realize that a thermocouple used to measure temperature will have a nonlinear response, while a Platinum resistor will have a linear response. Experiments, in which the data are automatically acquired by a computer, which corrects for the nonlinearity of temperature response of a thermocouple, could be profitably introduced in the second year of the M.Sc program.

In the last fifty years there has been a sea change in measurement techniques which has been ignored by our universities and colleges. The students still do experiments with very old techniques. We recognized that we should introduce the students to some relatively newer techniques of measurement made possible by the availability of inexpensive analogue chips. We intended to develop some simple stand alone analogue electronic circuits for carrying out the experiments. These analogue circuits are based on simple and low-cost IC chips, which are locally available. In building these circuits our aim has been to construct a low cost simple circuit with which the student can make reasonably good measurements. We have not gone in for sophistication in building the circuits. *It is emphasized that the aim of this course is to impart training in doing good experiments in Physics and not to teach electronics per se. The circuits developed are simple and are adequate for the purpose of carrying out the physics experiments in the manual. A purist in electronics is welcome to improve on these circuits.*

The first Refresher course was conducted in Goa in 2001. Since then we have been adding new experiments and making some improvements to the first circuits we built so that the circuits would become versatile. From 2001 to 2007 these courses were entirely sponsored by the Indian Academy of Sciences in Bangalore. From 2007 all three science academies in India have joined together in sponsoring the Refresher courses.

In 2006 we invited twenty-five senior teachers of Physics, nominated by the Indian Association of Physics Teachers, for a two-day workshop in Bangalore in which all the experiments were demonstrated. The teachers welcomed the initiative of the Academy and suggested that we should get some company to make a kit of these experiments commercially.

Following the suggestion of the nominees of the Association of Physics Teachers, we looked for a company that will fabricate the kits. We located Ajay Sensors and Instruments in Bangalore. This company has good experience in fabricating scientific instruments and has good machine shop and electronic facilities. The company fabricated the kits in consultation with Prof. Srinivasan and these were used successfully in the Refresher Course conducted at Pondicherry University from July 7 to 23, 2008. The Academies have now licensed Ajay Sensors and Instruments to produce the kits and sell them to colleges.

We were also encouraged by the fact that the Goa and Mysore universities introduced these experiments in the curriculum of the M.Sc courses they were conducting. The experience of the course we conducted for the teachers of the Kerala University in the Mar Ivanios College in Trivandrum in 2007 November served as a model for future refresher courses. We have conducted forty nine courses till the end of June 2013 and more courses are planned till August 2013. Eighty institutions including some IISERs, IITs, University of Mumbai-DAE Center for Excellence in Basic Sciences, many universities and autonomous institutions have included these experiments in the curriculum so far. More than 100 kits have been sold.

This manual contains detailed information on the experiments. The theeoretical background required for understanding the experiments is provided. The procedure for carrying out the experiment is explained. For some of the experiments it is necessary to make some mechanical fabrication. Details of these are also described. Sample data for each experiment are given. At the end of each experiment a few questions are added to test the understanding of the student. The manuscript is revised periodically. Even so there may be typographical errors. We would be grateful to the reader for pointing out these errors.

In the XXXVII course Dr. T.G. Ramesh of the Materials Science Laboratory, NAL, Bangalore, introduced experiments with the Thermo-emf Analyser, the Resistograph and the Differential Thermal Analysis set up developed at NAL by him and his colleagues. He introduced experiments on the Neutral temperature of the thermoemf of Fe-Cu thermocouple, Tracing the Paramagnetic to Ferromagnetic Phase transition in Ni with the resistograph and demonstrating phase transitions in KNO_3 and Copper sulphate using the DTA set up. Dr. Ramesh and Dr. Subha of NAL prepared the write up on these experiments which have been included in this manual. The Director of the National Aerospace Laboratories, Bangalore, very kindly gifted the equipment for these experiments to the Indian Academy of Sciences. New experiments are introduced in the course from time to time and the manual is updated to include these experiments.

The manual has been revised from time to time. Presentation of graphs and tables has been improved.

In all the courses conducted so far, the experiments worked extremely well. The participants were very happy with the experiments. The appreciative comments from the participants will encourage us to put in more efforts to add new experiments and improve the manual. We are very thankful to these participants.

We hope that these experiments will ultimately get into colleges.

Mysore
1 August 2013 R.Srinivasan

3. THE INDIAN ACADEMY OF SCIENCES, BANGALORE, KIT FOR EXPERIMENTAL PHYSICS

The following equipment is manufactured and sold by Messrs Ajay Sensors, Bangalore, under license from the Indian Academy of Sciences, Bangalore.

1. A regulated 30 V, 2A power supply
.
2. An On-OFF temperature controller using Pt 100 sensor
.
3. A constant current source working in two ranges: Low current range: 100 micro amps to 20 milliamps. Current remains constant to within 0.1% when the load varies, till the voltage across the load reaches 18 V. High Current: range: 20 mA to 0.3 amps continuous operation. Current remains constant within 3% as the load is varied till the voltage across the load reaches a value of about 18 V. For operation over a short interval the current can go up to 600 mA.

4. A DC differential amplifier with three inputs. Amplification can be 10 or 100. Offset adjust pot and reversing switch are provided. Useful for measuring thermo-emfs.

5. A capacitance meter for measuring capacitances up to 250 pfd. Useful for measuring dielectric constant of non-polar liquids and dipole moment of acetone. DC output voltage is proportional to capacitance.

6. A signal generator capable of giving square, triangular and sinusoidal output. Frequency from 20 Hz to 30 kHz in four ranges. RMS amplitude variable from 0 to 5 Volts (peak to peak amplitude from 0 to 14 V). Panel meter indicates frequency up to 10 kHz and amplitude up to second decimal place. Panel indication of frequency will be correct only to within 10%..

7. A power amplifier working in the audio range. The input is from the signal generator. It will drive a loudspeaker or a coil to generate an oscillating magnetic field.

8. An AC Bridge circuit with which Maxwell, DeSauty's and Maxwell-Wien bridges can be realized.

9. A circuit for measuring thermal relaxation time of a serial light bulb and verifying Debye's relaxation formula.

10. A lock in amplifier working from 400 Hz to 5 kHz. Provides a DC output of about 1 V for an AC rms input of 200 microvolts. Taps are taken to illustrate how phase sensitive detection works. An internal calibration circuit is provided.

11. A circuit for measuring high resistance by leakage.

12. An integrator to be used with the B-H curve and Search coil set ups.

13. Feigenbaum circuit to show bifurcation in Non-Linear dynamics.

14. Chua's circuit for non-linear dynamics.

15. A circuit for measuring k/e with a transistor.

Experimental set-ups to work with the above circuits

1. A furnace going up to 300 C to be used with the regulated power supply and temperature controller.

2. An insert to the furnace to measure thermo-emf of a copper constantan thermocouple and the forward voltage on a silicon diode at constant current to show their utility as temperature sensors.

3. An insert to measure the temperature coefficient of copper. It can be used with a sample of a semiconductor to measure its band gap.

4. An insert to measure temperature variation of capacitance of a ceramic capacitor and to verify the Curie Weiss law. A polymeric capacitor is also included to show the difference in behavior.

5. A variable load box to plot the load regulation curve of the constant current source with a current in the 1 to 5 mA range.

6. A calibration box for the DC differential amplifier
.

7. A set up to verify Stefan's radiation law and measure Stefan's constant.

8. A set up to measure thermal and electrical conductivity of copper and find the Lorentz number
.

9. A set up to measure thermal conductivity of Perspex.

10. A set up to measure thermal diffusivity of brass.

11. A capacitor box to verify the law of addition of capacitances using the capacitance meter.

12. A cylindrical capacitor to measure the dielectric constant of a non-polar liquid and the dipole moment of acetone using the capacitance meter.

13. A L-C-R box to (i) verify how the impedance of an inductance varies with frequency and find the inductance of a coil, (ii) to measure by how much the phase of the voltage, relative to the current, across an inductor varies with frequency and calculate the resistance of the inductor, (iii) to verify how the impedance of a capacitor varies with the frequency, (iv) to show that the impedance of a series resonant circuit is a minimum at the resonant frequency and (v) to show that the impedance of a parallel resonant circuit is maximum at the resonant frequency.

14. A box for passive filters to measure the characteristics of low pass, high pass and band pass filters.

15. A box for measuring a resistance below 1 ohm with the Lock in amplifier

16. A mutual inductance coil to show that the emf of the secondary differs in phase from the primary current by 90^0, to show that the emf of the secondary is proportional to the frequency and the current through the primary. This is done with a lock in amplifier. The mutual inductance is about a hundred micro-henries and this can be measured to an accuracy of 2 to 3% with a primary current of less than a milli-amp.

17. B-H curve set up.

18. A solenoid and search coil

19. A set up to determine Young's modulus by flexural vibrations of a bar.

20. A set up to measure rigidity modulus of a brass wire.

One may buy a standard kit consisting of electronic circuits 1 to 10 and experimental Set ups 1 to 15. The kit will have two constant current sources, two DC Differential Amplifiers and two signal generators. One may also buy individual items.

The items are guaranteed for a period of three years. If within this period anything goes wrong due to faulty manufacture and the item is not tampered with, it may be sent back to Ajay Sensors who will carry out the repairs free of charge and return the item to the consumer.

The electronic circuits will function well if the earthling in the laboratory is good. The voltage between the neutral and ground should be less than 1 Volt.

Messrs Ajay sensors will provide a price list on request. If one wants to buy the equipment required to carry out any one of the above experiments, Ajay Sensors will be happy to provide the list of equipment required. The company may be contacted at

Messrs Ajay Sensors and Instruments
45/17, Gubbanna Industrial Garden,
12th A Cross, 6th Block,
Rajaji Nagar,
Bangalore 560 010

E-mail: ajaysensors @ yahoo.com

SECTION I

BRIEF WRITE UP
ABOUT
ELECTRONIC EQUIPMENT

I.1 REGULATED POWER SUPPLY

1. INTRODUCTION:

A regulated power supply is a source of DC electrical power, the voltage of which can be controlled between zero and a maximum value. Such a power supply is required for heating a furnace and maintaining it at a given high temperature. This will enable experiments to be carried out on the temperature variation of some physical property of a material such as the electrical resistance.

The regulated power supply provided with the kit can go up to a voltage of 30 V DC and can provide a maximum current of 2 A. The maximum power that can be drawn from the supply is 60 W.

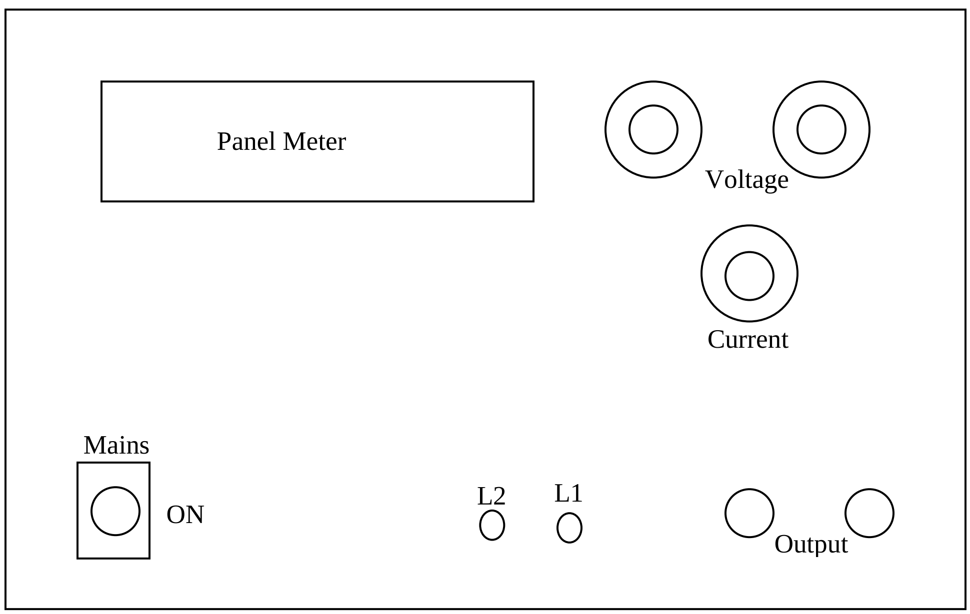


Figure I.1.1 Front Panel of Regulated Power Supply.

2. FRONT PANEL:

On the bottom left of the front panel is the mains-ON switch. On the top right there are two knobs with which the voltage output can be controlled. The left knob is for coarse adjustment of the voltage and the right knob is for fine adjustment. The voltage is indicated on the panel meter. Below the voltage control knobs is a current adjust knob. This knob sets the maximum current that can be drawn. Below this knob are the two banana plugs for connecting the load. On the left of the banana plugs are two indicator lights. As long as the current drawn is less than the value set by the current adjust knob, the voltage will appear at the output banana plugs and the green indicator light L2 will be on. When the current exceeds the set value, the indicator light L_2 will go off and a red light will glow at the indicator L1. If we want to continue, the current adjust knob is turned right. The red light will switch off and the green light at the indicator marked L_2 will be turned on. The voltage will be applied to the load.

I.2 TEMPERATURE CONTROLLER

Figure I.2.1 Temperature controller box

1. INTRODUCTION:

For studying temperature variation of a physical property of a material, it is necessary to put the sample in a furnace and adjust its temperature at different set values. A measurement is taken when the furnace reaches the set temperature and remains at this temperature for some time required for the sample to reach thermal equilibrium with the furnace. The furnace is heated with the regulated power supply described in the previous section. A temperature controller is used to set the temperature, which the furnace is required to reach, and to maintain the temperature within specified limits by controlling the heating power. With a Pt 100 temperature sensor the instrument can also be used as a temperature indicator.

On the top of each temperature controller a connection diagram is pasted. This diagram tells you to which terminals at the back of the temperature controller one should connect the Pt 100 leads, to which terminals one should connect the leads to the relay in the box and to which terminals the leads from 220 V AC power should be connected. Please follow this diagram while making connections.

2. METHOD OF FUNCTIONING OF THE TEMPERATURECONTROLLER

The temperature of the furnace is measured with a sensor. There are different types of sensors to measure the temperature, such as the thermocouple, the Platinum resistor etc. The temperature controller described here uses the Pt 100 sensor. This is a platinum resistor with a resistance of 100 Ohms at 0^0 C. The resistance of this sensor increases almost linearly with temperature in the range from -170^0 to 400^0 C at a rate of 0.4 Ohms per degree Centigrade change in temperature. This is a convenient thermometer to use.

The thermometer is placed within the furnace and monitors its temperature. If we want the temperature to be controlled, the regulated power supply is connected to the furnace through a relay in the temperature controller box as shown in Figure I.2.1 below.

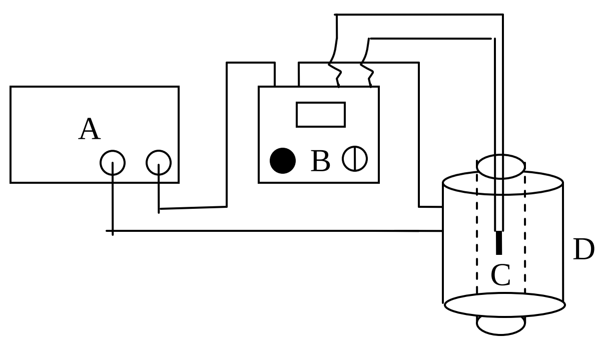


Figure I.2.2: Schematic diagram of a temperature controlled furnace
A: Regulated DC Power Supply; B: Temperature Controller
C: Temperature sensor; D: Furnace

A power chord is connected to two designated terminals at the rear bottom of the temperature controller.

The positive terminal of the regulated power supply is connected to one of the banana plugs on the furnace. The other banana plug on the furnace is connected to one of the pair of designated screw terminals (relay) at the back of the temperature controller. The other terminal of the (relay) pair is connected to the ground terminal on the regulated power supply. There is a relay in the temperature controller box. This relay is normally closed.

Then the power supply is connected to the furnace. When the power supply is switched on a current will flow through the heating coil of the furnace. The furnace gets hot.

A Pt 100 temperature sensor is placed in the furnace to measure its temperature. The two terminals of the Pt 100 resistor are connected to a second designated pair of screw terminals at the back of the temperature controller. One of the terminals should be shorted with a third terminal. This is indicated on the connections diagram pasted on the top of the controller box. The resistance of the sensor is converted to degrees centigrade and is shown on the panel meter of the temperature controller.

There is a button on the bottom left, and a knob on the bottom right, of the front panel of the temperature controller. By pressing the button and turning the knob we can set on the front panel of the controller the desired temperature the furnace should reach. When the button is released the indication on the front panel goes back to the actual temperature of the furnace. The reading on the panel meter increases as the furnace gets hot due to the passage of current through the heater. When the reading on the panel meter crosses the set temperature reading, the relay in the temperature controller box opens cutting off the current to the furnace. Because of its thermal inertia the furnace continues to heat to a temperature beyond the set value and then starts to cool. When the temperature of the furnace goes below the set value the relay in the controller box closes restoring the current to the furnace. The furnace starts to get hot. Thus the temperature of the furnace oscillates about the set value. The voltage across the furnace is adjusted so that the oscillation about the set temperature is about $\pm$ 0.5^0 C. Such a control is good enough for temperature measurement of physical properties of a material. The front panel meter indicates the temperature of the furnace with this accuracy as long as the temperature is below 200^0 C. Above 200^0 C the temperature is indicated to within 1^0C.

Sometimes one does not wish to control the temperature but one would like to heat the furnace slowly at about 1^0C/minute up to, let us say, 150^0C. Then the controller can be set at a temperature higher than 150^0 C and the voltage on the regulated power supply adjusted to give the desired heating rate. **One may also connect the furnace directly to the power supply bypassing the relay.**

The given temperature controller can be used with a Pt 100 sensor in the range 0 to 400^0C.

Note added: In a new version of the temperature controller the push bottom and knob to set the temperature are not there. The temperature is set by pushing one of the buttons on the front panel and setting its value.

I.3 FURNACE

1. INTRODUCTION:

A furnace is essential if properties have to be measured over a range of temperatures. Such a furnace is described below.

Figure I.3.1 Tubular Furnace

2. DESCRIPTION OF THE FURNACE

This is a tubular furnace with an inside diameter of 2 cm. Nichrome wire of resistance about 10 Ohms is wound on the middle portion of a ceramic tube of length 30 cm. This tube is put inside a cylindrical aluminium box with glass wool surrounding the ceramic tube to provide thermal insulation. Both ends of the ceramic tube are open. The furnace can reach a temperature of 300 C when powered by the regulated power supply described earlier. **When the furnace is heated the two open ends must be loosely plugged with a wad of cotton. This prevents convection air currents from taking away the heat supplied.** After the experiment, the wads are removed to allow the furnace to cool down fast. A stand is fixed to one side of the furnace so that inserts can be hung in the furnace.

Three inserts are provided:

1. An aluminum square-sided block containing a Pt 100 resistor, a Copper constantan thermocouple, a Si diode. With this one may study the calibration of the diode and thermocouple to measure temperature.

2. An aluminum block in which a Pt100 sensor, a copper coil of about 30 Ohms resistance with current and potential leads. With this one may measure the temperature coefficient of resistance of copper. **(An insert carrying, in addition, a silicon chip with current leads to measure the energy band gap of silicon can be provided as an optional item at extra cost).**

3. An aluminum block with a Pt 100 sensor, a ceramic capacitor of nominal value of 0.1 μfd capacitance and a capacitor with polymer as dielectric with the same nominal value. This insert is used to measure the temperature variation of capacitance and to verify the Curie-Weiss law.

CAUTION: IN ALL THE ABOVE INSERTS THERE ARE SOFT SOLDERED JOINTS. SO EXPERIMENTS WITH THESE INSERTS CAN ONLY BE PERFORMED UP TO A TEMPERATURE OF 150 C.

The furnace can be used with other custom designed inserts up to 300 C.

The furnace is mounted on a plastic table.

I.4. CONSTANT CURRENT SOURCE

Figure I.4.1   Constant Current Source

1. INTRODUCTION:

A constant current source is a versatile instrument used in many experiments described below.   It produces a constant DC current independent of the load over a range of load values.   As long as the potential difference across the load does not exceed a given value the current remains sensibly constant.   This will be seen from the specimen set of readings given in Section II.3.2.

2. DESCRIPTION OF THE SOURCE

The front panel of the commercial constant current source is shown in the figure I.4.1. On the bottom left of the panel is the switch marked Mains.   When the switch is down as shown in the Figure a red light will glow indicating that the instrument is on.   Power is now given to the current source.

The current source operates in two ranges: Low range: 100 μ amp to 20 m amp.
High range: 20 m amp to 0.3 amps in continuous operation;
up to 0.5 amp in intermittent operation

(The rating of 0.3 amps given above as the maximum current for continuous operation is a conservative rating. A given constant current source may go beyond this rating. Similarly the rating of 0.5 amps as the maximum current for intermittent operation is conservative)

If the current is set at more than 0.3 amperes for a long time in the high current range, the IC voltage regulator chips may become very hot and the current may start decreasing. **So one should use the instrument in this range above 0.3 amps only for a short time of a few minutes.**

There are two range switches to the right of the ON light. When these switches are put in the position marked LOW the instrument operates in the low current range. When the switches are set in the position HIGH, the instrument operates in the high current range.

There are three knobs, one in the right middle, and two at the top of the panel. The knobs can be turned ten turns. The knob marked low is used to set the current in the low range. **The current is at its minimum value when the knob is in the extreme left position.** The current increases when the knob is turned to the right. **Near the right extreme position the current will change rapidly as the knob is turned. The knob must be turned very slowly as the extreme right position is reached.** The two knobs on the right top control the high current. **Before switching on the instrument make sure that both the knobs are in the extreme left position.** The two knobs can be used to adjust the current at a precise value. **For example, to set a current of 0.300 A, first turn the left knob till you reach a current below 0.300 A (say 0.290 A). Then turn the right knob till you reach the desired current.** **<u>Do not turn the knobs fast at the right end of the pots. The current will increase very rapidly.</u>**

On the top left of the front panel is a meter, which reads the set current in milli-amperes. In the low current range the current will be indicated to the first decimal place. For a more accurate reading of the current in the low current range one should connect a DMM in the 20 milliamp range in series with the load or measure the voltage across a known resistance in series with the load.

The two banana plugs marked O/P on the bottom right of the panel are to be connected to the load.

Load regulation of the current is discussed in Section II.3.2.

I.5 DC DIFFERENTIAL AMPLIFIER

Figure I.5.1 DC Differential Amplifier

1. INTRODUCTION

The DC differential amplifier is used to measure small DC signals of a few microvolts. Thermocouples measuring temperature differences of a few degrees provide such signals. The DC differential amplifier can amplify ten or hundred times.

2. DESCRIPTION OF THE INSTRUMENT

The mains switch is at the bottom left corner of the front panel. When it is switched on a red light will come on. Three different sources of emf can be connected to three pairs of banana terminals marked I1, I2 and I3 on the bottom of the front panel. A selector switch at the center of the panel will select the source to be amplified depending on whether it is put in the position I1 or I2 or I3 marked next to the switch. The toggle switch to the left of the selector switch selects the amplification factor. When the switch is down, the amplified output is ten times the input. When it is up, the output is 100 times the input. **In all the experiments to be described in Section II, the thermo-emfs are amplified hundred times with the amplifier**. The pot to the right of the selector switch and marked OFFSET is used to adjust the offset of the amplifier to within a few milli-volts of zero. The switch marked REVERSE above the offset-adjust pot will reverse the connection of the source emf to the amplifier when it is toggled. The output is measured by connecting a DMM in the appropriate DC range to the banana terminals marked O/P.

3. MODE OF USE OF THE INSTRUMENT

First short the input terminals I1. Put the selector switch in position I1. Connect a DMM in the DC 200 mV range to the terminals marked O/P. Switch on the mains. The DMM will indicate a positive or negative reading. Turn the offset adjusting pot slowly to reduce the reading to within a few milli-volts of zero. **On throwing the reversing switch the sign of the reading will not change and the magnitude of the reading will remain nearly the same.**

.

Now the source, the emf of which is to be measured, is connected to the terminals I1. The DMM is set in the appropriate range and will indicate a reading in millivolts. This reading may have a plus or minus sign. We call the reading V_+ or V_- depending on the sign. The sign of the reading and its magnitude will change if the reversing switch is thrown to the other position. If the original reading was V_+, the reading on throwing the reversing switch will be V_- and vice-versa. In the amplification range 100,

$$V_+ = 100\ V_{signal} + V_{offset}$$
$$V_- = -100\ V_{signal} + V_{offset}$$

So

$$V_{signal} = (V_+ - V_-)/\ (2x100) \qquad \text{(I.5.2.1)}$$

We get rid of the offset voltage by the use of the reversal switch.

To illustrate this, let V_+ be 10.9 mV and V_- be -9.1 mV. Then

$$V_{signal} = (10.9 - (-9.1))/\ (2x100)\ \text{mV} = 100\ \mu\text{V}$$

If the sign of the output voltage does not change on toggling the reversing switch it means that the offset voltage is larger in magnitude than 100 V_{signal}. One can turn the offset pot so that the two signals are of opposite sign. Note that even if V_+ and V_- are of the same sign, the value of V_{signal} calculated using equation I.5.2.1will come out right.

If we have more than one source of emf to be measured, the sources are connected to the input terminals I1, I2 and I3 and the selector switch is turned to the appropriate positions and the measurements made using the reversing switch. Generally the offset voltage does not vary in sign or much in magnitude when different sources are selected by the selector switch. So it is enough to adjust the offset to within a few millivolts for one position of the selector switch.

In the experiments to be described in section II, we use the differential amplifier to measure thermo-emfs, which are in the microvolt to millivolt range. For this purpose an amplification factor of 100 is used.

4. CALIBRATION

A calibration circuit is provided in a small box. This box contains a 1.5 V AA cell in series with a high resistance and a potentiometer. The voltage across the potentiometer is brought to two pairs of banana terminals on the box. By adjusting the potentiometer one can get any voltage from 15 mV upwards at the terminals T1. When the switch is turned on, a light glows indicating the charge state of the cell.

Figure I.5.2 Calibration box for the DC Differential amplifier

The voltage at the terminals T1 is set close to 20 mV as measured on a DMM in the DC 200 mV range connected to terminals. The terminals T1 are connected to the banana terminals I1 on the differential amplifier. The voltage read on the output terminals of the differential amplifier (following the procedure mentioned in the previous page) is checked to be hundred times the voltage at the terminals of the calibration box. Usually the calibration will not change, once the differential amplifier is calibrated at the factory. But it is good to check the calibration say once in six months. If the calibration differs very much from the correct value, the amplifier should be sent back to the manufacturer for recalibration. But this contingency is remote unless the amplifier has been maltreated in some way.

The terminals T2 give a voltage approximately one-tenth of the voltage at T1.

I.6 CAPACITANCE MEASURING CIRCUIT

Figure I.6.1: Capacitance circuit

1. INTRODUCTION

The capacitance measuring circuit can be used to measure capacitances up to 250 pfd. It gives a DC output in volts proportional to the value of the capacitance. It is useful for measuring dielectric constant of non-polar liquids with a cylindrical capacitor and dipole moment of a polar molecule like acetone. These experiments are described in Section II.6.1 to 3.

2. DESCRIPTION OF THE INSTRUMENT

At the lower left corner of the front panel is the mains switch. When it is pressed down the instrument is activated. On the lower right corner are two banana terminals marked C. The unknown capacitance is connected to these two terminals. In the middle of the front panel are three preset pots marked R1, R1' and R2. Their values are preset in the factory. But one may change the values if one wishes by turning the screws. For a given value of the unknown capacitance, the DC output will vary as these screws are turned. At the top right are two banana terminals. A DMM in the appropriate DC range is connected to these terminals to measure the output.

3. PRINCIPLE OF CAPACITANCE MEASUREMENT:

Inside the instrument there is a capacitance of 2 nfd (i.e. 2000 pfd). This capacitance is connected in series with a resistance R1 that is in the 100 k to 500 k range. The unknown capacitor C is connected to another high resistance R2 in the same range. When a DC voltage 12 V is applied, both capacitors get charged. But the time constant of the two circuits will depend on the product $C_{std}R1$ and $C_{unknown}R2$. Since $C_{unknown}$ is more than ten times less than C_{std} and R1 and R2 are nearly of the same order, the time constant $\tau_{unknown}$ is less than the time constant τ_{std}. So the unknown capacitance charges faster than the standard capacitance.

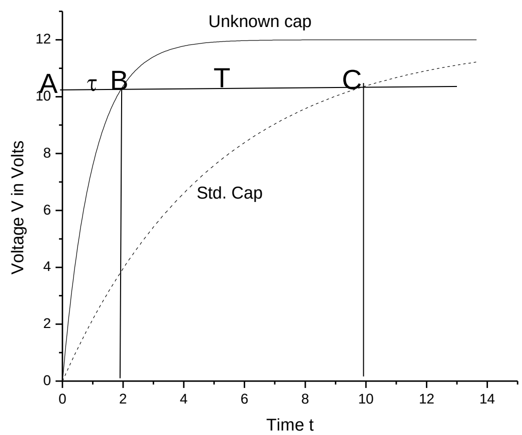


Figure I.6.2: Unknown capacitor charges faster than standard capacitor. Unknown capacitor reaches 9 V in time τ sec while standard capacitor reaches 9 V time T sec. T>τ.

While both the capacitors are charging, a fixed current passes through a transistor and produces a voltage of 10 Volts across a 1 k resistor. When the unknown capacitor reaches a voltage of 9 V a Schmitt trigger circuit switches off the current through the transistor. So current through the transistor flows for a time τ which is proportional to $\tau_{unknown}$ i.e. to $C_{unknown}R_2$. The standard capacitor C_{std} reaches 9 V at a later time T which is longer than τ since τ_{std} is larger than $\tau_{unknown.}$ The Schmitt triggers discharge both the capacitors when the standard condenser reaches 9V after a time T. The charging process begins again. The voltage across 1k appears in pulses of width τ as shown in Figure I.6.3.

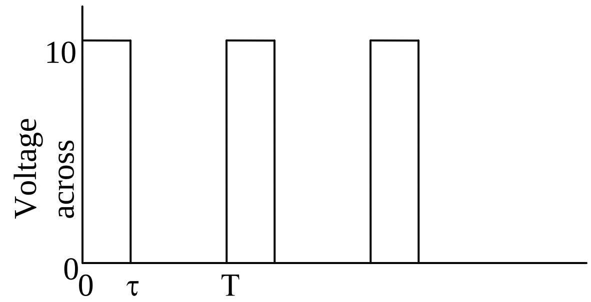


Figure I.6.3: Voltage pulse across 1 k resistor appears for a time τ in every T seconds.

Thus in 1 second the charge-discharge process takes place at a frequency 1/T that is in the range of a few kilo Hertz. The DMM reads the voltage across the 1 k resistor averaged over many charge-discharge periods. This average voltage will be $10\tau/T$. τ is proportional to $\tau_{unknown}$ i.e. to $C_{unknown}$ since R_2 is fixed. So the output DC voltage is proportional to the unknown capacitance. This is the principle on which the instrument works.

A box is provided with the instrument. This contains three capacitors 100 pfd, 47 pfd and 22 pfd connected at one end to a common terminal. These are nominal values. At the factory the resistances R1, R1' and R2 are adjusted so that when 100 pfd capacitor is connected to the terminals marked C, the DC voltage at the output terminals O/P is about 2 to 2.5 V. The linearity of the readings can be checked by connecting the 47 pfd and 22 pfd capacitors.

It is recommended that the settings of the resistors R1, R1' and R2 are NOT altered. They have been set to give a convenient output which is linear in the capacitance and which is suitable for measuring the dielectric constant of benzene and dipole moment of acetone with the cylindrical capacitor provided with the instrument.

CAUTION:

DO NOT USE THE INSTRUMENT FOR MEASURING DIELECTRIC CONSTANT OF POLAR LIQUIDS LIKE ACETONE OR WATER DIRECTLY. THE CIRCUIT WILL GET DAMAGED.

I.7 SIGNAL GENERATOR

Figure I.7.1 Signal Generator

1. INTRODUCTION:

The signal generator produces square, triangular and sinusoidal waves from 20 Hz to 30 kHz in four ranges. The upper value of 30 kHz is conservative. A signal generator may reach higher frequencies. For sinusoidal waves the rms amplitude can be varied from zero to 5 V maximum (i.e. peak to peak amplitude 14 V). There is a panel meter, which shows the frequency below 10 kHz and rms amplitude over the entire frequency range. The signal generator is used in many experiments to be described in Section II.

2. DESCRIPTION OF THE INSTRUMENT

On the bottom left corner is the mains switch. When it is pressed down an indicator light in the switch glows and the instrument is on. Next to the indicator is the knob of a range switch. There are four settings giving different ranges of frequencies. In the 0.1 kHz range the output frequency can be adjusted between 10 Hz and 100 Hz. In the 1 kHz range the frequency can be adjusted between 100 Hz and 1 kHz. In the 10 kHz range the frequency can be adjusted between 1 kHz and 10 kHz. In the 100 kHz range the frequency can be adjusted between 10 kHz and 30 kHz.

Next to the range switch is a selector switch to select the nature of the output, namely square waves (SQ), triangular waves (TRI) and sinusoidal waves (SIN). Next to the selector switch is a pair of banana terminals providing the output of the signal generator. Next to the banana terminals is a RCA socket from which also the output can be taken.

Above the output terminals is a pot marked amplitude. By turning the pot the rms amplitude of the output can be varied. There is a second pot marked frequency above the amplitude pot. By turning this pot the frequency can be varied from minimum to maximum value in a given range. There is a panel meter on the top with a toggle switch by its side. By putting the toggle switch to the position marked frequency the panel meter will give the frequency in a given range. The frequency meter is calibrated to read the frequency correct to ±10%. **The frequency reading will stabilize after a waiting time of 15 minutes. The frequency meter will not work beyond 10 kHz.** Beyond 30 kHz the output may be distorted with some spikes. For accurate measurement of frequency use a DMM which can read Hz (Like Meco 801 Auto). By putting the switch in the amplitude position the panel meter will read the RMS value of the output amplitude.

ALL **EXPERIMENTS WITH THE KIT ARE LIMITED TO FREQUENCIES BELOW 10 kHz.**

The signal generator will provide a current of a few milliamperes and so can be used directly to excite a circuit requiring such low currents. It cannot deliver large power.

I.8 POWER AMPLIFIER

Figure I.8.1 Power Amplifier

1. INTRODUCTION:

The power amplifier is used along with the signal generator for driving a loud speaker, or a coil of wire to produce oscillating magnetic fields.

2. DESCRIPTION OF THE INSTRUMENT

On the left bottom of the front panel is the 'Mains on' switch. When it is pressed down the instrument is on. The signal generator output is connected to the RCA socket marked IN on the front panel. The load (a loudspeaker or coil) is connected to the RCA socket marked OUTPUT on the front panel.

The power amplifier works in the audio range of frequencies i.e. from 30 Hz to 30 kHz. If a loud speaker is connected to the power amplifier it will emit a sound at the pitch of the input from the signal generator. The loudness of the sound increases as the amplitude of the input is varied by turning the amplitude knob on the signal generator.

The power amplifier will deliver a few watts of power (about 10 watts) to a matched load between 4 to 8 Ohms.

I.9 AC BRIDGE CIRCUIT

Figure I.9.1 AC Bridge Circuit

1. INTRODUCTION

There are three AC Bridge circuits, which can be realized with the above instrument. These are the Maxwell's bridge, the DeSauty's bridge and the Maxwell-Wien bridge.

An AC bridge is a Wheatstone's bridge with impedances connected in the four arms as shown in Figure I.9.2 below. The figure is drawn to accord with the symbols on the AC Bridge circuit. SG_{in} and SG_{out} are the two terminals to which the signal generator is to be connected. The terminals marked A and B are shorted with SG_{in} while the terminal C is shorted with SG_{out}. Between A and D_1 is the impedance Z_1. Between D_1 and D_2 is the DMM in the AC 200 mV range. In the front panel of the bridge D_1 and D_2 are connected to two terminals marked DMM. The DMM is to be connected between these two terminals. One can connect any impedance between A and D_1. For example one terminal of a coil is connected to D_1 internally and the other end of the coil is connected to a free terminal marked L_1. If the terminal A is connected to the terminal marked L_1 the coil takes the place of Z_1 in Figure I.9.2. In this fashion by connecting A to the terminals marked L_1, L_1', L_1" three different inductances can be connected as Z_1.

Similarly by connecting A to the terminals marked C_1, C_1' or C_1" one may connect three different capacitances as Z_1. In the third arm of the bridge circuit one end of the impedance Z_3 is connected internally to D_2. By connecting the free end of Z_3 to the terminal marked B one may connect the impedance Z_3 in the third arm of the bridge. In the third arm we may connect inductances L_3 or L_3', capacitances C_3 or C_3' and resistances R_3 or R_3'. Similarly by connecting the terminal marked C to L_4, L_4', C_4, C_4' or R_4, R_4' one may connect an inductance, capacitance or resistance as the impedance Z_4. The arm Z_2 is a variable resistor (pot). This pot is connected between D_1 and C when the switches SW_1 and SW_2 are pressed down. When they are up the pot is isolated and its resistance can be measured by connecting a DMM between the terminals marked T_1 and T_2.

External terminals are also provided marked EXT1, EXT2, EXT3 and EXT4 for connecting external inductances, capacitances or resistances in place of the built in coils capacitances and resistances**. In this case the terminals A, B and C are NOT connected to any other terminals and the switches SW_1 and SW_2 are in the up position.** At the terminals marked EXT2 one connects a dial resistance box and balance is obtained by adjusting the dials. In this way one may use impedances of one's choice.

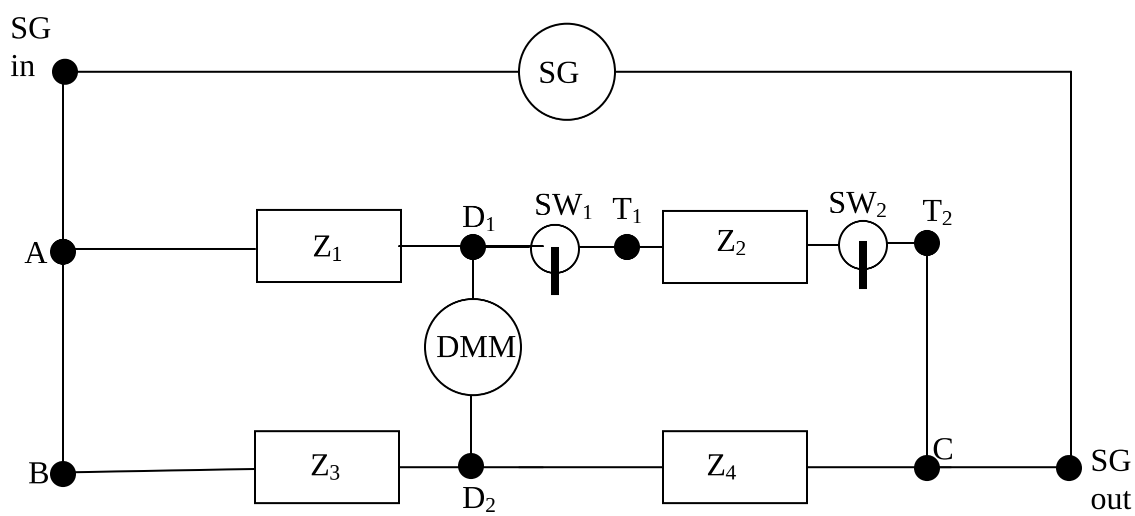
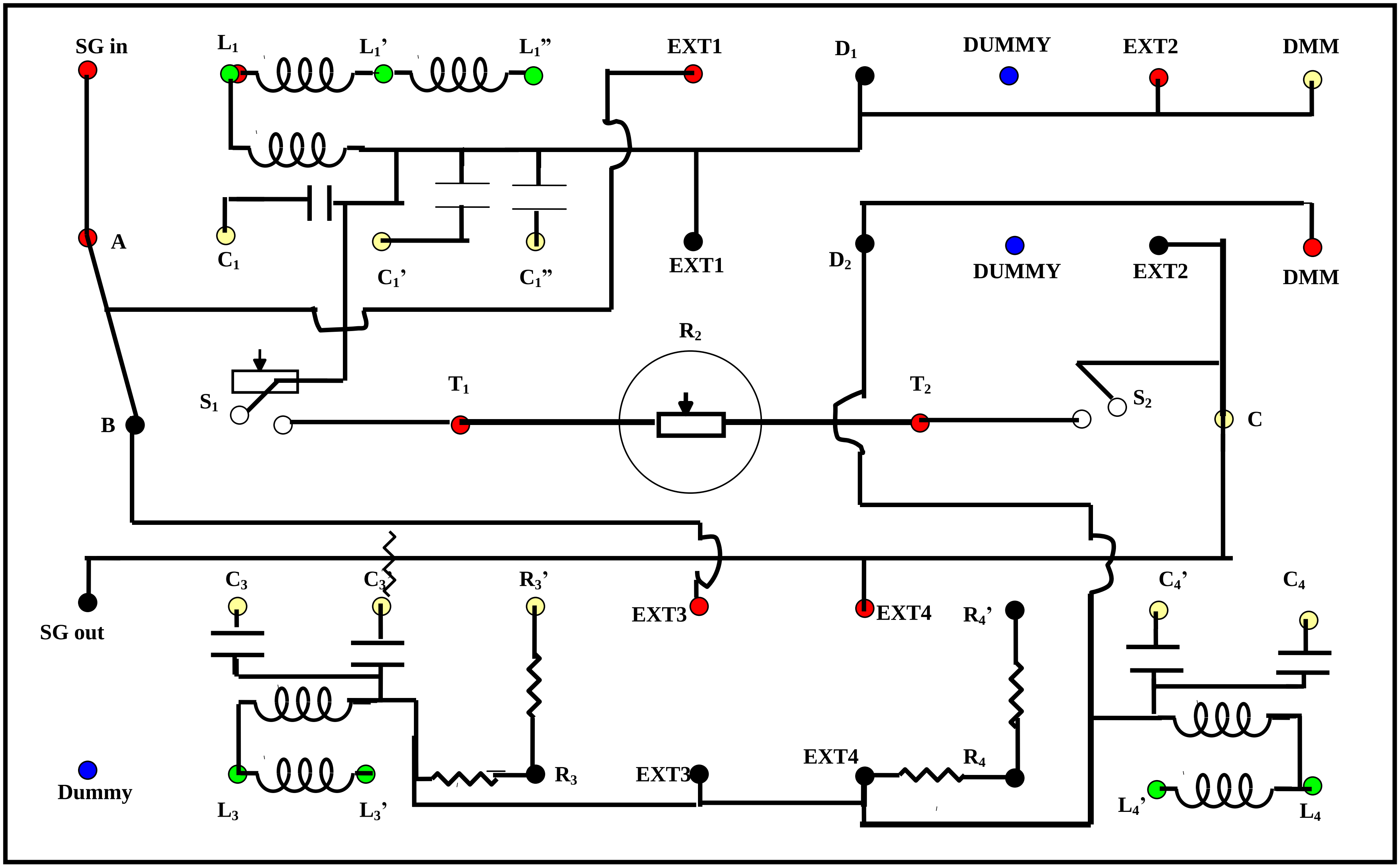


Figure I.9.2: Whetstone Bridge drawn to accord with the nomenclature on The AC Bridge circuit

Three AC bridges can be realized. By connecting A to L_1 (L_1' or L_1"), B to L_3 (or L_3') and C to R_4 (or R_4') one realizes the Maxwell's bridge. By connecting A to C_1 (C_1' or C_1"), B to C_3 (or C_3') and C to R_4 (or R_4') one realizes the DeSauty's bridge. By connecting A to L_1 (L_1' or L_1"), B to R_3 (or R_3') and C to C_4 (or C_4') one realizes the Maxwell-Wien bridge.


SG in
L1
L1'
L1"
EXT1
D1
DUMMY
EXT2
DMM
A
C1
C1'
C1"
EXT1
D2
DUMMY
EXT2
DMM
R2
S1
T1
T2
S2
C
B
C3
C3'
R3'
EXT3
EXT4
R4'
C4'
C4
SG out
Dummy
L3
L3'
R3
EXT3
EXT4
R4
L4'
L4

2. TROUBLESHOOTING:

If the bridge does not work check the following using a DMM in the kOHMs range.

1. Check that SG_{in}, A and B are connected together.
2. Check that SG_{out} and C are connected together.
3. Between D_1 and L_1, L_1' and L_1" one must obtain a resistance of approximately 35, 70, and 105 Ohms.
4. Between $D_1$and T_1 resistance should be zero when the switch SW_1 is pressed down.
5. Between D_1 and C the resistance must vary from zero to 2 k when switches SW_1 and SW2 are pressed down and the knob marked R_2 is turned.
6. Between D_1 and the black terminal of EXT1 and between D_1 and the red terminal of EXT 2 the resistance must be zero.
7. Between D_1 and the top terminal marked DMM resistance must be zero.
8. Between D_2 and L_3 (L_3') the resistance must be approximately 35 (70) ohms.
9. Between D_2 and R_3 (R_3') the resistance should be 220 (320) ohms.
10. Between D_2 and the black terminal of EXT3 resistance must be zero.
11. Between D_2 and the terminals marked L_4 (L_4') resistance must be approximately 35(70) ohms.
12. Between D_2 and terminals marked R_4 (R_4') the resistance must be 220 (440) ohms.
13. Between D_2 and the black terminal of EXT 4 resistance must be zero.
14. Between D_2 and the lower terminal marked DMM resistance must be zero.
15. Between terminal A and the red terminal marked EXT1 resistance must be zero.
16. Between black terminal of EXT2 and terminal C resistance must be zero.
17. Between terminal B and the red terminal of EXT3 resistance must be zero.
18. Between terminal C and the red terminal of EXT4 resistance must be zero.

This troubleshooting will help you to identify where the problem lies. DO NOT OPEN AND TRY TO RECTIFY THE ERROR. Inform the manufacturer who will rectify the fault.

I.10 LOCK-IN AMPLIFIER

Figure I.10.1 Lock-in Amplifier

1. INTRODUCTION:

The lock-in amplifier is a device, which can measure very small AC voltages in the presence of noise. It uses the principle of phase sensitive detection. It produces a maximum DC output when the signal to be measured is in phase with a reference signal at the same frequency. This lock-in amplifier can work in the frequency range 100 Hz to 5 kHz. An AC signal of 200 μV rms amplitude will produce a DC output of about 1 V.

2. DESCRIPTION OF THE FRONT PANEL

The mains switch is at the top left hand corner of the front panel. When the switch is pressed down the lock-in amplifier is powered and an indicator light in the switch comes on. On the right half of the front panel there are a number of RCA sockets. The bottom-most socket is labeled SIG GEN. **A signal generator is connected to this socket for internal calibration of the lock-in amplifier.** Above the sig gen RCA socket, we have two other sockets marked REF and REF'. These are sockets which can be connected to a two-channel oscilloscope to display the reference signal and the phase shifted reference signal respectively.

The reference signal is phase shifted by turning the knob of a pot marked PHASE SHIFT ADJUST. The RCA socket marked SIGNAL can be connected to an oscilloscope to show the amplified small AC signal with noise. The RCA socket marked PIN 13 shows the output at pin 13 of the chip AD 630. As the phase shift adjust knob is turned the pattern of the output at pin 13, as seen on an oscilloscope, changes. When the phase-shifted reference is in phase with the signal, the output at Pin 13 looks like the output of a full wave rectifier. **The RCA pin marked OUTPUT is connected to a DMM in an appropriate DC range. This measures the amplified DC output from pin 13 of the Lock-in chip.**

There are two toggle switches. **When both are put in CAL mode one can internally calibrate the lock-in as explained in Section II. When an external signal is to be measured the toggle switches are put in position EXT. In this mode, the external AC signal should be connected to the two Banana terminals (Red and Black) at the bottom of the front panel. The external reference signal should be connected to the RCA socket marked EXT REF.**

3. BLOCK DIAGRAM OF THE LOCK IN AMPLIFIER

In this section a block diagram of the lock in amplifier is given. All the front panel controls are marked on the diagram. The thick line gives external reference wiring. The broken line gives the wiring for the signal generated internally in the calibration mode when a signal generator is connected to the RCA socket marked SIG GEN. The internal calibration circuit consists of three resistances, 220 k, 10 Ohms and 5 k in series across which the AC voltage from the signal generator is applied. The voltage across 10 Ohms provides the internally generated signal used for calibration. The voltage across the 5 k provides the internally generated reference signal. When the switch SW1 is put in the position CAL the internally generated signal is connected to the signal amplifier. Similarly when the switch SW2 is put to the position marked CAL the internally generated reference signal is connected to the reference amplifier.

When measurements on external systems have to be done, the switches SW1 and SW2 are put in the positions marked EXT. The signal generated from the external circuit is connected to the red and black banana terminals marked EXT SIGNAL. Similarly the reference signal from the external circuit is connected to the RCA socket marked EXT REF. When the switches SW1 and SW2 are in the external position the external signal is connected to the signal amplifier and the external reference to the reference amplifier. The signal coming out of the signal amplifier is fed to pin 1 of AD 630 chip.

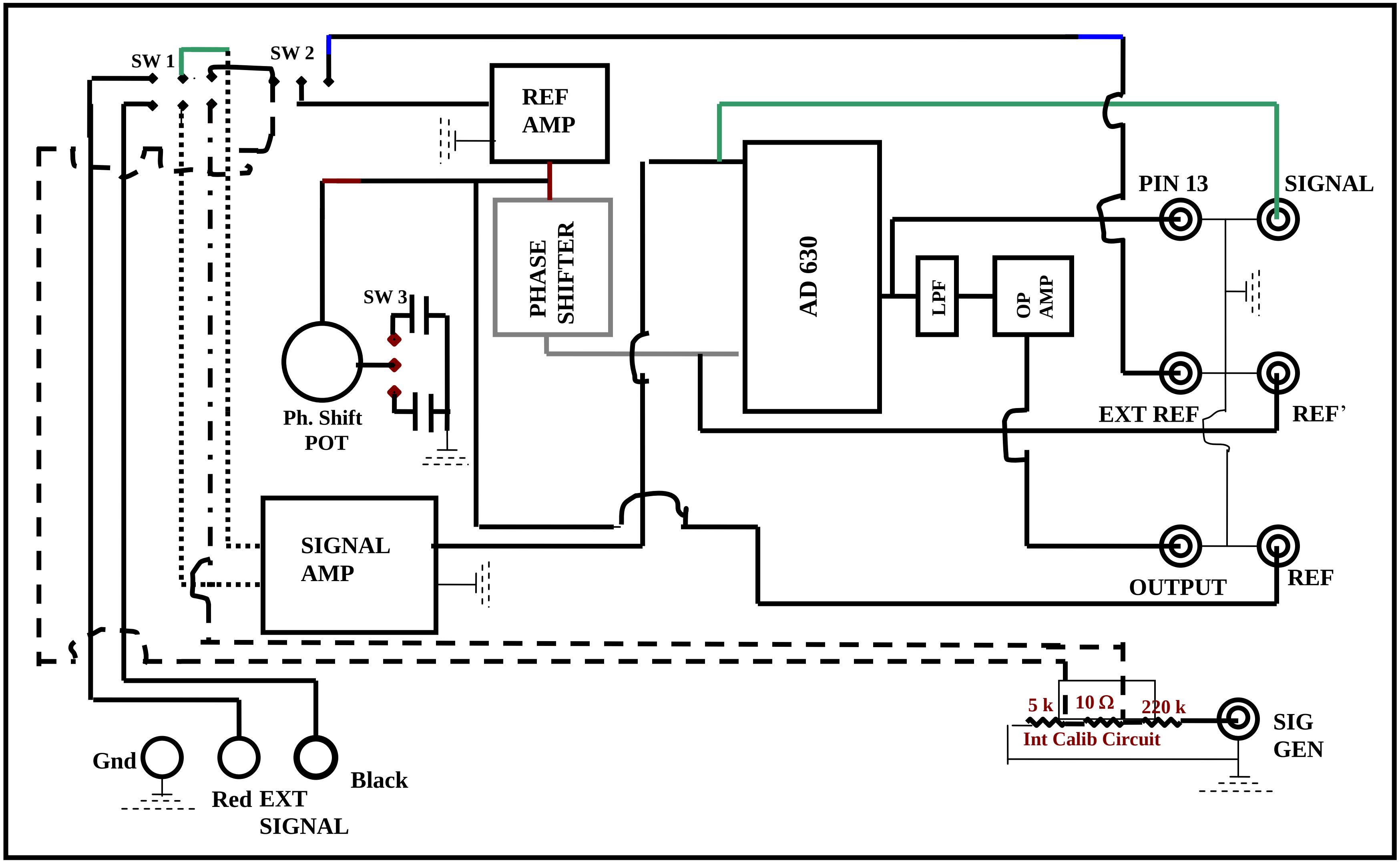


Figure I.10.2. SCHEMATIC DIAGRAM OF THE LOCK IN AMPLIFIER CIRCUIT

The reference signal after amplification goes to a phase shifter. Phase shifting is achieved by a RC network. R is a pot which can be adjusted by turning a knob marked phase shift. The maximum phase shift that one can achieve is given by the product of frequency f, C and R. At frequencies below 1 kHz, one has to use a larger capacitance to achieve a phase shift close to 180 degrees at the maximum resistance in the pot. At frequencies greater than 1 kHz, the use of the same capacitor will cause the phase to be shifted rapidly as the resistance in the pot is increased. To make the phase change more gradual in the frequency range 1 to 10 kHz, a smaller capacitance is used. The switch SW3 selects the capacitor. At frequencies below 1 kHz the switch is put in the up position and at frequencies greater than 1 kHz the switch is put in the down position.

The reference signal after phase shifting is connected to pin 9 of the AD 630 chip. Leads are taken from the input of the phase shifter and from pin 9 of AD 630 to two RCA sockets marked REF and REF' respectively. By connecting REF and REF' to two channels of an oscilloscope one can see how much the phase is shifted on turning the pot. When the signal to be measured and the phase shifted reference signal match in phase the output at Pin 13 of AD 630 looks like that of a full wave rectifier. A lead is taken from Pin 13 to a RCA socket marked PIN 13 on the front panel. By connecting it to an oscilloscope one may see how the output at pin 13 varies as the phase of the reference signal is shifted. This output of AD 630 is connected to a Low Pass Filter (LPF) and then to an output amplifier. The output of this amplifier is connected to a RCA socket marked OUTPUT. A DMM in the appropriate DC V range connected to this socket measures the DC output of the lock in amplifier.

A lead is taken to a RCA socket marked signal on the front panel. By connecting it to an oscilloscope one can see the trace of the signal.

The principle of Phase sensitive detection is described in Section II Chapter 7.1. The internal calibration of the lock in amplifier is described in Section II, Chapter 7.2. Two experiments with the lock-in are described in Chapters 7.3 and 7.4 in Section II.

At frequencies greater than about 1 kHz there is a distortion in the Reference signal as seen at REF or REF'. However the lock-in will measure only the first harmonic component of the signal independent of the distortion. For student experiments this distortion does not matter.

IF THERE IS A PROBLEM IN THE LOCK IN AMPLIFIER PLEASE SEND IT TO THE MANUFACTURER FOR REPAIR.

I.11 CIRCUIT FOR HIGH RESISTANCE BY LEAKAGE AND SPLIT POWER SUPPLY

1. INTRODUCTION

Figure I.11.1 shows the front panel of the box containing the circuit for high resistance by leakage cum split power supply.

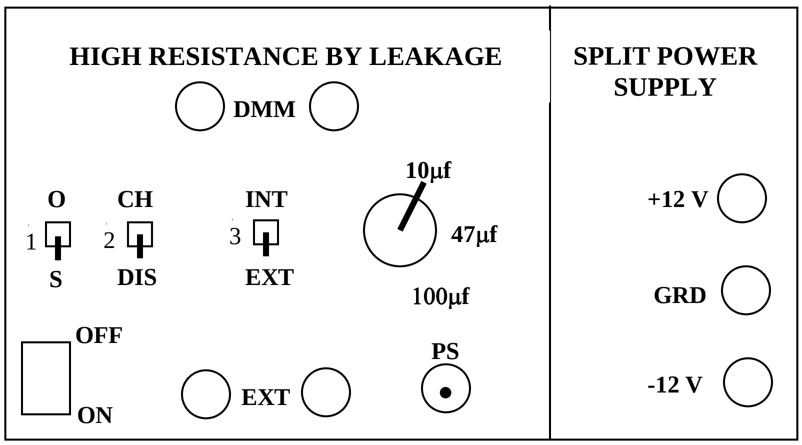


Figure I.11.1: Front panel of box containing the circuit for High Resistance by leakage cum split power supply

The front panel is divided into two areas. The area marked Split Power Supply contains three banana terminals marked +12 V, GRD and –12 V. One can use this to provide split power to a separate electronics circuit.

The main area marked High Resistance by leakage has all the controls on the front panel to measure a high resistance. From a regulated power supply inside the box, a potential divider provides about +2 V to charge a capacitor. The capacitor to be charged can be selected by a band switch. Three tantalum capacitors are provided, one of 10 μf capacitance, the second of 47 μf capacitance and the third of 100 μf capacitance. There are three toggle switches marked 1,2and 3. When the second one is put in the position marked **CH** the capacitor selected by band switch gets charged to approximately 2 V. When this toggle switch is put in the position marked **DIS** the capacitance discharges through a high resistance. There is a high resistance of 130 megohm inside the box. By putting the third toggle switch in the position **INT** the capacitor discharges through this resistance in series with the first toggle switch. When the first toggle switch is put in position **O** the capacitor discharges through its internal resistance. When the switch is put in position **S** the capacitor discharges through the 130 Meg in parallel with its internal resistance. One may also connect a high resistance externally to the banana terminals marked **EXT**. If the third toggle switch is put in position marked **EXT** the capacitance discharges through the external resistance in series with the first toggle switch.

The voltage across the capacitor will decrease exponentially with time with a time constant CR where C is the capacitance and R is the high resistance, when the capacitor is discharging. This voltage cannot be measured with an ordinary multimeter because the input impedance of the multimeter will be of the same order or less than the high resistance being measured and will short the high resistance. There is a cathode follower circuit inside. The input impedance of the operational amplifier is higher than the resistance to be measured. By pressing the push switch marked **PS** the voltage across the capacitor will be applied to the input of the cathode follower. The output voltage of the cathode follower can be measured by connecting a DMM in the DC 2 Volts range to the Banana terminals marked **DMM**. The push switch, when pressed, connects the voltage across the condenser to the input of the cathode follower for a short time when the reading on the DMM is taken. For the rest of the time the input of the cathode follower is isolated from the condenser terminals. This is an added precaution to reduce the effect of the input impedance from altering the measured value of the high resistance.

Taking a measuring time of 3000 seconds as the upper limit, one may measure a resistance from 10 Megohm to 50 Megohm using 100 μfd capacitor, a resistance of 50 Megohms to 100 Megohms with the 47 μfd capacitor and a resistance of 100 Megohms to 500 Megohms with the 10 μfd capacitor. Of course one can measure any high resistance with any of the capacitors provided one takes readings for a time equal to or greater than the time constant.

I. 12 INTEGRATOR FOR B-H CURVE AND SEARCH COIL EXPERIMENTS

1. INTRODUCTION

Any time varying pulse of voltage v (t) can be integrated by the integrator. The output will be a DC voltage proportional to the integrated value of v (t). Such an integrator is used in the study of B-H curve of a ferro-magnetic material and in the calibration of a search coil to measure a magnetic field.

The front panel of the integrator circuit is shown in Figure I.12.1.

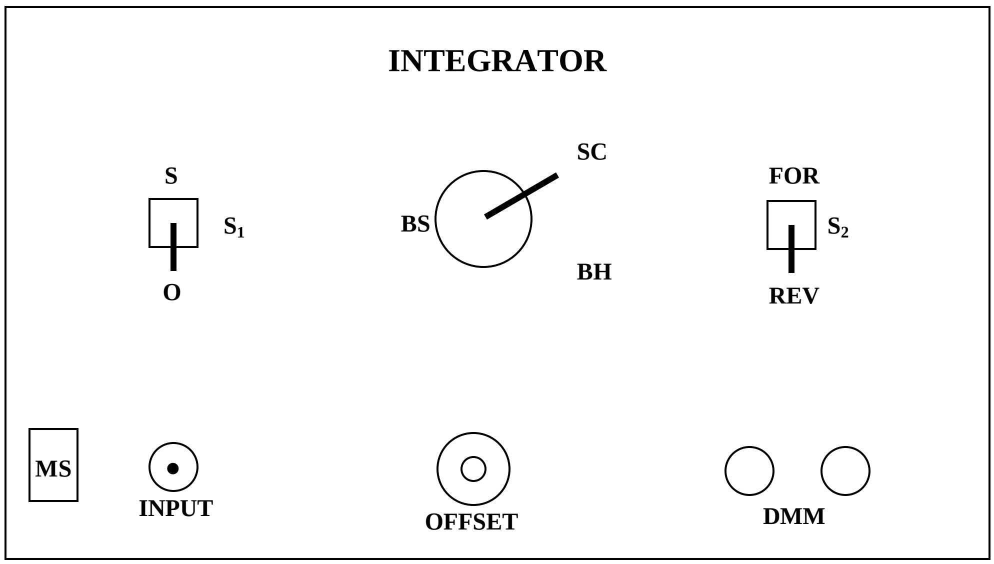


FIGURE I.12.1 Front panel diagram of the Integrator Circuit

The mains switch **MS** is in the lower left corner. The input signal from the search coil or secondary coil in the BH curve experiment is connected to the RCA socket marked **INPUT.** There are two toggle switches. If the switch $\mathbf{S_2}$ at the right is put in the position marked **FOR** the input signal is directly connected to the integrator input. If the switch is put in position marked **REV** the input signal is reversed before it is connected to the integrator input. Since we use a peak detector in the final stage, an output will be obtained only for one polarity of the input signal. If the input signal polarity is opposite, the reversing switch helps to change the polarity of the input to the integrator so that a positive voltage appears at the peak detector. The second toggle switch $\mathbf{S_1}$ is used to short the output of the peak detector when it is put in the position marked **S**. When we need to take a measurement this switch should be put in the position marked **O**. For the B-H curve experiment the band switch **BS** is put in the position **BH**. For calibrating a search coil with a solenoid the band switch is put in the position **SC**. An off-set adjust potentiometer is used to adjust the offset to zero when the toggle switch is in position **S**. The output is measured by a DMM to read DC voltage to two decimal places. The DMM is connected to the banana terminals marked **DMM.**

The pulsed voltage that is to be integrated is connected to the integrator input. The integrator circuit is shown in Figure I.12.2.

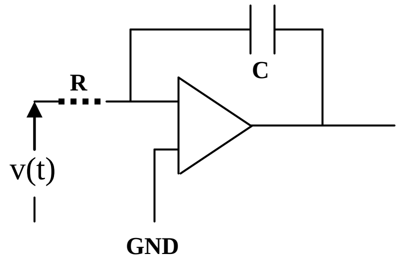


Figure I.12.2 Basic Integrator circuit

The voltage pulse v (t) produces a current

$$I(t) = v(t)/R \tag{I.12.1}$$

This charges the capacitor C. If the voltage across the capacitor is V (t) then

$$C dV/dt = I = v(t)/R \tag{I.12.2}$$

So

$$\int_{ti}^{tf} v(t)\, dt = CR\,(V_f - V_i) \tag{I.12.3}$$

The subscript i refers to the initial value and the subscript f to the final value. The change in voltage of the capacitor is proportional to the integral of the pulsed input voltage.

For the band switch in the **BH** position C = 47 μf and R = 220 Ohms. For the band switch in the **SC** position C = 1 μf and R = 220 Ohms. In the SC position, the signal is initially amplified before it is applied to the integrator.

After the integrator there are two stages of amplification, each amplifying the integrator output by a factor of 2.2. Finally the amplified output is fed to a peak detector shown in Figure I.12.3.

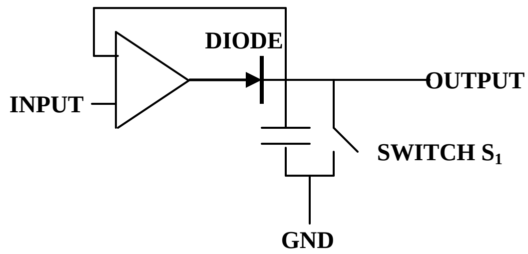


FIGURE I.12.3 Peak detector

The output will be a DC voltage corresponding to the peak value of the input provided the input is of the proper sign. If the input is of the improper sign, the reversing toggle switch $\mathbf{S_2}$ can be put in the other position to make the input of the proper sign to give an output. The condenser will discharge slowly. After taking the reading of the output, the switch $\mathbf{S_1}$ is put in the position **S** (Figure I.12.1) to short the condenser and discharge it completely. When this switch is put in the position **O** (Figure I.12.1) the peak detector is ready to measure.

I.13. FEIGENBAUM CIRCUIT

1. INTRODUCTION:

The Feigenbaum circuit performs the iteration

$$X_{n+1} = rX_n(10-X_n) \qquad (I.13.1)$$

where r is a parameter going from 1 to 5 and X_n is a positive number between 0 and 10.

As r is varied the value of X undergoes a series of bifurcations before it reaches chaos at r =4.

The circuit described here is taken from "Some Experiments in Chaotic Dynamics" by Keith Briggs, American Journal of Physics, 55, 1084-1089, (1987), with a few minor modifications. The principle of the circuit is described by the block diagram shown in Figure I.13.1.

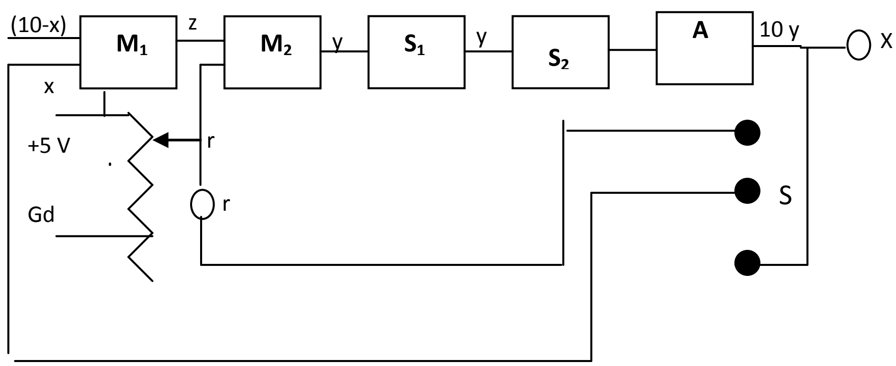


Figure I.13.1: Block diagram of the Feigenbaum circuit.

The parameter r is the voltage at the variable point of a potentiometer to which 5 volts are applied at both ends. If the switch S is pushed up this voltage r appears at the input of the multiplier M_1 and serves as the starting value X_0 of the iteration. A voltage (10-X) is generated within the box where X is the voltage at the input of multiplier one. The output of multiplier one is

$$Z = X(10-X)/10 \qquad (I.13.2)$$

This output is the input to the second multiplier M_2. r is the other input to M_2. The output of multiplier M_2

$$Y = r(X/10)\,(1-(X/10)) \qquad (I.13.3)$$

The sample and hold chip S_1 receives this value of Y, holds it for a short time till it receives the next value of Y from M_2 during the iteration process. When the next value of Y is received it passes the old value of Y to the second sample and hold chip S_2 . This chip retains the value till it receives the next value from S_1 during the iteration. It then passes the old value of Y to the amplifier which amplifies this value ten times. This new value of X = 10Y comes to the switch S. If S is pressed down, the iteration starts. The iteration takes place around 250 times in one second. The value of r can be read on a DMM. The iterated value of X after much iteration can be seen on a storage oscilloscope.

The front panel of the Feigenbaum circuit is shown in Figure I.13.2.

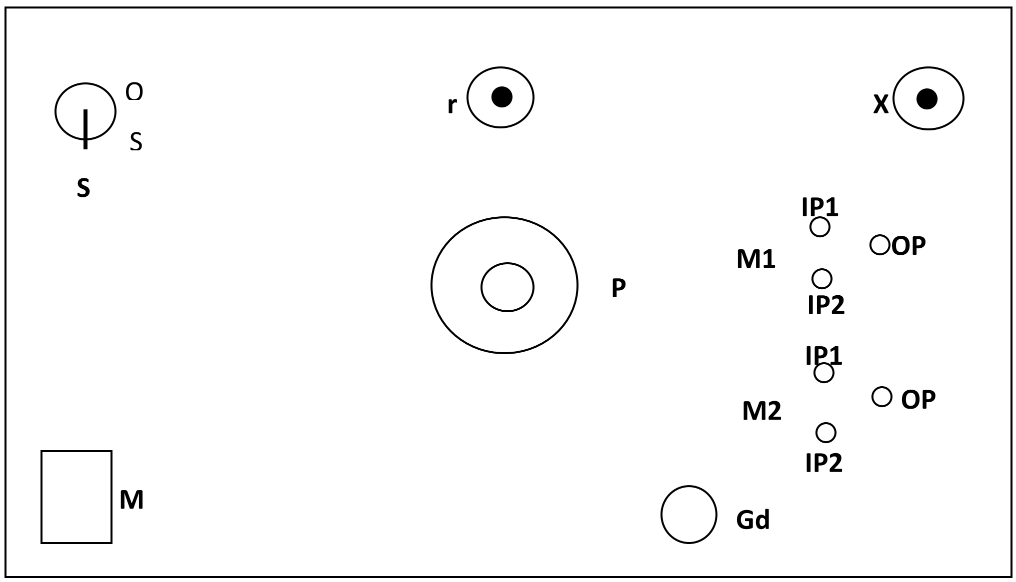


Figure I.13.2: Front Panel of Feigenbaum Circuit

In the above diagram M is the main switch. P is a ten turn potentiometer which sets the value of r. This value of r can be measured by connecting the RCA socket marked **r** to a DMM in the 20 V range. The inputs and output at multiplier M_1 can be measured by connecting the prongs of a DMM to the corresponding banana terminal and the banana terminal marked Ground on the front panel. The inputs and output of multiplier M_2 can be measured similarly. The value of X after much iteration can be measured on a storage oscilloscope connected to the RCA socket marked X.

The storage oscilloscope we use in Bangalore is made by a Korean company GWINSTEK. It has two channels. Channel 1 is connected to the RCA socket X. Channel 1 is put in the DC mode and the scale is set at 2 Volts per division. The horizontal menu button is pressed and the button against roll is pressed. Then the trace on the oscilloscope runs from right to left. To measure the value of X the button marked Cursor is pressed. Two horizontal yellow lines appear on the screen. Press the button against Y_1. Then one of the cursors Y_1 can be moved up or down by rotating a knob. When the cursor is brought to coincide with the trace of X on the screen, the value of X is indicated in a window marked Y_1.

If an oscilloscope of a different make is used one must first learn the function of the different controls on the oscilloscope.

I.14. CHUA's CIRCUIT

1. INTRODUCTION:

The Chua's Circuit is a basic circuit to illustrate period doubling and the different types of attractors one will get in a non-linear AC circuit.

The Chua's circuit is shown in Figure I.14.1.

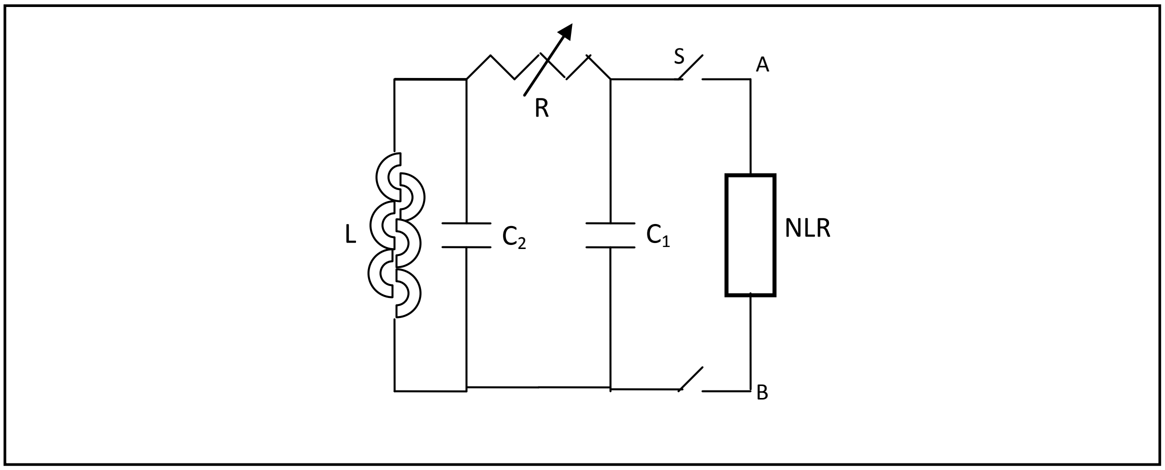


Figure I.14.1: Chua Circuit

This circuit is an oscillatory circuit. It has an inductance L connected to two capacitances C_1 and C_2 in parallel. The two capacitances are connected through a potentiometer R. The heart of the Chua circuit is the non-linear negative resistance device NLR. How such a non-linear negative resistance device can be constructed with an Operational Amplifier will be described in II.10.2. A switch S can isolate the nonlinear negative resistance device from the rest of the circuit. In the open position of the switch one can connect a power supply to the NLR and measure its current voltage characteristic. When the switch is closed the NLR is connected to the oscillatory circuit. Now if R is reduced from a maximum value the circuit first starts oscillating at a certain value of R. A plot of the AC voltage across C_1 against the AC voltage across C_2 on an oscilloscope in the XY mode will show a single loop. As the resistance is further decreased many loops will appear having different shapes which will be described in II.10.2.

The Chua's circuit is described in detail in "Robust Op-amp realization of Chua's circuit" by Michael Peter Kennedy in http://www.physics.smu.edu/scalise/chaoscircuit.pdf. The circuit parameters described in this article are used in the construction of Chua's circuit.

The front panel of the Chua's circuit is shown in Figure I.14.2.

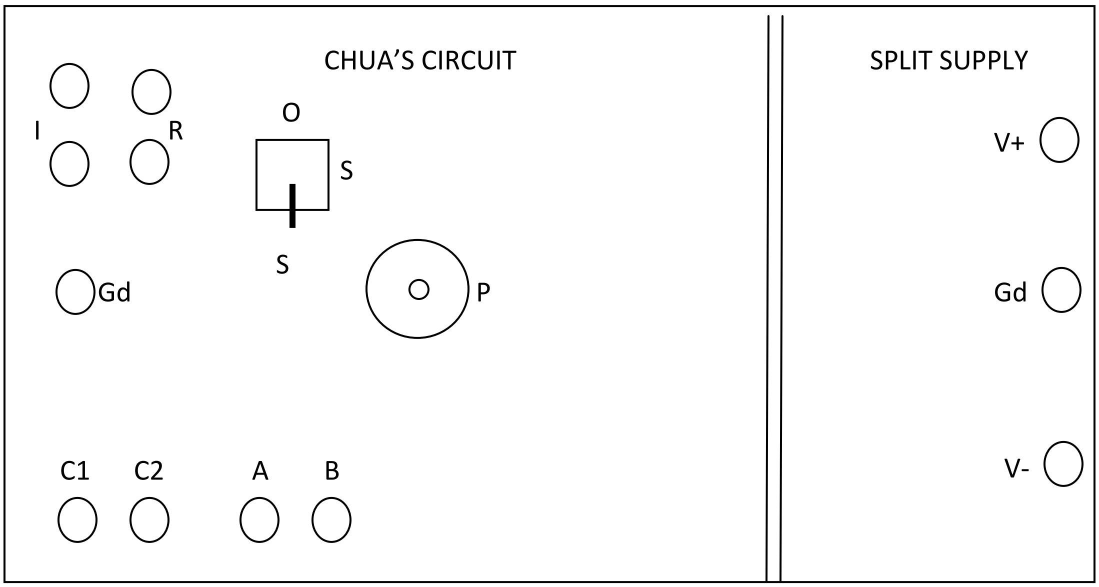


Figure I.14.2: Front Panel of Chua's Circuit

The front panel is divided into two parts. The left part is marked Chua's circuit and has all the terminals and controls for doing this experiment. The right part is a split power supply giving +8,0 and -8 volts. This can be used as a split power supply for other electronic experiments.

The main switch is at the back of the cabinet. The switch S can be put in the O or S position. In position O the NLR is not connected to the rest of the circuit. In this position one may connect a DC power supply to the banana terminals marked A and B through a milli-ammeter and measure the I-V characteristic of the NLR. One can measure at terminals marked I the voltage across a 10 Ohms resistor in series with the NLR. If an ammeter is not available this voltage measures the current I through the NLR. **When measuring IV characteristic the main switch at the back of the cabinet must be on.**

When the switch S is put in position S, the NLR is connected to the rest of the Chua circuit. At that time the power supply should be disconnected from A and B.

The voltage across C1 is measured between the terminal marked C1 and the terminal marked Gd. Similarly the voltage across C2 is measured between the terminal marked C2 and the terminal marked Gd. These two voltages must be connected to two channels of an oscilloscope put in the XY mode.

The terminals marked R serve to measure the resistance of the potentiometer.

The different figures that one gets on an oscilloscope when the potentiometer setting is changed are discussed in II.10.2.

I.15 BOX FOR MEASURING k/e USING A TRANSISTOR

1. INTRODUCTION:

In a transistor the collector current I_{CE} varies with the base-emitter voltage V_{BE} as

$$I_{CE} = I_o \exp(e\, V_{BE}/\, k\, T) \qquad \text{(I.15.1)}$$

Here e is the magnitude of the electronic charge, k the Boltzmann's constant and T the absolute temperature of the transistor. A measurement of I_{CE} as a function of V_{BE} provides a simple and convenient method to measure the ratio k/e of two fundamental constants.

2. BOX FOR DETERMINING k/e

The front panel of the box is shown in Figure I.15.1.

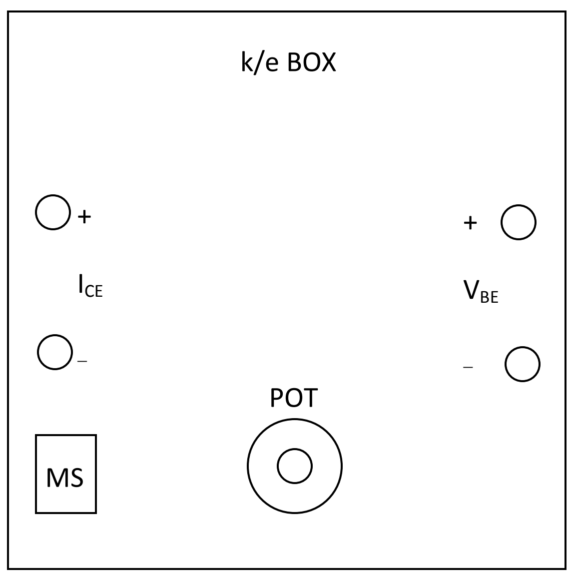


Figure I.15.1: Front Panel of the k/e box

The main switch is at the bottom left of the panel. When the pot is turned the base-emitter voltage V_{BE} changes. V_{BE} is measured at the banana terminals marked V_{BE} by connecting a DMM to read DC 2 Volts. $I_{\cancel{CE}}$–, the collector current, is measured by connecting a DMM to measure DC micro/milli-amps.

SECTION II

EXPERIMENTS

II.0 INTRODUCTION TO ERROR ANALYSIS

1. INTRODUCTION

In any experiment we use different types of instruments to make measurements. These instruments have their limitations and any measurement made with an instrument is subject to errors. During the measurement one tries to keep certain environmental parameters constant. However there could be small variations in the environmental parameters, which may lead to an error in the measurement. Measurements are made on samples, the composition of which may not be exactly defined. Results will then vary from sample to sample. It is necessary to understand the magnitude of such errors to estimate how accurate the results of a given measurement are.

2. PRECISION AND ACCURACY

Often precision and accuracy are confused. Precision deals with reproducibility of a given value measured by an instrument in repeated trials. To take a crude example a colour blind person will identify a certain shade of blue as green. Every time this shade of blue is shown to him he will identify the shade as green. He is precise in his statement. But he is not accurate. To take another example an ammeter, the calibration of which is in error by 10%, will always read a current of 2 amperes as having a value of 2.2 amperes. Here again the ammeter will give the same value for the current however many times the experiment is repeated. The ammeter is precise. But it is not accurate as the true value of the current is 2 amperes and the ammeter reads it as 2.2 amps.

3. TYPES OF ERRORS

One can distinguish between different types of errors:

1. Reading error of an instrument:

   A meter scale is graduated in millimeters. If one wants to measure the length of a rod with this scale, we place one end of the rod to coincide with the zero reading on the meter scale, and then look what the reading is at the other end. We may find the other end of the rod is between 50.0 cm and 50.1 cm. We cannot read the length to better than 1 mm with this scale. We say the reading error of the scale is 1 mm.

   The meter scale is an analogue instrument. One may subjectively imagine the 1 mm interval on the scale to be subdivided into two equal intervals and subjectively estimate that the end of the rod lies between the marked division 50.0 cm on the scale and the imaginary division 50.05 cm. In this case the experimenter can say that the reading error is 0.05 cm. Thus the reading error of an analogue instrument is subjective and varies from person to person. The maximum reading error is the interval between two successive divisions on the scale. But a person can estimate subjectively a reading error less than this.

Suppose one measures a current on a digital multimeter. The reading will fluctuate between 2.44 and 2.45 A, where the last unit on the panel meter will be fluctuating between 4 and 5. If the current is between 2.445 and 2.455 A, the meter will indicate 2.45 A while if the current is between 2.435 and 2.445 A, the meter will indicate 2.44 A. The rounding off error is therefore half of the unit in the last digit. If one wants to measure the current to a higher precision one should use a panel meter, which will read to three decimal places.

2. Calibration error

In an experiment one uses a variety of sensors and diverse instruments for measurement. These sensors and instruments are generally calibrated against standards. However there could be errors in calibration. A meter scale divided into millimeters may not be accurately 1 m in length. Even assuming the graduations are uniform, a length measured with such a scale will be either *systematically* more or less than the actual length. Similarly a thermometer may have a calibration error so that all temperatures measured with that thermometer will always differ from the actual value *in the same sense and to the same percentage*. The detection of systematic error in an instrument is difficult. One will have to measure certain standard quantities with such instruments to detect systematic errors. *Small calibration errors will not change the verification of a physical principle.* Suppose one wants to verify that the heat conducted along a rod is proportional to the temperature gradient along its length. If our thermometer has a calibration error the temperature gradient will be in error. But all values of temperature gradient for different amounts of heat transported will suffer the same percentage of error in the same sense. So a plot of the quantity of heat conducted against the erroneous temperature gradients measured with this thermometer will still lead to a linear relation. Only the value of thermal conductivity will differ by a small percentage from the actual value. Small calibration errors are not very important for student experiments.

3. Random errors:

Random errors arise due to factors beyond our control. Environmental parameters may undergo sudden random changes during the course of the measurement. In electrical measurements the starting or stopping of a motor near the place of the experiment may affect the reading transitorily due to electromagnetic interference. Random errors are always present in any experiment and lead to deviations in the value of the measured quantity from one run to the other. We shall consider an example of random error in the experiment to be described below.

4. ANALYSIS OF DATA – AN ILLUSTRATION

All experimental measurements are subject to different types of error. To illustrate these different types of errors, consider the following simple experiment to measure the acceleration due to gravity. I want to drop a ball from a height, h, and measure the time, t it takes to reach the ground. From this measurement I can calculate g from the formula

$$h = \frac{1}{2} g t^2 \tag{II.0.1}$$

The experimental arrangement is shown in Figure II.0.1.

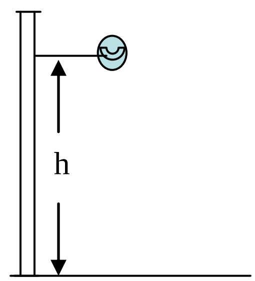


Figure II.0.1. Arrangement to measure g

There is a stand with a horizontal rod and a clip. A ball is fixed to the clip. With a spring latch I can open the jaws of the clip to release the ball. A pointer is attached to the rod to read the height of the rod from the bottom of the stand on a scale fixed to the stand.

I adjust the height of the center of the ball above the ground to be 1000 mm. This implies several things. I must know the diameter of the ball. I must position the ball every time within the clamps at the same position. I must know the distance between the marker and the center of the ball when it is clamped. All these factors I may know correct to 1mm. Then when I say the ball is clamped so that its center is at a height 1000 mm, I mean that the height of the center of the ball from the ground is between 999.5 mm to 1000.5 mm. This uncertainty in the position of the ball is called the reading uncertainty of my scale. In analog measurements one may use one's judgment to say that the marker is between 999.75 to 1000.25 mm. In this case the reading uncertainty is only ± 0.25 mm.

I measure the time on a digital clock. The last digit on the clock is in units of 0.001 s. A time interval between 0.5005 and 0.5015 s will be indicated as .501 s on the clock while a time interval between 0.5005 and 0.4995 s will be indicated as 0.500 s. The reading error here is half the unit in the last digit.

I drop the ball from a height of 1000 mm and record the time it takes for the ball to reach the ground. My sense impressions convey to me when I have released the ball and then I start the clock. When the ball has reached the ground, I hear the thud and stop the clock. These sense impressions are subjective and are subject to random variations. I repeat the observations twenty times and record the time t of fall. I get the following data:

Table II.0.1

Number	Time s	Number	Time s
1	0.450	11	0.453
2	0.453	12	0.458
3	0.457	13	0.450
4	0.450	14	0.452
5	0.452	15	0.454
6	0.450	16	0.451
7	0.452	17	0.449
8	0.454	18	0.447
9	0.453	19	0.452
10	0.451	20	0.453

We see that the time interval measured varies randomly between 0.447 and 0.458 s. This difference is much larger than the reading error of the clock. This variation arises due to random errors. These errors are caused by the delays in sensory impressions. There may also be errors caused by air currents etc.

We plot a histogram in which we count the number of times a given value for t occurs in the above table. This is shown in Figure II.0.2 as the discontinuous curve joining various points.

From these observations we can calculate the mean value $<t>$ of time of fall. This is defined by

$$<t> = \Sigma_i^{N} t_i/N \qquad \text{(II.0.3)}$$

where the sum is over the N observations (in this example N is 20). This mean value of $<t>$ comes out to be 0.4522 s. Then we calculate the sum of squares of the deviations of the measured values from the average value.

$$\text{Sum of deviations squared} = \Sigma_i^{N} (t_i - <t>)^2 \qquad \text{(II.0.4)}$$

This is always positive though the deviations may have either sign.

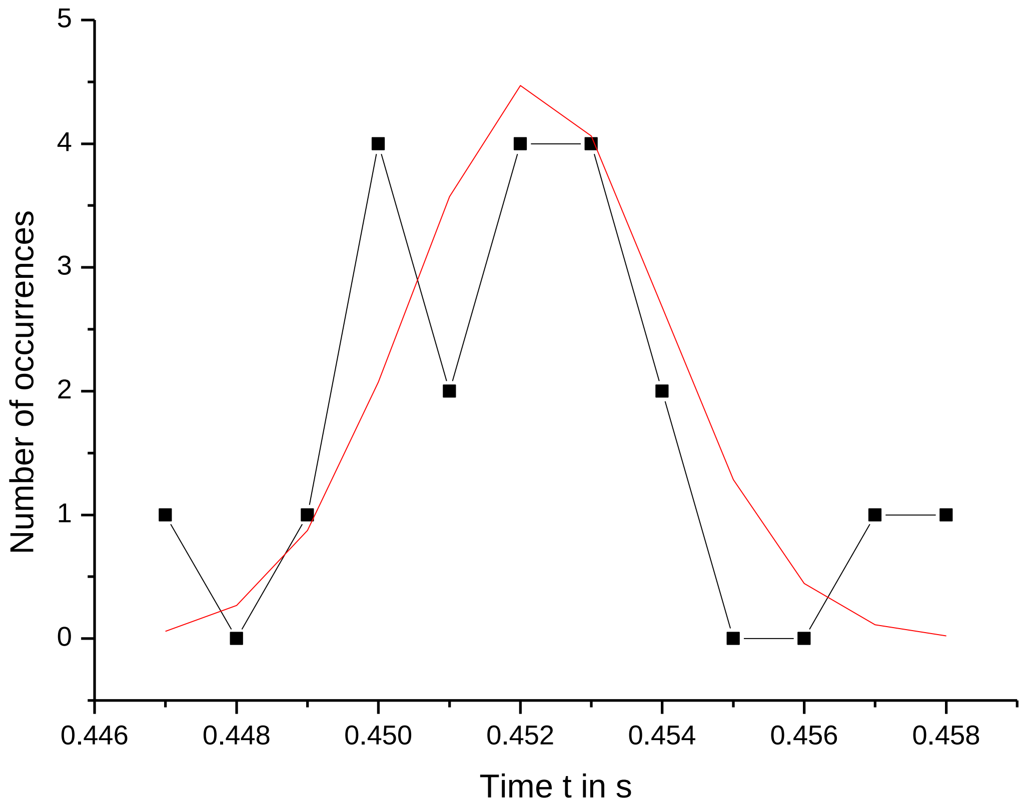


Figure II.0.2: Shows the number of times a given value occurs. This is the black curve. The continuous curve shows a normal distribution

From the sum of errors we can calculate the variance defined as

$$\text{Variance} = \text{Sum of deviations squared}/\ (N-1) \qquad \text{(II.0.5)}$$

This comes out to be $6.34 \times 10^{-8}\ s^2$. The standard deviation σ is defined as

$$\text{Standard deviation } \sigma = (\text{Variance})^{1/2} \qquad \text{(II.0.6)}$$

This comes out to be 0.0025 s.

In Figure II.0.2 the continuous curve follows the equation

$$N = N_0 \exp - (((t - \langle t \rangle)/\sigma)^2) \qquad \text{(II.0.7)}$$

In drawing the above curve we take $<t>$ to be 0.4522 and the standard deviation σ to be 0.0025 and N_0 to be 4.5. This curve is called the Normal distribution or the Gaussian distribution. We see that this curve roughly represents the histogram of the twenty trials. .If we take a very large number of readings then the histogram will approximate more and more closely to the Normal distribution. *From the normal distribution it follows that the probability for the measured value of t to lie between* $<t>-\sigma$ *and* $<t>+\sigma$ *is 70%.*

We then give the result of our measurement of the time of fall as

$$t = <t> \pm \sigma \qquad \text{(II.0.8)}$$

From our measurements

$$t = 0.4522 \pm 0.0025 \text{ s} \qquad \text{(II.0.9)}$$

or

$$t = 0.452 \pm 0.003 \text{ s} \qquad \text{(II.0.10)}$$

since the reading error of the clock is 0.0005 s.

5. PROPAGATION OF ERRORS

Let us now calculate the value of g taking h to be 1 m and t to be 0.452 s from the equation

$$g = 2h/t^2 \qquad \text{(II.0.11)}$$

g comes out to be 9.789 m/s^2. The error in h and the error in t will contribute to an error in the calculated value of g. Let us now calculate the maximum possible error in g due to error in h and error in t. Taking the logarithm of both sides of equation (II.0.11) and differentiating we get

$$\Delta g/g = \Delta h/h - 2\,\Delta t/t \qquad \text{(II.0.12)}$$

Here $\Delta g/g$ is the fractional error in g due to the fractional error $\Delta h/h$ and the fractional error $\Delta t/t$. The fractional errors in h and t can be positive or negative. So the *maximum* possible error in g is obtained from the equation

$$\Delta g/g = |\Delta h/h| + 2\,|\Delta t/t| \qquad \text{(II.0.13)}$$

The reading error Δh is 0.5 mm and h is 1 m. The random error plus the reading error in t is 0.003 s and t =0.452 s. So substituting in (II.1.1) we get

$$\Delta g/g = 0.5/1000 + 2*(0.003/0.452)$$
$$= 0.0005 + 0.0133 = 0.0138$$

The percentage error in g is obtained by multiplying the fractional error by 100. So the percentage error in g is 1.38%. The major contribution to this error comes from the error in the time of fall. We then say

$$g = 9.789 \pm 0.130 \text{ m/s}^2$$

This example shows how errors propagate. Any quantity, which occurs to a power m in an equation, will contribute m times to the fractional error. If m is large then we should take special care to measure that quantity more accurately than the other quantities.

This gives a brief introduction to error analysis. The value of taking many reading to get an accurate value of standard deviation cannot be over-emphasized.

II.1. EXPERIMENTS IN MECHANICS

II.1.1 YOUNG'S MODULUS OF BRASS BY FLEXURAL VIBRATIONS OF A BAR

1. INTRODUCTION:

Consider a bar of length *l*, width b and thickness d clamped at one end. If the thickness is small, the bar can be bent easily in the direction of its thickness (downwards in Figure II.1.1). In the bent bar there is a line called the neutral line, the length of which is not changed by bending Lines above the neutral line are extended by bending, while lines below the neutral line are compressed by bending.

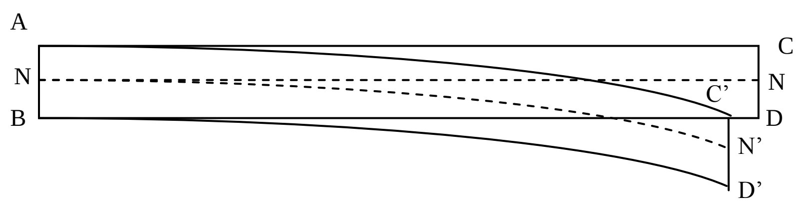


Figure II.1.1: A bar ABCD is clamped at one end AB and is bent downwards at the end CD to take the position C'D'. The dashed line NN before bending becomes the line NN' after bending. But it does not change its length. This is called the neutral line. Any line above NN suffers extension on bending while a line below NN suffers compression.

The neutral plane is the plane passing through NN perpendicular to CD. Take any cross section of the bar perpendicular to its length. The neutral plane will intersect the cross section along a line EF (Figure II.1.2).

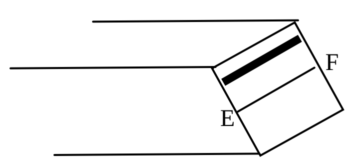


Figure II.1.2: Cross section of the bent bar. EF is the line of intersection of the neutral plane with the cross section.

All elements of the bar above EF will be extended while all elements below EF will be compressed. The tensile stress on an element will increase as the height of the element above EF increases. Similarly as one moves from EF to the bottom of the cross section the compressive stress will increase. Any cross section is subjected to a bending moment which is given by Y Ak^2/R where Ak^2 is the geometric moment of inertia of the cross section, R is the radius of curvature of the neutral line at the point x where the cross-section of the bar is made and Y is the Young's modulus. For a rectangular bar of width b and thickness d, this geometric moment of inertia is $bd^3/12$. For a bar of circular cross section of radius a, it is $\pi a^4/4$.

When this bar is set in flexural vibrations the equation of motion is

$$\rho A\partial^2 y/\partial t^2 = - YAk^2\, \partial^4 y/\partial x^4 \qquad \text{(II.1.1.1)}$$

Here y is the displacement at a point x along the length of the bar, ρ is the density and A is the area of cross section of the bar.

Assuming harmonic vibration with a frequency ω the equation becomes

$$YAk^2\, d^4y/dx^4 - \rho A\omega^2 y = 0 \qquad \text{(II.1.1.2)}$$

For a rectangular bar $Ak^2 = bd^3/12$ and A = bd and the equation becomes

$$d^4y/dx^4 - \alpha^4\, y = 0 \qquad \text{(II.1.1.3)}$$

where

$$\alpha^4 = \quad 12\rho\omega^2/\,(Yd^2) \qquad \text{(II.1.1.4)}$$

The general solution of this equation is

$$y(x) = A\cosh(\alpha x)+B\sinh(\alpha x) +C\cos(\alpha x)+D\sin(\alpha x) \qquad \text{(II.1.1.5)}$$

Note that there are four constants A, B, C and D which have to be determined from the boundary conditions.

The boundary conditions are:

At the clamped end x = 0 (i) y = 0 and (ii) dy/dx =0
At the free end $x = l$ (iii) $d^2y/dx^2 = 0$ and (iv) $d^3y/dx^3 = 0$ (II.1.1.6)

Applying the first two boundary conditions we get

$$A+C = 0$$

and

$$B+D = 0 \qquad \text{(II.1.1.7)}$$

Applying the boundary conditions (iii) and (iv) at the free end $x = l$ we get

$$A(\cosh\alpha l + \cos \alpha l) + B(\sinh \alpha l + \sin \alpha l) = 0 \qquad \text{(II.1.1.8)}$$

And

$$A(\sinh\alpha l - \sin\alpha l) + B(\cosh \alpha l + \cos \alpha l) = 0 \qquad \text{(II.1.1.9)}$$

There will be non-trivial solutions only if

$$(\cosh\alpha l + \cos \alpha l)^2 - (\sinh^2\alpha l - \sin^2\alpha l) = 0 \qquad \text{(II.1.1.10)}$$

Or $$2(1+\cosh \alpha l \cos \alpha l) = 0 \qquad \text{(II.1.1.11)}$$

Or $$-\cos \alpha l = 1/\cosh \alpha l \qquad \text{(II.1.1.12)}$$

The first two solutions of equation (II.1.1.12) are

$$\alpha_1 l = 1.875 \text{ or } \omega_1 = (1.875/l)^2 (Yd^2/12\rho)^{1/2} \qquad \text{(II.1.1.13)}$$

and

$$\alpha_2 l = 4.694 \text{ or } \omega_2 = (4.694/l)^2 (Yd^2/12\rho)^{1/2} \qquad \text{(II.1.1.14)}$$

The lowest natural flexural vibration frequency of the bar is ω_1 and the next higher frequency is ω_2. **For a given bar clamped at one end the two frequencies are in the ratio**

$$\boldsymbol{\omega_2/\omega_1 = (4.694/1.875)^2 = 6.26}$$

The lowest frequency is proportional to the thickness of the bar and **inversely proportional to the square of its length.**

If one determines the frequency ω_1 one can obtain the Young's modulus of the material of the bar.

2. AIM OF THE EXPERIMENT:

To verify that the fundamental frequency of vibration of a steel bar clamped at one end is inversely proportional to the square of its length and to measure theYoung's Modulus of the bar.

3. APPARATUS REQUIRED:

Signal generator, Power amplifier, Young's modulus setup, multimeter measuring frequency in the range 1 to 1000 Hz.

4. EXPERIMENTAL SET UP

A brass bar, one foot in length is clamped between two metal plates so that its breadth is vertical. At the free end of the bar a small magnet is fixed with superglue. The length of the bar from the clamped to the free end can be changed by pushing the bar into the clamping plates. The vibrations of the bar are excited by a coil with a core consisting of steel wires insulated from one another with shellac varnish. If a solid iron core is used it will get hot due to eddy currents. The coil can be moved on a horizontal rail, so that, as the length of the bar is changed, the coil can always be positioned to excite the bar at its free end. The coil is connected to the output of a power amplifier. The input of the power amplifier is connected to a signal generator. Since the power amplifier delivers sufficient power only if the load is between 4 to 8 ohms, a nichrome coil is connected in series with the copper excitation coil to make the total resistance approximately four ohms and the output of the power amplifier is connected to this load. A DMM connected to the output terminals of the signal generator is used to measure the frequency of the excitation current exactly.

5. PROCEDURE:

The bar is clamped so that the length of the bar from the free end to the edge of the clamp is 20 cm. The excitation coil is positioned opposite the free end and the distance between the coil and the bar is adjusted (to be approximately 1 cm). The signal generator is put in the frequency range 10 to 100 Hz. The amplitude and frequency knobs of the signal generator are turned to the extreme left. The signal generator and the power amplifier are switched on. The amplitude knob on the signal generator is slowly turned to the right till the DMM connected to the banana plugs shows a stable reading of the frequency around 13 Hz. The output amplitude as seen on the panel meter of the signal generator should be around 3 V. The frequency knob is slowly turned to the right and the end of the steel bar is observed. At one point it starts vibrating and as the frequency is slowly increased the amplitude reaches a maximum value and then starts decreasing with a further increase in frequency. Note the frequency value f_1 on the DMM when the amplitude reaches the maximum value. This is the fundamental frequency for the length l of the bar. Then turn the frequency adjust knob to a frequency nearly six times the fundamental frequency. Reduce the distance between the excitation coil and the free end of the bar. Otherwise the amplitude of vibration will be too small to detect. Now adjust the frequency till the free end of the bar vibrates with maximum amplitude. If the apparatus is on a wooden table one can hear a loud noise at resonance. This is the overtone frequency f_2.

The screws on the clamp are loosened and the bar is pushed in so that the length is changed to 18 cm. Move the coil on the rail so that the coil comes opposite the free end of the bar. Again measure the fundamental and overtone frequencies by adjusting the frequency knob of the signal generator and measuring the frequency at which the amplitude of vibration becomes a maximum. Repeat the measurements by decreasing the length of the bar by 2 cm each time till you reach a length of 8 cm.

Remove the bar from the clamp and measure its thickness d with a screw gauge. Do not assume the thickness to be the value given in the sample data below. Different bars may have different thicknesses.

A specimen set of readings are shown below in Table II.1.1.1.

Plot a graph of $1/f_1$ against l^2. Such a plot is shown in Figure II.1.1.2. Measure the slope α of the graph. Then the Young's modulus is calculated from the relation

$$Y = (4\pi^2/1.875^4)\ (12\rho/d^2)\ (\alpha)^2 \qquad \text{(II.1.1.15)}$$

The slope α is 0.590 m^2/s and Y is 2.14×10^{11}Pa (1 Pa = 1 Newton/m^2) = 214 Giga-Pascals.

Next calculate the value of f_2/f_1. Find the average value. This is 6.2 which agrees with the theoretical value.

TABLE II.1.1.1

YOUNG'S MODULUS OF BRASS

Density of brass ρ =				8500	kg/m^3		
Thickness of bar d =				9.00E-04	m		
length l cm	f_1 Hz	l^2 m^2	$1/f_1$ in s	f_2 Hz	f_2/f_1		
20	12.6	0.04	0.079365079	81.9	6.5		
18	16.1	0.0324	0.062111801	102.6	6.37		
16	20.2	0.0256	0.04950495	140.4	6.95		
14	26.1	0.0196	0.038314176	178.3	6.83		
12	33.9	0.0144	0.029498525	219.5	6.47		
10	48	0.01	0.020833333	316.1	6.59		
8	74.3	0.0064	0.01345895	515.6	6.94		
				Average	6.66		
Slope of l^2 vs $1/f_1$, α =		0.52	m^2/s				
Young's Modulus Y = $(4\pi^2/1.875^4)$ $(12\rho/d^2)(\alpha)^2$					1.09E+11	Pascals	or 109 Gpa

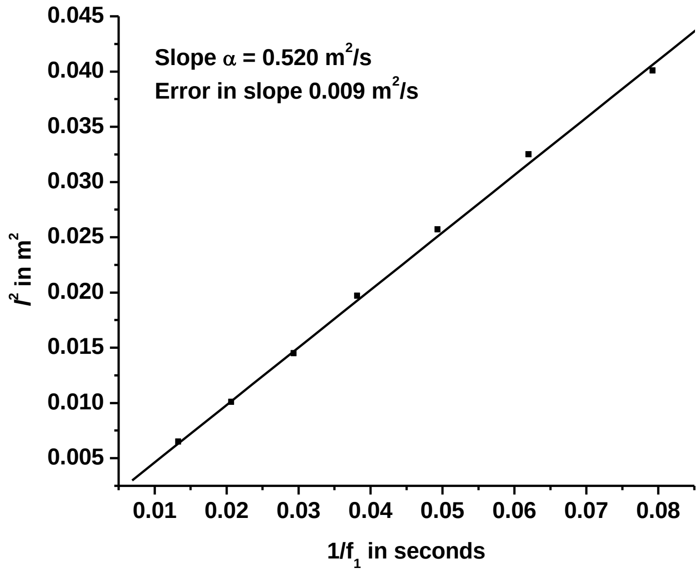


Figure II.1.1.2: Plot of l^2 against 1/f

For brass the Young's modulus varies from 100 to 120 GPa depending on its composition.

Questions:

1. For a sonometer wire the frequency is inversely proportional to the length of the wire while for the flexural vibrations of the bar the frequency is inversely proportional to the square of the length. Why is it so?

2. I want a platform to be mounted on pillars so that sensitive instruments on the platform are not affected by vibrations due to a large motor in the same hall rotating at 500 rpm. Should I choose the dimensions of the pillar such that the natural vibration frequency of the pillar is very different from the rotational frequency of the motor? Justify your answer.

3. A reed is a bar of small thickness, width and length. I have two reeds of the same material, clamped at one end, and I want their fundamental frequencies to differ by an octave. If the reeds have identical thickness and width, what should be the ratio of their lengths?

II.1.2 RIGIDITY MODULUS OF BRASS

1. INTRODUCTION

A poly crystalline material with random grain orientation is isotropic in behavior. Such a material has three elastic moduli. These are

1. Young's modulus represented by the symbol Y
2. Rigidity Modulus represented by the symbol n
3. Bulk modulus represented by the symbol. K

Young's modulus is defined as the longitudinal stress σ_L divided by longitudinal strain ε_L. Longitudinal stress is the force along its length per unit area of cross section of the material and longitudinal strain is the ratio of the increase in length δl to the original length of the material. In the previous experiment we described the measurement of Young's modulus by flexural vibrations of a bar.

When a rectangular parallelepiped of a material is sheared it becomes a piped with a parallelogram as the two sides. This is shown in Figure 1.

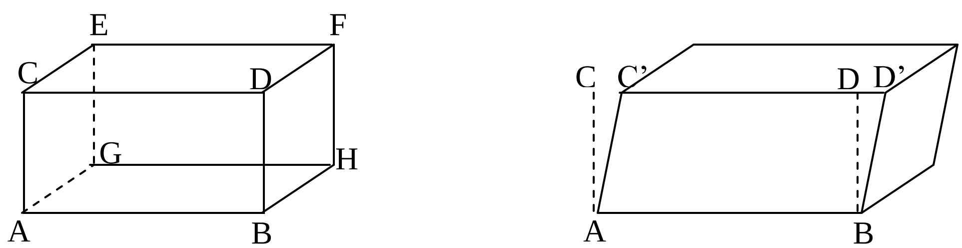


Figure I.2.1: Rectangle ABCD is sheared to become a parallelogram ABC'D' by applying a shearing force parallel to CD.

Shear strain is defined as the displacement CC' in the X direction divided by AC in the Y direction. This is tangent of the angle CAC'. For small strains, this is the angle CAC' in radians since tan $\theta = \theta$ for small shear strains. This shear is brought about by applying a force in the X direction on the face CDFE and an opposite force of equal magnitude to the face ABHG. The ratio of this tangential force to the area of the face CDFE is called the shear stress. For small strains the ratio of shear stress to shear strain is the rigidity modulus.

When the rectangular parallelepiped is subjected to a uniform hydrostatic pressure p, all the sides shrink slightly in length. The volume of the material changes by $-\Delta V$, the minus sign indicating that under hydrostatic pressure the volume is reduced. The volume strain is $-\Delta V/V$ and the bulk modulus is defined as $K = -p/\,(\Delta V/V)$.

In addition there is a fourth elastic property called Poisson's ratio. When a material is elongated along the X axis it suffers a contraction in a perpendicular direction. The Poisson's ratio is $\sigma = -(\Delta y/y) / (\Delta x/x)$.

For an isotropic material these four constants Y, n, K and σ are not independent. If we know any two of them the other two can be calculated. For example if we know Y and n, K and σ can be calculated from the following relations.

$$Y = 2n(1+\sigma) \tag{II.1.2.1}$$

And

$$Y = 3K(1-2\sigma) \tag{II.1.2.2}$$

2. COUPLE PER UNIT TWIST OF THE WIRE

The rigidity modulus can be determined by a dynamic method by measuring the frequency of oscillations of a pendulum of the wire when torsional oscillation is excited. The torsional pendulum is a long wire of the material, the rigidity modulus of which is to be measured, with a disc at one end. The pendulum is clamped at the other end. If the disc is twisted and let go the pendulum executes torsional oscillations.

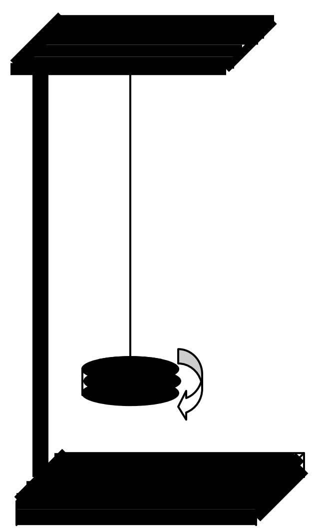
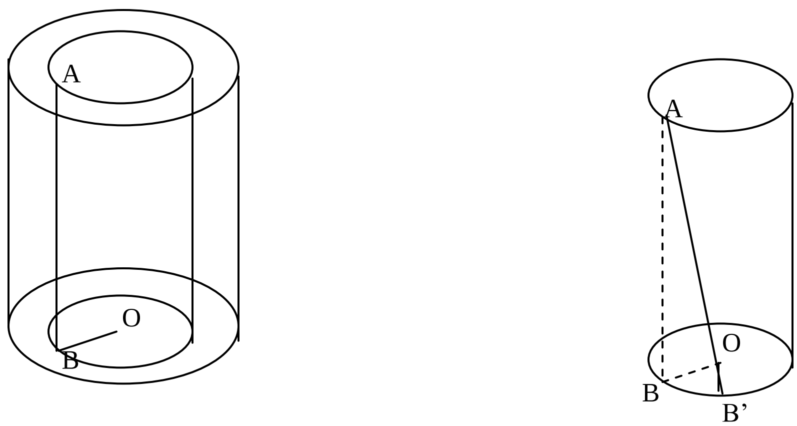


Figure II.1.2.2: Tensional oscillation of the wire. The circular disc is set in oscillation by twisting it and letting it go.

Consider a solid cylinder of radius R and length l. This is the outer cylinder shown in Figure II.1.2.3 (a). Consider an inner concentric section of radius r and same length. This is shown as the inner cylinder in the same figure. Take a line AB of length l on this cylinder. OB is its radius r ($r<R$). When one end is twisted through an angle θ, this line AB moves to AB' and AB' makes an angle BAB' with the original line. Let this angle be ϕ. This is shown in Figure II.1.2.3 (b).



(a) (b)

Figure II.1.2.3: A cylinder of length l and radius r is clamped at the top. (a) AB is a line on the surface parallel to the length of the cylinder. O is the center of the circular base and OB is the radius r.

(b) The top face of the cylinder is held fixed and the bottom face is twisted through an angle θ. The original point B goes to B'. Angle BOB' is θ. Angle B'AB is ϕ.

From the figure it is evident that the displacement BB' is equal to $r\theta$, where θ is the angle of twist BOB'. BB' is also equal to $l\phi$. So

$$r\theta = l\phi \qquad \text{(II.1.2.3)}$$

The angle $\phi = r\theta/l$ is the shear strain. The shear force applied to the bottom face is perpendicular to the radius. If we take a small element of area $(rd\xi)\, dr$ between r and r+dr, and polar angles ξ and $\xi+d\xi$ the shearing force on that area is

$$dF = n\,\{(rd\xi)\, dr\}\, \phi = n\,(r^2/l)\theta dr\, d\xi. \qquad \text{(II.1.2.4)}$$

This force is tangential to the circle. The torque due to this force about the axis of the cylinder is

$$d\tau = dF\ r = n\ (r^3/l)\ \theta dr\ d\xi \qquad (II.1.2.5)$$

To get the total torque on the cylinder of radius R, we integrate over $d\xi$ from 0 to 2π and over dr from 0 to R. This gives

$$\tau = (\pi n R^4/2l)\ \theta \qquad (II.1.2.6)$$

τ/θ is the couple per unit twists of the wire. .

3. EXPERIMENTAL SET-UP

A brass wire 1.5 mm diameter and 50 cm in length is clamped at its two ends in a frame. There are two moveable clamps which can be moved symmetrically on either side of the center of the wire so that the length of the wire between the two clamps can be varied. The brass wire at its center carries a cylindrical disc of brass which is 0.5cm in thickness and 3.5 cm in diameter. It is brazed to the brass wire. Two magnets are attached to this disc on two diametrically opposite prongs. Two coils are wound around an iron core and placed in front of the magnets in such a way that when one coil pulls one magnet, the other coil pulls the other magnet in opposite direction. When the coils are excited by an AC current the wire will be subject to a torque oscillating at the same frequency as the frequency of the AC current. This excites forced torsional oscillations of the wire. The arrangement is shown in Figure II.1.2.4.

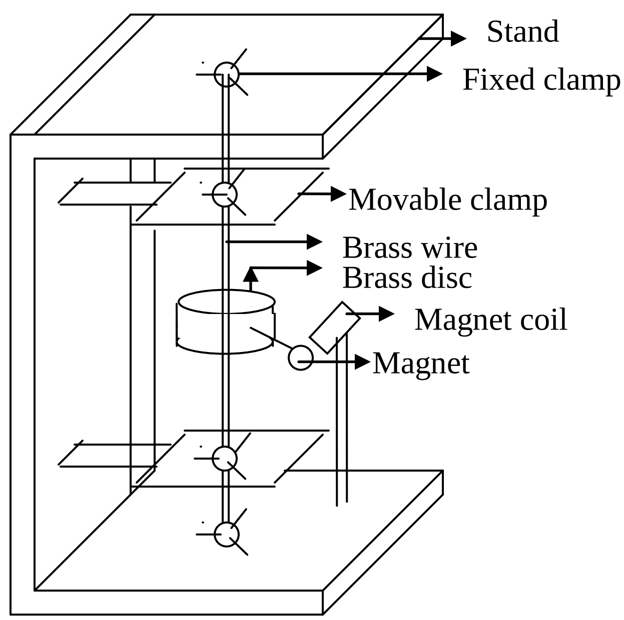


Figure II.1.2.4: Experimental arrangement for Rigidity Modulus

4. PROCEDURE:

Fix the movable clamps on either side of the center of the wire so that they are at a distance of 20 cm from the center of the brass wire. Connect a signal generator to the power amplifier. The output of the power amplifier is connected to the wires from the coils on the experimental set up. When the signal generator is turned on and the signal amplitude is kept around two volts, the power amplifier sends a current through the two magnet coils in series. The coils will exert a force on the magnets fixed to prongs attached to the central brass disc. This will exert a torque on the brass wire and will make it execute forced torsional oscillations at the frequency of the signal generator output. The central disc has a moment of inertia $mR'^2/2$ about the axis of the wire. Here m is the mass of the disc and R' is its radius (3.5 cm). The mass of the disc is $\pi R'^2 t \rho$, where t is the thickness of the disc (0.5 cm), and ρ is the density of brass (8500 kg/m^3). Each of the two magnets has a mass m' and the distance between them is d'. The total moment of inertia I about the axis of the wire is

$$I = mR'^2/2 + 2m'(d'^2/4) \qquad \text{(II.1.2.7)}$$

The torque per unit twist of the brass wire is $(\pi n R^4/2l)$ x2 where n is the rigidity modulus of the wire, R its radius and l is the length of the movable clamp from the center of the wire. The factor 2 arises because each half of the wire from the center to a fixed clamp produces this torque per unit twist. The equation of motion of the wire is

$$I\, d^2\theta/dt^2 + (\pi n R^4/l)\,\theta = T \exp(\boldsymbol{i}\omega t) \qquad \text{(II.1.2.8)}$$

Here θ is the twist of the wire at its center, T is the torque exerted by the coils on the wire and $\omega = 2\pi f$, where f is the frequency of the AC excitation. When the frequency of the excitation coincides with the natural frequency f_{nat} one gets the maximum amplitude of oscillation. The natural frequency is the solution to the equation

.
$$I\, d^2\theta/dt^2 + (\pi n R^4/l)\,\theta = 0 \qquad \text{(II.1.2.9)}$$

Writing $\theta = \theta_0 \exp(\boldsymbol{i}\omega_{nat}\ t)$, we get

$$f_{nat} = (1/2\pi)\,(\pi n R^4/\,l\, I)^{1/2} \qquad \text{(II.1.2.10.)}$$

This natural frequency will vary as $(1/l)^{1/2}$ as the distance between the center of the wire and the movable clamps is changed.

We vary the frequency of the signal generator. Resonance is indicated by the maximum amplitude of vibration of the disc. At resonance the frequency of the signal generator is f_{nat}. The length is changed from 20 cm to 8 cm in steps of 2 cm and the natural frequency is measured by observing resonance. **Measure the diameter of the wire with a screw gauge.**

A sample set of data is shown in Table II.1.2.1.

Table II.1.2.1

Density of brass $\rho =$				8500	kg/m^3	
Diameter of brass wire				1.50E-03	m	
Moment of inertia of disc				6.258E-06	kgm^2	
Mass of each magnet				2.00E-03	kgm^2	
Distance between magnets				7.00E-02	m	
MI of magnets about axis				4.90E-06	kgm^2	
Total MI I=				1.12E-05	kgm^2	
l in cm	f in Hz	1/*l in* m^{-1}	f^2 in 1/s^2			
20	18.8	5	353.44			
17.5	20.4	5.714286	416.16			
15	22.5	6.666667				
12.5	24.6	8	605.16			
10	27	10	729			
7.5	32.6	13.33333	1062.76			
5	41.7	20	1738.89			
slope α	91.4	m/s^2				
Rigidity modulus n = $(4\pi I/R^4)\ \alpha$			4.05E+10	Pascals	40.5	GPA
Poisson's ratio σ = (Y-2n)/2n			3.42E-01			
Bulk Modulus			Y/(3*(1-2σ))	1.15E+11	Pascals	115 Gpa

A plot of f^2vs $1/l$ is shown in Figure I.2.1.4.

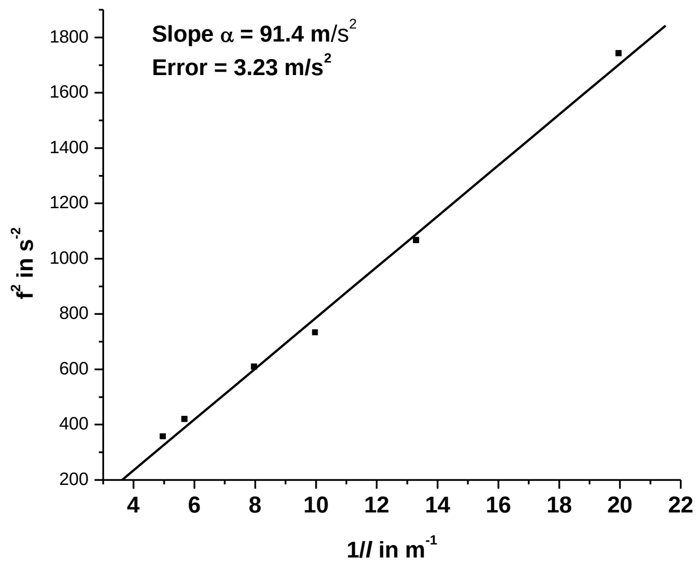


Figure II.1.2. 4 f^2 vs. $1/l$

From the Young'smodulus found in the previous experiment and the rigidity modulus found in this experiment the Poisson's ratio and the bulk modulus are calculated and given in Table II.1.2.1.

The elastic moduli of brass taken from Internet have the following values:

Young's modulus Y	100 t0 125 GPa
Rigidity modulus n	37 to 45 GPa
Bulk modulus K	120 to 140 GPa

NOTE: Brass comes in different compositions of Zinc and Copper. It is possible that the composition of the brass bar used in Young's modulus experiment is different from the composition of the wire in the rigidity modulus experiment. If the Poisson's ratio is calculated from these values, it may come out greater than 0.5. Poisson's ratio for a give material cannot be greater than 0.5. If this happens it is likelythat the compositions of the bar and the wire are different.

I

QUESTIONS:

1. The natural frequency of a torsional pendulum of length l and moment of inertia I is 20 Hz. I decrease the length to l/2 and increase the moment of inertia to 2I. What is the new frequency of the torsional pendulum?

2. The rigidity modulus of material A is twice the rigidity modulus of material B. I take two wires of materials A and B of equal length. If the torque for twisting wire A through an angle θ is τ, what is the torque required to twist the material B through the same angle.

3. I want to measure the amplitude of the angular twist of the torsional pendulum. Can you suggest a simple method of doing this?

4 I give an impulse τdt to the torsional pendulum. dt is small compared to the period T of the pendulum. Calculate the throw θ of the pendulum by equating the kinetic energy the pendulum acquires due to the torque to its potential energy at the end of the throw. Does this relate to the theory of the ballistic galvanometer?

II.2 EXPERIMENTS IN HEAT

II.2.1 CALIBRATION OF A Si DIODE AND A COPPER-CONSTANTAN THERMO-COUPLE AS TEMPERATURE SENSORS

1. INTRODUCTION:

For temperature measurement in the laboratory one uses a thermometer. Thermometry is based on the variation of some property of the thermometer with temperature. The Platinum resistance thermometer is based on the variation of the electrical resistance of high purity Platinum wire with temperature. The Platinum resistance thermometer is convenient because it has an appreciable temperature coefficient of resistance and the variation of resistance with temperature is very nearly linear over a wide range of temperature. For example in the temperature controller described in Section I a platinum resistor with a nominal resistance of 100 Ohms at 0^0C is used. The resistance of this Pt100 thermometer increases by 0.4 Ohms for every degree rise in temperature.

Two other convenient thermometers are thermocouple and diode thermometer. In a thermocouple we have two junctions between two dissimilar materials like copper and constantan. When the junctions are maintained at two different temperatures T_1 and T_2, an emf appears across the thermocouple leads. This emf is not a linear function of temperature difference over a wide range of temperature. The value of the thermo-emf vs. T_1, when T_2 is held constant, is a curve which can be fitted to a polynomial. The slope of this curve at any temperature gives **thermoelectric power** of the thermocouple at that temperature. For example near room temperature a copper-constantan thermocouple has a thermopower of $40\mu V/^0C$. Because the thermo-emf is small, one has to amplify the output with a DC differential amplifier like the one described in Section I. The advantage of the thermocouple is its small thermal capacity. A thermocouple will track rapidly changing temperatures because its thermal capacity is small.

Another thermometer is a diode. One can send a small current through a diode (about 1 to 2 mA) in the forward direction and measure the voltage drop across the diode. If the current through the diode is kept constant, this forward voltage decreases as the temperature increases nearly linearly over a limited temperature region. This decrease is measurable with a digital voltmeter. The diode thermometer can be used for controlling the temperature. However the forward voltage changes with the age of the diode. For precision thermometry diode is seldom used.

In this experiment we study the temperature variation of the thermo-emf of a copper-constantan thermocouple and the temperature variation of the forward voltage of a silicon diode at a constant current.

2. APPARATUS REQUIRED

Regulated power supply, temperature controller, furnace, insert containing diode and Copper-constantan thermocouple, Constant current source, DC Differential amplifier, and two multimeters reading from DC 200 mV range up.

2. EXPERIMENTAL PROCEDURE

Connect the stabilized power supply to the furnace as described in Section I Chapter 2. The furnace has an insert. This is a length of aluminium rod of square cross section of 1cmx1cm. An axial hole of 3 mm diameter is bored in the aluminum block. On one end a Pt 100 sensor is inserted into the block. One junction of a copper-constantan thermocouple is inserted in the hole from the other end. The other junction of the copper-constantan thermocouple is put inside a copper tube and this is fixed to the terminal block from which the insert hangs into the furnace. On one of the lateral faces of the aluminium block, the Si diode is fixed with the araldite. Care is taken to see that there is no shorting of any of these elements with the block. The aluminum block is suspended inside the furnace from the terminal disc on a stand fixed to the furnace.

The Pt 100 terminals are connected to the appropriate terminals on the terminal block at the back of the temperature controller as indicated on the information sheet pasted on top of the temperature controller.

On the terminal disc of the insert to the furnace we have a RCA socket (marked TC) which is connected to the thermocouple leads, a pair of banana terminals (marked Pt 100) connected to the Pt100 sensor and another pair of banana terminals (marked Si diode) connected to a Silicon diode.

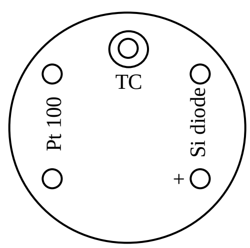


Figure II.2.1.1 Terminal Block of the insert

The output terminals of a constant current source are connected to the Si diode, the positive to the terminal marked + on the terminal block. The current will flow in the forward direction. The two switches on the current source must be in the Low current mode. The constant current is switched on and the LC pot is set to give a current of 1 or 2 ma through the diode. (If this current is not achieved, it means the current is not flowing in the forward direction and the marking of +on the terminal block is wrong. Interchange the connections to the diode.) A DMM connected in the DC 2 V range to the Si diode terminals measures the forward voltage across the Si diode.

The RCA socket on the terminal block is connected to the terminals I_1 of the DC differential amplifier. The DC differential amplifier measures the amplified voltage V_+ and V_- of the thermocouple in the two positions of the reversing switch. If the offset is adjusted to a few millivolts, the two readings V_+ and V_- will have opposite signs. If not adjust the offset adjusting potentiometer till V_+ and V_- have opposite signs with magnitudes differing by a few millivolts. Switch on the Differential amplifier. Connect a DMM in the DC 200 mV range to the output terminals of the Differential amplifier.

Switch on the stabilized power supply and adjust the voltage across the heater to be about 7 V.

Put a wad of cotton to close lightly the top of the furnace tube. Otherwise there will be convection currents of air in the furnace and the furnace will not reach the required temperature even when the applied voltage reaches 20 V.

The temperature of the furnace will increase slowly at the rate of about 1 to 1.5 degrees per minute. As the temperature rises one will have to increase the applied voltage slowly to maintain this rate of heating. This applied voltage will not exceed about 12 V to reach a temperature of 120 ^{0}C.

Measure the voltage across the Si diode at intervals of 10^{0}C. Similarly measure at intervals of 10^0 C the output of the differential amplifier in the two positions of the reversal switch to record the amplified thermo-emf values V_+ and V_- of the Copper-constantan thermocouple. Go up to 130 ^{0}C. Since the contacts to the Si diode and the thermocouple are soft soldered, the solder will melt around 180^0 C.

At the end of the experiment do not forget to switch off the power supply and all the other equipment.

A sample set of readings for Si diode is given in Table II.2.1.1. The current through the diode is kept at 1 mA.

Table II.2.1.1

Forward voltage as a function of temperature
for a Si diode
Forward current 1 mA

			Sum of	
Temp C	V_{for} Volts	V cal	Error^2	
37.5	0.578	0.578	7.56E-08	
50.2	0.55	0.551	2.26E-06	
62.2	0.527	0.526	2.97E-06	
77.3	0.495	0.494	3.46E-06	
89.9	0.467	0.468	3.97E-06	
105	0.435	0.436	4.69E-06	
117.5	0.41	0.409	4.97E-06	
135.2	0.372	0.372	4.98E-06	
149.8	0.34	0.341	6.73E-06	
		Variance	8.41E-07	
		Std Dev	9.17E-04	V

In the above table the first column gives the temperature in Celsius as recorded on the temperature indicator. The second column gives the forward voltage across the silicon diode as indicated on the DMM.

In Figure II.2.1.2 a plot of the forward voltage, V_{for}, as a function of temperature t in ^{0}C is shown. The points can be fitted well to the equation

$$V = 0.6574-0.00211*t \qquad \text{(II.2.1.1)}$$

The quality of the fit is shown by calculating V using this equation at the temperatures of measurement. The calculated values are given in the third column of Table II.2.1.1. The fourth column gives the sum of $(V_{meas} – V_{cal})^2$ upto the row in which this value is entered. The value in the fourth column at the last temperature measured gives the sum of error squared for all measured values. The variance is obtained by dividing this value by (N-1) where N is the number of measurements taken. The standard deviation is the square root of the variance. The standard deviation of the measured value is within ±1 mV. Since the diode voltage changes by 2.11mV for every degree temperature change, this standard deviation will correspond to an error in temperature of 1/ (2.11) ~ 0.5^{0}C. With this calibration graph, one can measure temperature with the Si diode with an accuracy of ±0.5^{0}C.

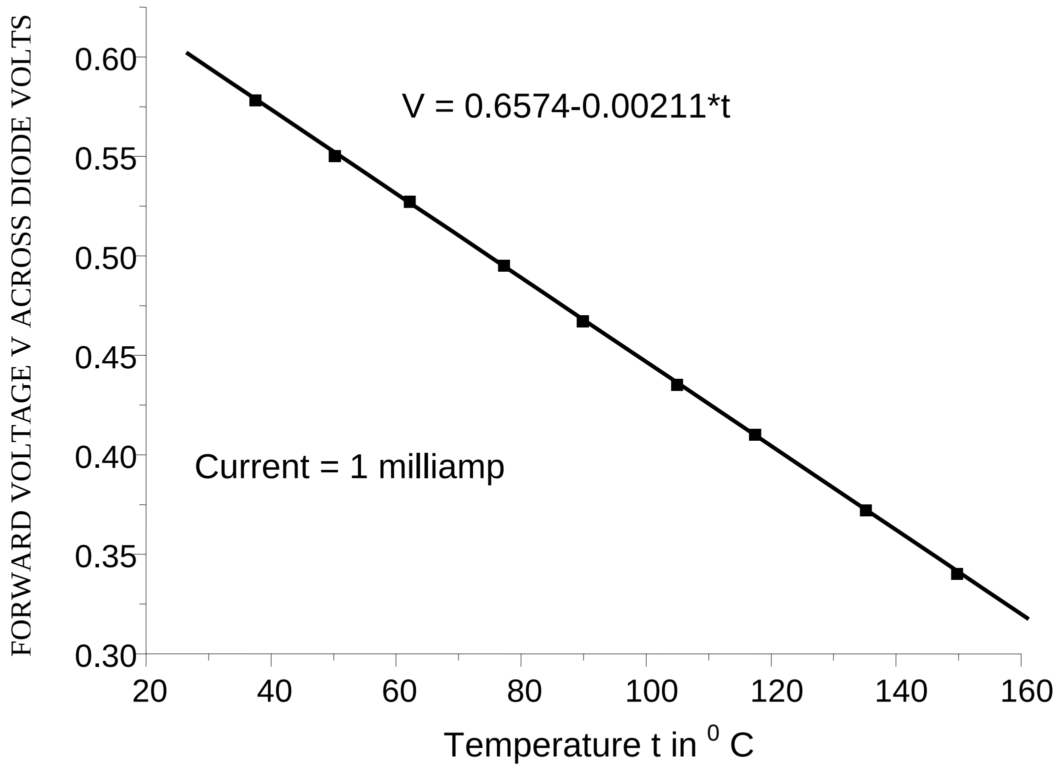


Figure II.2.1.2 Forward voltage across a Si diode as a function of temperature. Current is 1 milliampere.

NOTE: The readings shown above are sample readings. The forward voltage will vary with the current and the diode. However for all diodes the slope will be between 1.8 mV/^{0}C to 2.2 mV/^{0}C.

The thermo-emf of a copper constantan thermocouple was measured on a DC Differential amplifier with the same set up. The results are given in Table II.2.1.2

Table II.2.1.2
Calibration of a copper-constantan thermocouple
Reference Junction at room temperature

	THC	THCrev	THCcorr	TH-emf	TH-emf cal	Sum of	mV/K	
Temp C	mV	mV	mV	mV	mV	Error^2	Thermopower	
37.5	44	-35	39.5	0.395	0.400	2.71E-05	0.040	
50.2	92	-89	90.5	0.905	0.920	2.44E-04	0.041	
62.2	141	-150	145.5	1.455	1.421	1.40E-03	0.042	
77.3	215	-195	205	2.05	2.066	1.66E-03	0.043	
89.9	253	-273	263	2.63	2.617	1.84E-03	0.044	
105	325	-332	328.5	3.285	3.291	1.87E-03	0.045	
117.5	385	-385	385	3.85	3.861	2.00E-03	0.046	
135.2	441	-495	468	4.68	4.688	2.06E-03	0.047	
149.8	540	-539	539.5	5.395	5.386	2.15E-03	0.048	
					Variance	2.68E-04		
					Std Dev	1.64E-02	mV	

First column gives the temperature in C indicated by the temperature indicator. The next column gives the amplified thermoemf V_+ measured in mV on the DMM connected to the Differential amplifier. The third column gives the value of V_- when the reversing switch is toggled. The fourth column gives $(V_+ - V_-)/2$, which is corrected for offset as explained in Section I. The fifth column gives the thermo-emf of the thermocouple which is $((V_+ - V_-)/2)/100$, where 100 is the amplification factor of the differential amplifier. Figure II.2.1.3 shows a plot of thermo-emf against temperature.

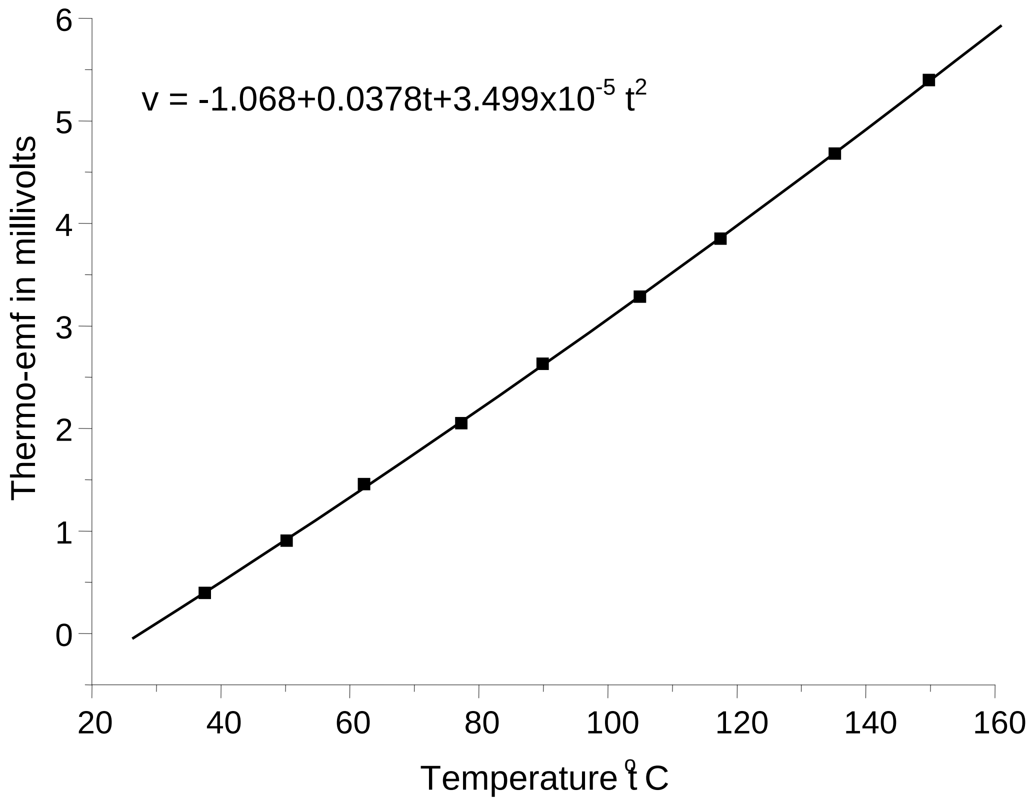


Figure II.2.1.3 Variation of thermoelectric power of Cu-Constantan Thermocouple

This variation is fitted with a second-degree polynomial, which gives

$$V \text{ (in mv)} = -1.0680+0.0378*t+3.499\times10^{-5}\, t^2 \qquad \text{(II.2.1.2)}$$

where t is the temperature in ^{0}C. The calculated value of the emf using this formula is given in column 6. The sum of the square of the error in the fit is calculated in column 7. The variance and standard deviation are given in the Table II.2.1.3. The standard deviation is about 16 microvolt, which means that the equation will give the temperature within $\pm 0.4^0$. The calculated thermo-power, dV/dt, from equation (II.2.1.2) is given in the last column of the table.

The aim of the experiment was to show how different sensors could be used to measure temperatures.

QUESTIONS:

In this experiment you have used three different temperature sensors: Platinum resistance thermometer with the temperature controller, Si diode and the thermocouple.

1. The response of the Platinum thermometer is linear with temperatur**e.** Of the other two sensors, which one shows a linear response?

2. Can you use a Si diode thermometer to control the temperature?

3. Which of the two sensors, the Si diode and the thermocouple, will have a lower mass? Which thermometer will respond fast when the temperature changes rapidly?

4. Can you think of any other property that can be used to measure temperature?

5. Can you suggest a way of controlling the temperature of a bath using Si diode thermometer?

II.2.2. STEFAN'S CONSTANT OF RADIATION

1. INTRODUCTION:

A hot body emits radiation that will be absorbed by another body. The amount of energy radiated depends on its surface area, the absolute temperature of the body and a property depending on the nature of the surface of the body. This property is called emissivity and is denoted by ε. This is the ratio of the energy radiated by unit area of the given surface to that radiated by unit area of a perfectly black surface when both the surfaces are at the same temperature. The energy radiated per second, Q, is given by

$$Q = \sigma \, \varepsilon \, A \, T^4 \qquad \text{(II.2.2.1)}$$

Here A is the surface area of the body. σ is called the Stefan-Boltzmann constant and is one of the important constants of physics.

$$\sigma = (\pi^2/60)\, k^4/\, \hbar^3 c^2 \qquad \text{(II.2.2.2)}$$

k, $\hbar$ (= $h/2\pi$) and c are the Boltzman constant, the Planck's constant divided by 2 π and the velocity of light in vacuum respectively.

A body can lose heat to its surroundings by conduction, convection and radiation. To measure the Stefan's constant we must reduce the fraction of heat lost by conduction and convection to a few percent of the total heat lost, so that radiation dominates. This is achieved by the following steps: (a) blackening the surface of the body and that of the enclosure to make ε have the maximum value of nearly unity, (b) increasing the resistance to heat conduction by using very thin wires for the electrical measurements and by using a nylon thread of poor conductivity to suspend the sample and (c) using a closed enclosure to reduce convection losses due to currents of air.

2. EXPERIMENTAL ARRANGEMENT:

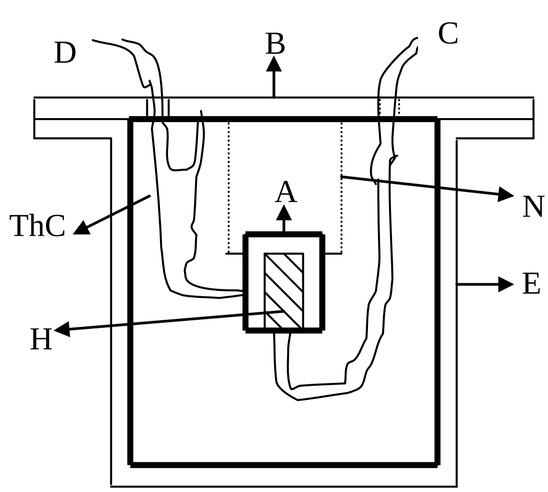


Figure II.2.2.1: A schematic of the experimental arrangement

Figure II.2.2.1 shows a schematic of the set up for measuring Stefan's constant. A is a light weight aluminum cylinder about 3 cm long and 2 cm in outer diameter with wall thickness of 0.5 mm. To heat the sample we require a heater which can generate about 1 W power. A simple way of making the heater is the following. A Teflon tube of outer diameter 10 mm and wall thickness 0.5 mm is cut to a length of 2.6 cm. A series of five 1mm diameter holes are made around the circumference of the tube at the top and bottom. Five 100-Ohm ¼ W resistors are taken and the leads of each resistor are inserted into the top and bottom holes in the Teflon tube so that the five resistors are connected in parallel and lying flat on their length on the surface of the Teflon tube. All the leads will be inside the Teflon tube. All incoming leads are twisted together and all outgoing leads are twisted together. The total resistance of the heater is 20 (ie. 100/5) ohms and it can generate up to 1 W of heat. A pair of copper leads is soldered to the pair of twisted leads. These are insulated by winding a thin Teflon tape around them and are brought out from the top of the aluminum cylinder. Two or three layers of thin Teflon tape are wound on the outer surface of the resistors. The Teflon tube is placed within the aluminum cylinder and the inter-space between the wall of the aluminum cylinder and the outer wall of the Teflon tube is filled with aluminum foil to provide a good thermal link.

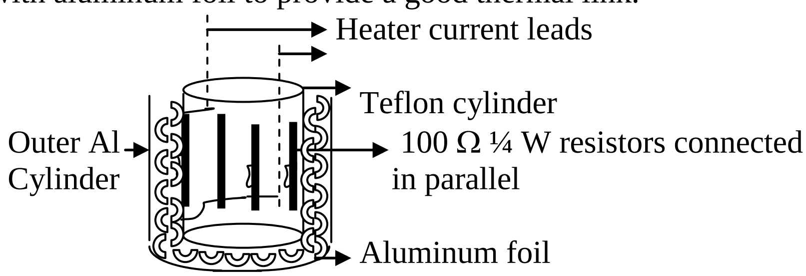


Figure II.2.2.2: Mounting of the heater in the Al cylinder

A copper constantan thermocouple is made by taking about 5 cm of insulated constantan wire (about SW 40) and two long insulated copper wires of the same gauge. After removing the insulation of the wires to a length of 3 to 5 mm at the ends, two copper constantan junctions are made by twisting the bare wires. When the solder is just melting on the soldering iron the two junctions are pushed into the molten solder and pulled out. A drop of superglue (a very quick and strong rapid-cure adhesive available in small tubes) is applied on the Aluminum cylinder near the middle where it is desired to place one of the thermocouple junctions. In a few minutes the liquid dries over the surface to form a thin electrically insulating layer. A junction of the thermocouple is placed at this point and a thin insulated copper wire is tied around it to hold it tight against the cylinder. Another drop of superglue is applied on the thermocouple junction. One should be careful not to apply too much superglue. If the glue forms a thick layer between the junction and the surface of the aluminium cup, there will be an appreciable temperature drop in this layer of glue. The thermocouple will measure a temperature lower than the actual temperature of the aluminium surface. After a few minutes the superglue sets and forms a tight joint between the thermocouple and the aluminium cylinder. The aluminum cylinder can then be painted black on the outside with enamel paint. The paint must be allowed to dry. The cylinder is hung by a thin nylon thread from the top lid of a small stainless steel can E. The inside of the can is also painted black. On the top lid D banana sockets are fixed for the current leads and a RCA socket for the thermocouple leads. The second junction of the thermocouple is fixed to the lid D of the stainless steel can E.

The copper constantan thermocouple has a thermo-electric power α of 40 micro-volts per degree Kelvin near room temperature. For a 10-degree difference in temperature of the two junctions the thermo-emf will be about 400 microvolts. This is amplified a hundred times by the DC amplifier described in Section I.

The Stefan's constant box is shown in Figure II.2.2.3.

Figure II.2.2.3 Stefan's constant box

3. Apparatus required:

Constant current source, DC differential amplifier, Stefan's constant box, and a multimeter reading in DC 200 mV range.

4. PROCEDURE:

The connection diagram for the Stefan's constant experiment is shown in Figure II.2.2.4. A constant current source in the high current mode is connected to the heater terminals of the Stefan's constant box. The two switches on the constant current source are put in the high current mode. The input terminals I_1 of a DC differential amplifier are connected to the RCA socket for thermocouple outlet on the Stefan's constant box. The band switch on the DC differential amplifier is set at I_1 and the toggle switch in the position X100. The output terminals of the DC differential amplifier are connected to a digital multimeter (DMM) set to range DC 200 mV.

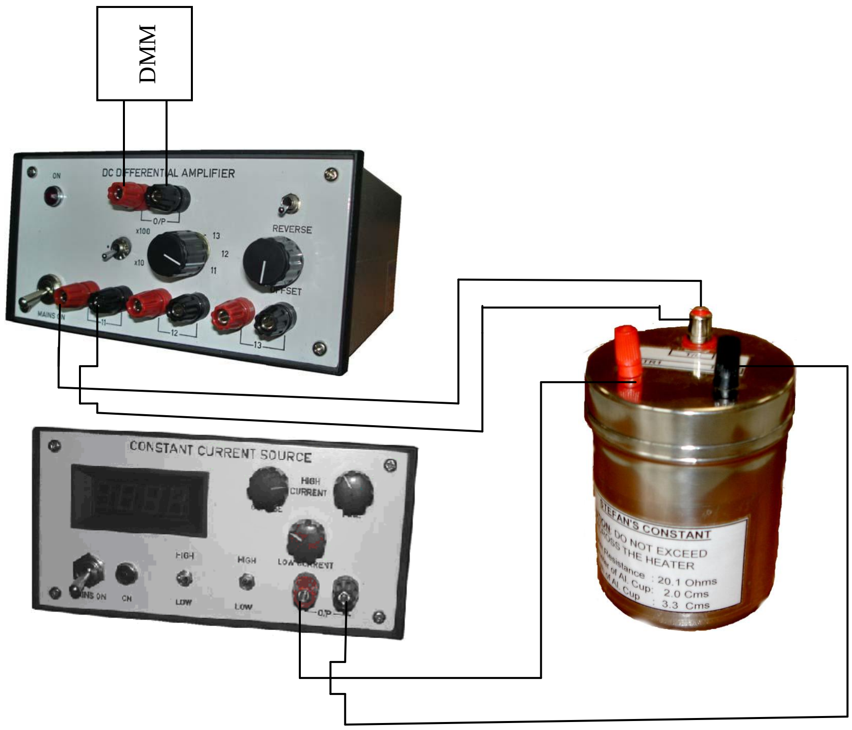


Figure II.2.2.4 Connection diagram for Stefan's constant experiment

Adjust the current through the heater at 200 mA. **This is the maximum current you should pass through the heater. Do not exceed this current.** A voltage will appear on the DMM which will increase in magnitude with time. Wait for one hour for the steady state to be reached. Note the value of the output DC voltage in milli- volts V_+. (It is assumed that this value is positive. Otherwise toggle the reversing switch to get a positive voltage). *The reversing switch on the differential amplifier is thrown to the other position and the reading* V_- *is noted. The amplified thermo emf corrected for the offset of the amplifier is*

$$V_{corr} = (V_+ - V_-)/2 \qquad \text{(II.2.2.3)}$$

Then reduce the current in steps of twenty mA till you reach 140 mA. For each value of current wait for 45 minutes before taking a reading of V_+ and V_-.

The total power dissipated in the resistor is

$$Q = I^2R \qquad \text{(II.2.2.4)}$$

where I is the current through the heater and R is the resistance of the heater. The resistance of the heater is marked on the Stefan's constant box.

The temperature difference between the cylinder and the surrounding can is

$$T_2 - T_1 = \ (V_{corr} \text{ in milli-volts}/\ \alpha\ \mu\ )x10^3 = V_{corr} \text{in mV}/ \qquad (II.2.2.5)$$

Here α is taken as 40 micro-volts per degree Kelvin and μ = 100 (amplification of Differential amplifier). The factor 10^3 arises because we are dividing millivolts by microvolts. The room temperature T_1 is measured with a mercury thermometer. Knowing T_1 and $\Delta T = (T_1 - T_2)$ from measurement, T_2 is calculated. **All temperatures should be expressed in Kelvin.**

If all the heat is lost only by radiation then

$$Q = \sigma\ \varepsilon A\ (T_2^4 - T_1^4) \qquad (II.2.2.6)$$

Here ε is the emissivity of the black surface. For different types of black paints the emissivity varies between 0.9 and 0.95, the latter being the value for enamel paints. We take 0.95 as the emissivity of the black surface since we have used enamel paint.

The area A of the aluminium cup radiating heat is

$$A = 2\pi r\ (r+l) \qquad (II.2.2.7)$$

where r is the radius and l the length of the cylinder. The values of r and l are given on the stainless steel box.

A graph is plotted with $(T_1^4 - T_2^4)$ on the X axis and Q on the Y-axis. A straight line is fitted to the points on a computer and the slope of the straight line is found. The Stefan's constant is calculated from the slope knowing the area A of the aluminum cylinder.

A sample set of readings is given below on Table II.2.2.1. In this table the Column 1 gives the current through the heater in milli-amperes. The heater power Q in watts is given in column 2. Column 3 gives $V_{+\ in}$ mV. Column 4 gives V_- in mV. Column 5 gives V_{corr} in mV. From this the temperature difference $\Delta T = T_2 - T_1$ is calculated by dividing V_{corr} by 4.0 and this is given in column 6. Knowing T_1, the temperature T_2 of the surface of the aluminum block is calculated and given in column 7. Column 8 gives $(T_2^4 - T_1^4)$ in units of 10^9 K^4. Figure II.4.2.4 below gives a plot of Q against $(T_2^4 - T_1^4)$.

Table II.4.2.1

T_1	299	K					
Dia of Al	$2x10^{-2}$	m					
Height	$3.3x10^{-2}$	m					
Area	$2.7x10^{-3}$	m^2					
Resistance	20.1	Ohms					
amp	Watt	mV	mV	mV	^0K	^0K	K^4
I	Q	V_+	V_-	Vcorr	ΔT	T_2	T_2^4-T_1^4
0.203	0.8283	180	-176	178	44.5	343.5	5.93E+09
0.182	0.6658	162	-155	158.5	38.66	337.66	5.01E+09
0.161	0.521	127	-123	125	30.49	329.49	3.79E+09
0.141	0.3996	101	-94	97.5	23.78	322.78	2.86E+09
0.121	0.2943	77	-68	72.5	17.68	316.68	2.06E+09
0.101	0.205	54	-48	51	12.44	311.44	1.42E+09

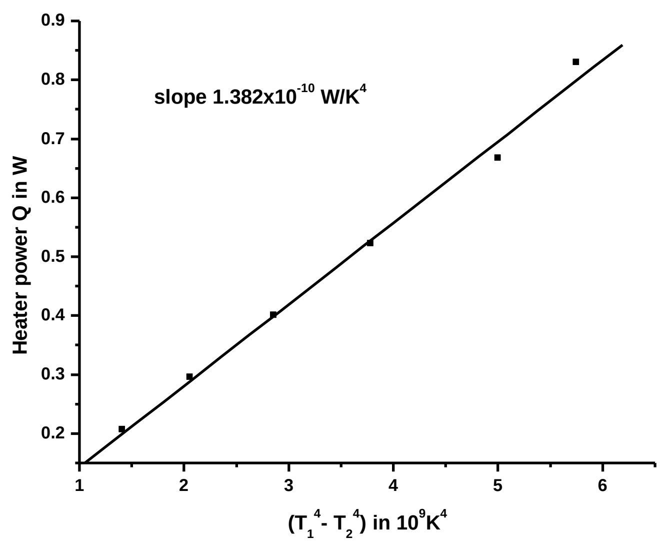


Figure II.2.2.4 Plot of Q against ($T_1^4 – T_2^4$)

From the figure the slope of the curve is $1.382x10^{-10}$ W/K^4. The Stefan's constant is calculated from the area of surface (given in Table II.2.2.1) and the emissivity of black paint taken to be 0.95. It comes out to be 5.38×10^{-8} W/m^{2}K^4 in reasonable agreement with the actual value 5.67×10^{-8} W/m^{2}K^4.

The sources of systematic errors in this experiment are the following:

1. The emissivity of the black paint used may be lower than 0.95. This is the most important source.
2. Some heat may be lost by conduction and convection.

If one would like to measure the emissivity of a polished surface, one may have a second set up with a thin copper cylinder with the outer surface electroplated with Nickel. The above measurements can be repeated. Knowing the value of σ, the emissivity of the electroplated nickel surface can be found.

5. TIME CONSTANT:

In all heat experiments time constant plays an important role. The temperature grows from its initial value at room temperature to its final value as

$$T_1 + \theta(t) = T_1 + \theta(\infty)\,[1 - \exp(-t/\tau)] \qquad \text{(II.2.2.8)}$$

where $\theta(\infty)$ is $(T_2\text{-}T_1)$, the steady state value of the increase in temperature and τ is the relaxation time. Theoretically the steady state temperature is attained after infinite time. But in practice, if one waits for a time t longer than 5τ, the temperature is within 1% of its steady state value. It is necessary to understand the factors governing τ so that the experiment can be designed with a value of τ of the order of 10 minutes. Then the reading of the thermocouple for a given heater power can be taken after forty five minutes to one hour and the experiment can be repeated for at least four different heat inputs within three hours.

At a time t, part of the heat input goes to raise the temperature of the block and part is lost as radiation, conduction and convection. If the temperature rise is not too large we may lump all the heat loss mechanisms into a factor $C\theta$. This is the Newton's law of cooling.

$$ms\,d\theta/dt + C\theta = Q \qquad \text{(II.2.2.9)}$$

Here ms is the total thermal capacity of the metal block including the thermal capacity of the heater and thermometer.

The solution to this equation is

$$\theta(t) = \{Q/C\}\,[1\text{-}\exp\{-(C/ms)\,t\}] \qquad \text{(II.2.2.10)}$$

From this equation we identify Q/C as $\theta(\infty)$ and (ms/C) as τ. For reducing the relaxation time τ we should reduce the mass of the cup. That is why it is preferable to use a lightweight cup made of aluminum or thin copper.

To calculate what mass we should choose we assume heat is lost only by radiation. Then

$$C\theta = \varepsilon\sigma A\,[(T_1+\theta)^4 - T_1^4] \approx 4\varepsilon\sigma A T_1^3\,\theta \qquad \text{(II.2.2.11)}$$

when θ is small. So

$$C \approx 4\varepsilon\sigma A T_1^3 \qquad \text{(II.2.2.12)}$$

and

$$\tau \approx ms/(\,4\varepsilon\sigma A T_1^3) \qquad \text{(II.2.2.13)}$$

Taking σ to be $5.7\text{x}10^{-8}$ W/m^{2}K^4, ε to be 1, T_1 to be 300K, A = $2\text{x}10^{-8}$ m^2

$$C \approx 0.0125$$

So if τ is to be less than 600 s,

$$ms < C\text{x}600 = 7.5 \text{ J/K}$$

The specific heat of aluminum is 0.910 J/gK, means that the mass of the aluminum cylinder should be less than about 8 gm. A cylinder of 2 cm diameter, length l of 3 cm and wall thickness t of 0.5 mm will have a volume $(2\pi r l + \pi r^2)t = 1.09$ cc. Density of Aluminum is 2.7 g/cc. The mass of the cylinder in the experiment is about 2.9 g. To this must be added the weight of the Teflon cylinder and the five resistors. This is the reason why you should wait for at least 45 minutes for steady state to be reached.

Questions:

1. Why is the inside of the wall of a thermos flask silvered?

2. Liquid nitrogen is stored in an evacuated double walled stainless steel container. The radius of the outer of the two walls is 10 cm and its length is 1 meter. The wall is at room temperature (300 K). The container is filled with liquid nitrogen. Its boiling point is 77 K. Assuming the emissivity of stainless steel to be 0.1, calculate the heat reaching the liquid nitrogen from the outer wall. If the latent heat of vaporization of Liquid nitrogen is 160 KiloJoules/litre of liquid nitrogen calculate the boil off rate of liquid nitrogen.

3. If we put 10 layers of aluminized mylar foils between the outer wall and the inner wall of the doubled wall vessel will it reduce the heat radiation reaching the liquid nitrogen? If the emissivity of the foils is taken to be the same as the emissivity of stainless steel by what factor will the heat reaching liquid nitrogen change?

II.2.3 MEASUREMENT OF THE ELECTRICAL AND THERMAL CONDUCTIVITY OF COPPER TO DETERMINE ITS LORENTZ NUMBER.

1. INTRODUCTION:

In a pure metal, the conduction of heat is almost entirely due to electrons. Therefore one would expect the thermal conductivity of a metal to be related to its electrical conductivity. There is a relation called the Wiedeman-Franz relation. This relation states that

$$K/\sigma = L\,T \qquad \text{(II.2.3.1)}$$

Here K is the thermal conductivity, σ is the electrical conductivity, T is the absolute temperature and L is called the Lorentz number. On the free electron theory of metals this Lorentz number is a universal number related to the charge on an electron and the Boltzmann's constant

$$L = (\pi^2/3)\,(k/e)^2 \qquad \text{(II.2.3.2)}$$

The Wiedeman Franz relation is found to be satisfied by many metals like copper, silver and gold especially at and above room temperature. A measurement of L therefore allows one to get an approximate value for the ratio of k/e.

2. EXPERIMENTAL ARRANGEMENT

This arrangement is shown in Figure II.2.3.1.

A 3/8" OD copper tube of length 15 cm and of refrigerator tube quality is used for this purpose. In the middle 3 cm length a heater of about 22 Ohms is kept. The heater is made of a 2 W resistor of about 22 Ohms which is placed inside the tube near its center. Leads are taken from the two ends of the resistor. A thin layer of Teflon tape is wound to insulate the wires from shorting with the copper tube. The interstices between the Teflon tape and the copper tube is filled with aluminum foil. **With such a heater the maximum current that can be passed should not exceed 260 mA.** The exact resistance of the heater is given on the thermal conductivity box.

Two Copper constantan thermocouples are made. The two junctions of each thermocouple are fixed with superglue on each side of the copper tube. The distance between the junctions of each thermocouple is d_1 (about 6 cm). The exact value of d_1 is given on the thermal conductivity box. Two copper current leads CL1 and CL2 are soldered to the copper tube at its two ends. Between the two current leads two more voltage leads VL1 and VL2 are soldered to the copper tube. The distance between the voltage leads is d_2 (about 15 cm). **The value of d_2 is given on the thermal conductivity box.**

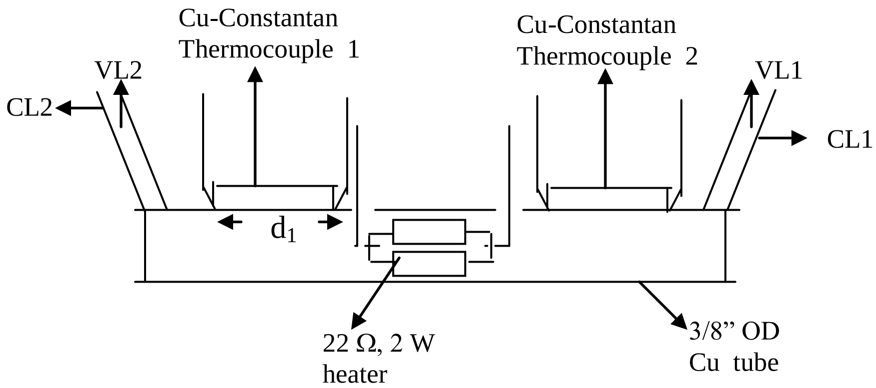


Figure II.2.3.1: Copper tube with heater, thermocouples 1 and 2, current Leads CL1 and CL2 and voltage leads VL1 and VL2

A layer of cotton is wound on this copper tube. On this a layer of paper is wound. On the paper an aluminum foil is wound. Again a layer of paper and a layer of aluminum foil are wound. In this process care is taken to bring the wires out of the wrappings. The cotton and the aluminum foil wrappings serve to reduce heat loss by convection and radiation.

The copper tube with its wrapping is laid on a loose bed of cotton in a box. It is covered loosely with cotton and the leads are soldered to terminals on the front panel of the box. The front view of the box is shown in the Figure II.2.3.2.

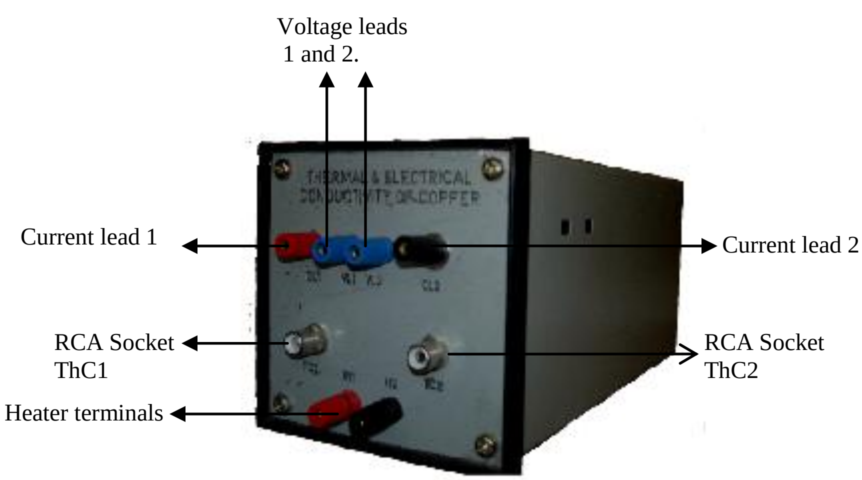


Figure II.2.3.2 Setup for thermal and electrical conductivity of copper

The red and black banana terminals (marked H H) at the bottom are the heater connections. The two RCA sockets on either side are the thermocouple leads for the two differential thermocouples ThC_1 and ThC2. The four banana terminals at the top are for measuring the electrical conductivity. The two extreme banana terminals (marked CL1 and CL2) are connected to the current leads and the two banana terminals in between (markedVL1 andVL2) are connected to the voltage leads.

3. APPARATUS REQUIRED:

Constant current source, thermal conductivity of copper box, DC differential amplifier, and a multimeter reading in DC 200mV range.

4. PROCEDURE FOR MEASURING THERMAL CONDUCTIVITY

1. Check that the two switches in the constant current source are in the high position and the coarse and fine potentiometers in the high current mode are rotated anticlockwise to their extreme positions. Connect the banana terminals on the constant current source to the banana terminals marked HH on the box.

2. Switch on the differential amplifier. Put the selector switch on the differential amplifier in the position I_1. Short the input terminals I_1. Connect a DMM in the DC 200 mV range to the output terminals of the differential amplifier. Adjust the offset adjust potentiometer so that the offset voltage is about 1 mV. When the reversing switch is thrown the output reading should **not** change sign.

3. Connect the RCA socket ThC1 on the Thermal conductivity box to Input terminals I_1 on the differential amplifier and the RCA socket marked ThC2 on the thermal conductivity apparatus to the input terminals I_2 of the differential amplifier.

4. Switch on the constant current source and set the current at 260 mA. With this current wait for 45 minutes before taking a reading. Steady state is attained in this time.

5. Note the DMM reading V_{1+} in mV for the thermocouple ThC1. Throw the reversing switch on the differential amplifier and note the reading V_{1-}. Put the selector switch in the position I_2 and measure V_{2+} and V_{2-}.

6. Reduce the current in steps of 20 mA till you reach a current of 200 mA. For each value of the current, wait for 30 minutes and take the readings V_{1+}, V_{1-}, V_{2+} and V_{2-}.

From these readings the thermal conductivity is calculated as follows. The heat input when a current I passes through the heater is

$$Q = I^2R \text{ Watts} \qquad \text{(II.2.3.3)}$$

The resistance R of the heater is marked on the thermal conductivity box.

Assuming no heat loss due to convection and radiation, part Q' of this heat is conducted along the left half of the tube where ThC1 is located and the remaining part Q" = Q-Q' is conducted along the right half of the tube where the thermocouple ThC2 is located. If the amplified thermo-emf of ThC1 measured is V_{1+} and V_1- (when the reversing switch is thrown) the amplified thermo-emf voltage corrected for any off set is $(V_{1+} - V_{1-})/2$. The amplification factor is 100. So the thermo-emf is $(V_{1+}-V_{1-})/(2x100)$ millivolts. The thermoemf in microvolts is $10 \times (V_{1+}-V_{1-})/2$, since 1 mV = 1000μV. Since the thermoelectric power of Copper –constantan thermocouple is 40 microvolt/degree K, the above voltage will correspond to a temperature difference between the two junctions of ThC1 by

$$\Delta T_1 = [10 \times (V_{1+}-V_{1-})/2]/40 = [(V_{1+}-V_{1-})/2]/4.0 \text{ K} \qquad \text{(II.2.3.4)}$$

In the steady state

$$Q' = K (A/d_1) \Delta T_1 \qquad \text{(II.2.3.5)}$$

Similarly from the measurement of V_{2+} and V_{2-} of ThC2 we have the temperature drop ΔT_2 across the junctions of ThC2 given by

$$\Delta T_2 = [10 \times (V_{2+}-V_{2-})/2]/40 = [(V_{2+}-V_{2-})/2]/4.0 \text{ K} \qquad \text{(II.2.3.6)}$$

So

$$Q'' = K (A/d_1) \Delta T_2 \qquad \text{(II.2.3.7)}$$

since we have kept the distance between the two junctions of ThC1 and ThC2 the same at the value d_1.

Adding the two equations

$$Q = Q'+Q'' = K (A/d_1) (\Delta T_1 + \Delta T_2) \qquad \text{(II.2.3.8)}$$

Or $$K (A/d_1) = Q/ (\Delta T_1 + \Delta T_2) = Q/ \Delta T \qquad \text{(II.2.3.9)}$$

For different heat inputs we find the right side and take the average value of $[Q/\Delta T]$. From the outer diameter OD (9.5 mm) of the copper tube and the wall thickness t (0.75 mm) given on the top of the box we calculate the inner diameter of the tube as

$$ID = OD - 2t \qquad \text{(II.2.3.10)}$$

So the area of cross section of the tube is

$$A = (\pi/4) (OD^2 - ID^2) = 0.207 \text{ cm}^2$$

Taking d_1 to be 6 cm, $d_1/A = 6x10^{-2}$ m/ $0.207x10^{-4}$ m^2 = 2910/m
You should calculate for d_1 marked on the box.

A sample set of readings are given below:

Table II.4.3.1
Thermal conductivity of Copper

d1	6	cm	d1/A	2910	m^{-1}						
R	22	Ohms									
I amp	Q W	V_{1+} mV	V_{1-} mV	V_{1corr} mV	ΔT_1K	V_{2+} mV	V_{2-} mV	V_{2corr} mV	ΔT_2 K	ΔT K	$Q/\Delta T$
0.26	1.487	19.4	-27.4	23.4	5.85	26.3	-34.3	30.3	7.58	13.43	0.111
0.24	1.267	18.0	-21.7	19.85	4.96	24.0	-28.2	26.1	6.50	11.46	0.111
0.22	1.065	21.5	-10.7	16.1	4.03	26.3	-18.4	22.35	5.59	9.62	0.111
0.2	0.88	25.0	-3.3	14.15	3.54	27.8	-9.0	18.4	4.60	8.14	0.108
										Average	0.110

The value of thermal conductivity is

$$K = (Q/\Delta T) \times (d_1/A) = 320 \text{ W/mK.}$$

For very pure copper the thermal conductivity is 401 W/mK. However copper tubes used for refrigeration are called de-oxidized high phosphorus containing copper – DHP copper. It has a copper (+Ag) content of 99.85% with Phosphorous between 0.013 to 0.04%. From the Plumber's Handbook 8th Edition page 7, published by Australian Copper industries, the thermal conductivity of this type of copper is between 305 to 355 W/mK. From the Internet, I found that Minerex AG, a company supplying tubes of DHP, has given the thermal conductivity as 340 W/mK.

I have measured thermal conductivity of copper with eight different thermal conductivity boxes. All of them give a value between 310 and 330 W/mK. So we may take the correct thermal conductivity of copper used in the conductivity box as 320 ±10 W/mK.

5. PROCEDURE FOR MEASURING THE ELECTRICAL CONDUCTIVITY

Connect the constant current source between CL1 and CL2. Connect the terminals I_3 on the differential amplifier to VL1 and VL2 and set the selector switch on the Differential amplifier to I3. Pass a current I of about 300 mA. Note the out put voltage of the Differential amplifier in the DC 200 mV range of the DMM. Ohm's law is instantaneous. So there is no need to wait. Let this voltage be V_{1+}. Throw the reversing switch on the differential amplifier and note the output V_{1-} of the amplifier. Then the potential difference across the voltage leads corrected for offset is $[(V_{1+}-V_{1-})/2]x10$ in microvolts. Increase the current in steps of 50 mA up to 600 mA and find the potential difference across the voltage leads. Draw a graph between the potential difference in

micro-volts and the current. Fit a straight line. The slope of the line gives the resistance R in micro-ohms. **NOTE THE VALUE OF d_2 GIVEN ON THE BOX.**
A sample set of readings are given below:

Table II.4.3.2
Electrical conductivity of copper

d_2	15	Cm		
d_2/A	7.25E+03	m^{-1}		
I	V_+ mV	V_- mV	V_{corr} mV	Pot Diffce in μV
0.3	7.7	-3.3	5.5	55
0.35	8.5	-4.3	6.4	64
0.4	9.5	-5.1	7.3	73
0.45	10.2	-6.2	8.2	82
0.5	11.3	-7.2	9.25	92.5
0.55	12.3	-8.3	10.3	103
0.6	13.3	-9.3	11.3	113
slope	194	micro-ohms		
Electrical conductivity			3.82E+07	
$K/\sigma T$			2.87E-08	

Figure II.2.3.3 shows a plot of the potential difference in micro-volt against the current in amperes.

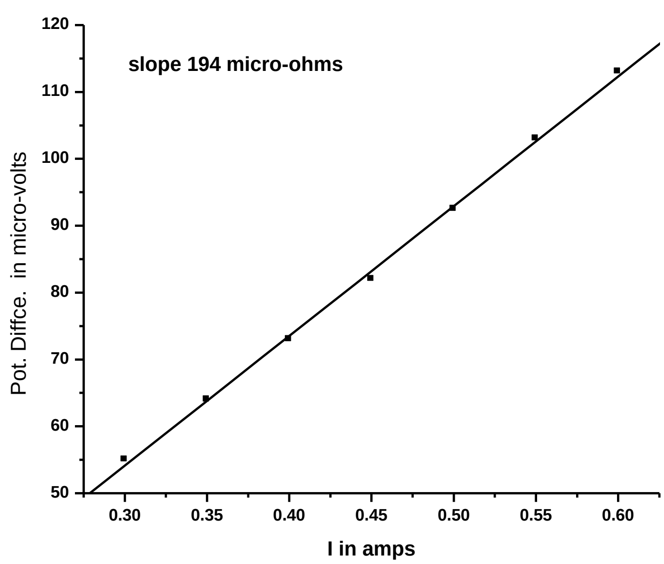


Figure II.2.3.3: Plot of Potential difference in micro-volts against current in amps.

The slope of the graph is 194 micro-ohms. The electrical conductivity σ is calculated from

$$\sigma = (1/\text{slope}) \times (d_2/A)$$

σ comes out to be 3.82×10^7 Siemens. For pure copper σ is 6×10^7 Siemens. The Plumbers Handbook gives the specific electrical resistance for *annealed* DHP copper as 0.0192 to 0.0238 microhm-meter. This implies a conductivity from 4.3 to 5.2×10^7 Siemens. We get a lower electrical conductivity probably because the tube is not annealed.

The Lorentz number is $K/\sigma T = 319/(3.82\text{x}107\text{x}300) = 2.87\times10^{-8}\ (V/K)^2$.

The Lorentz number on the free electron theory is $2.44\times10^{-8}\ (V/K)^2$

Note: The value of the heater resistance R, the distances d_1 and d_2 for each setup is noted on the box. These may be different from the values of R, d_1 and d_2 used in the sample data given here.

Questions

1. Do you think one can use this technique of measuring thermal conductivity for a poor conductor of heat?

2. Estimate the value of k/e from equation (II.4.2.2) and compare with the actual value.

II.2.4 THERMAL CONDUCTIVITY OF A POOR CONDUCTOR

1. INTRODUCTION:

One may use a steady state method for measuring the thermal conductivity of any material. In the steady state method one gives a heat power Q at one end of the specimen and measures the temperature gradient ΔT at two points separated by a distance d. Then

$$Q = KA\, \Delta T/d \qquad \text{(II.2.4.1)}$$

Here A is the area of cross section of the material. To obtain a measurable temperature difference of a few degrees C and at the same time to keep the heat power to about 1 W, one must decrease the area of cross section A and increase the distance d for a good conductor like copper. That is why in the experiment on thermal conductivity of copper we used a tube (area of cross section is determined by the diameter and thickness of the tube; area is small) and measured the temperature difference between two points separated by a distance of a few centimeters. The surface area over which heat is lost by radiation and convection is large in this case. However for a good conductor most of the heat is conducted and only a small part is lost by radiation. This loss can be reduced by lagging the tube to reduce convection and by surrounding the lagging by a layer of aluminum foil to reduce radiation.

For a poor conductor (K < 1 W/mK) a small amount of heat will produce a large temperature gradient. To make the material to conduct this heat we have to increase the area over which heat is conducted. At the same time to keep the temperature difference to a few degrees Centigrade we have to reduce the thickness. For poor conductors therefore one uses a plate method in which the plate has a surface area of a few square centimeters and a thickness of the order of 1 mm.

2. DESCRIPTION OF THE SET-UP:

A copper cup of about 2.54 cm (1”) diameter is made into two halves C as shown in Figure II.2.4.1.

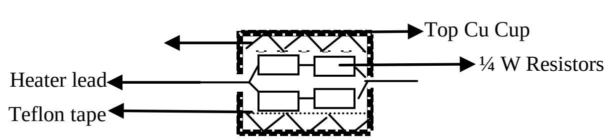


Figure II.2.4.1: Construction of the heater

A heater H made of six 120-Ohm ¼ W resistors in parallel is insulated by a thin layer of Teflon tape and is placed in the cup between two pads D of aluminium foil. The foil helps to conduct the heat away from the heater on both sides. Two circular discs B are cut from 1” diameter Perspex rod. The discs have equal thickness between 1 to 2 mm. These discs are clamped between the heater on one side and two square copper plates A on the other side as shown in Figure II.2.4.2. The height of each half of the cylindrical cup containing the heater is 5 mm. Two junctions of a copper constantan thermocouple E are fixed with super glue to the upper square copper plate and near the top of the cylindrical surface of the upper half of the copper heater cup. Another pair of junctions of a copper constantan thermocouple is similarly fixed to the bottom square copper plate and near the bottom edge of the bottom half of the heater cup. The two plates of Perspex are smeared with a very thin layer of Vaseline and clamped by 4 bolts and nuts on the square plates.

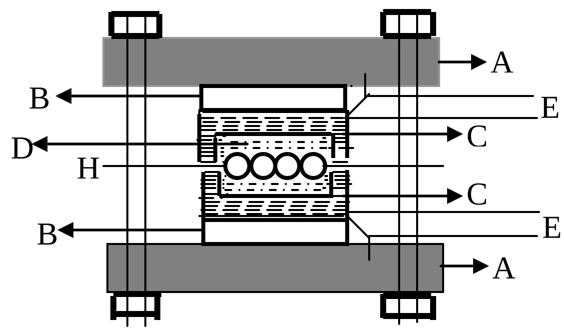


Figure II.2.4.2: Set up for thermal conductivity of
Poor conductors
A: Copper base plate each 5 cm square, 5 mm thick
B: Perspex discs 2.5 cm dia and about 0.2 cm thickness
C: Two halves of a copper cup 2.5 cm dia, outer height 5 mm
Inner height 3 mm
D: Aluminium foil pads
H: Heater made of six 120 Ohm ¼ W resistors in parallel
E: Cu-Constantan thermocouples ThC1 and ThC2

The thin layer of Vaseline under pressure provides an acceptable thermal contact. The lateral surface of the heater cup is wrapped with a layer of cotton, a layer of aluminium foil and a second layer of cotton. This is not shown in figure. The heater leads are brought to a pair of banana terminals on a box in which the assembly is kept. The terminals of the two thermocouples are likewise brought to two RCA sockets on the box. The box is loosely filed with cotton to prevent convection.

Note that the differential couple measures the sum of the temperature drop across the Perspex plate and the temperature drop across the copper plates from the surface to the point at which the thermocouple junctions are fixed. But because the thermal conductivity of copper is about a thousand times larger than the thermal conductivity of Perspex, the latter contribution to the measured temperature drop is small. One may take the measured thermo-emf as arising solely from the temperature drop across the Perspex plate.

3. APPARATUS REQUIRED:

Constant current source, the thermal conductivity of a poor conductor box, DC differential amplifier and a DMM reading in the EDC 200 mV range.

4. PROCEDURE:

Connect a constant current source to the heater terminals. Connect the two thermocouple outputs to I_1 and I_2 terminals of the DC Differential Amplifier. Connect a multimeter in the DC 200 mV range to the output of the Differential amplifier.

Set a current of about 260 mA from the constant current source. **DO NOT EXCEED 260 mA OF CURRENT.** For this current wait for forty five minutes and measure the output V_{1+} on the DMM for Thermocouple 1. Press the reversing switch on the DC differential amplifier and measure the output V_{1-} on the DMM for the same thermocouple. Then change the band switch on the differential amplifier to I_2 and repeat the measurements of V_{2+} and V_{2-} for thermocouple 2. Then decrease the current in steps of 20 mA till you reach a current of 200 mA. For each value of the current wait for half an hour for steady state to be reached. Measure the outputs V_{1+}, V_{1-} for thermocouple 1 and V_{2+}, V_{2-} for thermocouple 2 as before.

A sample set of readings is given in Table II.2.4.1 below. The diameter of the Perspex plate is 2.54 cm. The thickness of each Perspex plate is 0.170 cm. The resistance R of the heater is 20.5 Ohms. **NOTE: Values of R, diameter of the plate and its thickness are marked on each box. Use these values in your measurement.**

From the DMM outputs V_+ and V_- for any thermocouple, the output V_{corr} , corrected for offset is given by

$$V_{corr} = (V_+ - V_-)/2. \qquad \text{(II.2.4.2)}$$

Since the amplification factor of the amplifier is 100 and the copper-constantan thermocouple has a thermo-$_{\text{electric}}$ power of $40\mu V/^0C$, the temperature drop across plate1 is

$$\Delta T = V_{corr}\, x1000/\ (100x40) = V_{corr}/4\ \ {}^0K. \qquad \text{(II.2.4.3)}$$

From the measurements of the two thermocouples we find the temperature drop ΔT_1 across plate 1, and the temperature drop ΔT_2 across plate 2. Both the plates have the same area A and thickness d. So

$$Q = (KA/d)\ (\Delta T_1 + \Delta T_2) \qquad \text{(II.2.4.4)}$$

A specimen set of readings are given below.

TABLE II.2.4.1

Sample data on the thermal conductivity of Perspex

Diameter of perspex plate			2.54	Cm							
Thickness			0.174	Cm							
d/A	3.434	m^{-1}									
R	20.5	Ohms									
I	Q	V_{1+}	V_{1-}	V_{1corr}	ΔT_1	V_{2+}	V_{2-}	V_{2corr}	ΔT_2	ΔT	$Q/\Delta T$
amp	watts	mV	mV	mV	C	mV	mV	mV	C	C	W/K
0.125	0.320	9.4	-7.8	8.6	2.15	9.6	-7.7	8.65	2.16	4.31	0.074
0.150	0.461	12.8	-11.9	12.35	3.09	15.5	-11.3	13.4	3.35	6.49	0.072
0.175	0.628	18.8	-16.5	17.65	4.41	17.9	-16.4	17.15	4.29	8.70	0.072
0.200	0.820	22.1	-22.3	22.2	5.55	23.7	-22.8	23.25	5.81	11.36	0.072
0.225	1.038	28.8	-27.3	28.05	7.01	29.7	-27.9	28.8	7.20	14.21	0.073
0.250	1.281	35.3	-35.1	35.2	8.80	35.1	-35.7	35.4	8.85	17.65	0.073
										Average	0.073
K=	0.25	W/mK									

For different values of the current, $Q/(\Delta T)$, (where $\Delta T = \Delta T_1 + \Delta T_2$), must remain constant. The last column gives the value of $Q/\Delta T$ and this is found to be constant within ±0.001. Find the average value of $Q/\Delta T$. Then the thermal conductivity is

$$K = (d/A)\,(Q/\Delta T). \qquad \text{(II.2.4.5)}$$

The value of thermal conductivity comes out to be 0.25 W/mK. This is about the value of K for Perspex from Internet.

NOTE:

The dimensions of the Perspex disc are chosen so that one can pass the same currents through the heater as in the case of thermal conductivity of copper set up (hereafter called set-up A). So one can connect the constant current source to the heater on set-up A in series with the heater on the set-up B (which is for the thermal conductivity of poor conductor). The waiting time for both set-ups to reach steady state is about the same. After this waiting time connect the thermocouples ThC1 and ThC2 on set up A to I_1 and I_2 of the differential amplifier and measure the emfs.

Remove the connections to I_1 and I_2 of set up A and connect the thermocouples of set-up B to I_1 and I_2 and measure the thermo-emfs of set up B. Then change the current through the heater, wait for 30 minutes and repeat the procedure. This way the two experiments can be done simultaneously with a large saving in time.

II.2.5 THERMAL DIFFUSIVITY OF BRASS

1. INTRODUCTION:

The thermal diffusivity of a material is defined by the ratio $\kappa/\rho c$ where κ is the thermal conductivity, ρ, the density and c the specific heat of a material. If heat is generated at a point in the material, the speed with which it diffuses out from the point is determined by this ratio. The dimension of this ratio is ((Joule/s.m.K)/ [(kg/m^3) (Joule/kg K)] = m^2/s. It indicates the spread, m^2 per second, of the locally applied heat. For a poor thermal conductor like glass, κ= 1W/mK, ρ = 2500 kg/ m^3 and c = 1 kiloJ/kg K and the thermal diffusivity is 4×10^{-7} m^2/s. For copper the diffusivity has a much higher value. Heat spreads much faster in copper than in glass.

In steady state measurements of heat transport only the conductivity plays a role. The diffusivity has no role to play. But if the heat supplied varies as a function of time then the diffusivity determines how the temperature varies with space and time in the medium. For such a case, the equation satisfied by the temperature T for a one-dimensional problem is obtained as follows.

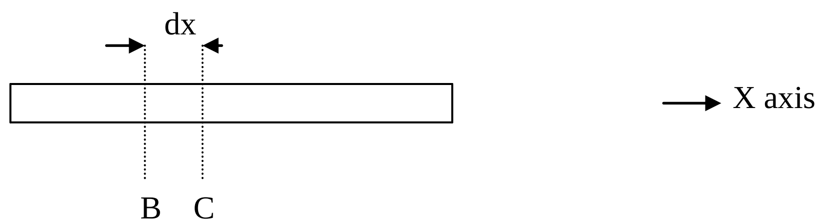


Figure II.2.5.1

Let the cross-section of the rod be A. Heat is propagated along the length of the rod, which is taken along the X direction. Consider two sections of the rod at B and C separated by an infinitesimal distance dx. The temperature at the cross section at B is T. The amount of heat crossing the section at B in the direction of the X-axis is $-\kappa A dT/dx$. The heat passing out through the section at C is $-\kappa A\,[\partial T/\partial x + \partial^2 T/\partial x^2\,dx]$. So the net amount of heat flowing out of the element of length dx in one second is given by

$$\{-\kappa A\,[\partial T/\partial x + \partial^2 T/\partial x^2\,dx]\} - (-\kappa A\,\partial T/\partial x) = -\kappa A\,\partial^2 T/\partial x^2\,dx \qquad \text{(II.2.5.1)}$$

The mass of material between the two sections is ρA dx. If the temperature varies with time then the amount of heat used up in one second in heating this element is

$$\rho A\,dx\,c\,\partial T/\partial t \qquad \text{(II.2.5.2)}$$

In addition there may be radiation from the surface to the surroundings. Using the simple Newton's law the heat lost from the surface area of the element between the sections B and C is

$$\varepsilon P\, dx\, (T - T_0) \qquad \text{(II.2.5.3)}$$

where T_0 is the temperature of the surroundings and P is the perimeter of the cross section of the rod.

The total heat used up by the element in one second is

$$\rho A\, dx\, c\, \partial T/\partial t - \kappa A \partial^2 T/\partial x^2\, dx + \varepsilon P\, dx\, (T - T_0) \qquad \text{(II.2.5.4)}$$

If an amount of heat λ Adx is supplied for unit time, the law of conservation of energy tells us that the heat supplied must be equal to the heat used up. So

$$\lambda\, Adx = \rho A\, dx\, c\, \partial T/\partial t - \kappa A \partial^2 T/\partial x^2\, dx + \varepsilon P\, dx\, (T - T_0) \qquad \text{(II.2.5.5)}$$

or

$$-\kappa \partial^2 T/\partial x^2 + \rho\, c\, \partial T/\partial t + \varepsilon\, (P/A)\, (T - T_0) = \lambda \qquad \text{(II.2.5.6)}$$

This is the partial differential equation satisfied by the temperature.

Let us consider a case where we put a heater on one of the end faces (face at x = 0) of the rod and there is no distributed heat source along the rod. Then $\lambda = 0$ all along the rod. Then the equation satisfied by the temperature T is

$$\kappa \partial^2 T/\partial x^2 - \rho\, c\, \partial T/\partial t - \varepsilon\, (P/A)\, (T - T_0) = 0 \qquad (0<x<L) \qquad \text{(II.2.5.7)}$$

$$\partial^2 T/\partial x^2 - (\rho c/\kappa)\, \partial T/\partial t - \varepsilon\, (P/A\kappa)\, (T - T_0) = 0 \qquad \text{(II.2.5.8)}$$

Let us assume that over the end face the heat flowing per unit time Q varies as $\exp(-\iota\omega t)$ i.e.

$$Q = Q_0 \exp(-\iota\omega t) \qquad \text{(II.2.5.9)}$$

Then the temperature also will vary as

$$T(x, t) = \theta(x) \exp(-\iota\omega t) + T_0 \qquad \text{(II.2.5.10)}$$

Then $$d^2\theta/dx^2 + \iota(\omega\rho c/\kappa)\, \theta - \varepsilon\, (P/A\kappa)\, \theta = 0 \qquad \text{(II.2.5.11)}$$

Writing ε (P/Aκ) = a and ($\omega\rho c/\kappa$) = b equation (11) can be written as

$$d^2\theta/dx^2 - (a-\iota b)\theta = 0 \qquad \text{(II.2.5.12)}$$

This equation has to be solved subject to certain boundary conditions. One condition will be determined by

$$-\kappa A\,(\partial T/\partial x)_{x=0} = Q \qquad \text{(II.2.5.13)}$$

The other boundary condition can be imposed by keeping the end of the rod at L at a fixed temperature T_0.

Writing $a\text{-}ib = \eta^2$ and putting $u = \eta x$ (II.2.5.14)

$$d^2\theta/du^2 - \theta = 0 \qquad \text{(II.2.5.15)}$$

The solution of this equation is

$$\theta(x) = C\exp(\eta x) + B\exp(-\eta x) \qquad \text{(II.2.5.16)}$$

The temperature T (x, t) varies as

$$T(x,t) = [C\exp(\eta x) + B\exp(-\eta x)]\exp(-\iota\omega t) + T_0 \qquad \text{(II.2.5.17)}$$

$$\eta = (\alpha-\iota\beta) = (a-\iota b)^{1/2} = (a^2+b^2)^{1/4}\exp(-\iota\phi/2) \qquad \text{(II.2.5.17a)}$$

where $\tan\phi = b/a$ (II.2.5.17b)

$$T(x,t) = [C\exp(\alpha x)\exp(-\iota\beta x) + B\exp(-\alpha x)\exp(+\iota\beta x)]\exp(-\iota\omega t) \qquad \text{(II.2.5.18)}$$

$$\alpha = (a^2+b^2)^{1/4}\cos(\phi/2) \qquad \text{(II.2.5.19 a)}$$
$$\beta = (a^2+b^2)^{1/4}\sin(\phi/2) \qquad \text{(II.2.5.19 b)}$$

α and β have the dimension of inverse of length. If $\alpha L>5$, we may take $T = T_0$ at L and put C = 0 in the term C exp ($\alpha+\iota\beta$) x in (II.2.5.18). We assume this to be true and put C = 0.

Then the temperature distribution is given by

$$T(x, t) = B \exp(-(\alpha-\iota\beta)x) \exp(-\iota\omega t) + T_0 \qquad \text{(II.2.5.20)}$$

The temperature gradient at x = 0 is then given by

$$dT/dx = - [(\alpha-\iota\beta)] B \exp(-\iota\omega t) \qquad \text{(II.2.5.21)}$$

The boundary condition at x = 0 is

$$-\kappa A\, |dT/dx| = Q_0 \exp(-\iota\omega t) \qquad \text{(II.2.5.22)}$$

This gives

$$\begin{aligned} B &= [Q_0 / (\kappa\, A\} (\alpha^2+\beta^2)^{1/2}] \qquad \text{(II.2.5.23)} \\ &= [Q_0 / (\kappa\, A\} (a^2+\beta^2)^{1/4}] \end{aligned}$$

So the temperature distribution is given by

$$T(x, t) = [Q_0/ (\kappa A\, (a^2+b^2)^{1/4}] \exp(-\alpha x) \exp -\iota(\omega t -\beta x) \qquad \text{(II.2.5.24)}$$

The temperature is not in phase with the heating. The amplitude of the temperature oscillation is a function of x and varies as

$$\mathrm{Amp}\,(T(x)) = [Q_0/ (\kappa A\, (a^2+b^2)^{1/4})]\exp(-\alpha x) \qquad \text{(II.2.5.26)}$$

The ratio of amplitudes at x_1 and x_2 is

$$\mathrm{Amp}\,(x_2)/\mathrm{Amp}(x_1) = \exp[-\alpha\, (x_2-x_1)] \qquad \text{(II.2.5.27)}$$

The difference in phase between the temperature oscillations at two points x_1 and x_2 is

$$\phi(x_1) - \phi(x_2) = \beta\, (x_1-x_2) \qquad \text{(II.2.5.28)}$$

One can find α and β and hence the thermal diffusivity of the material by measuring the ratio of amplitudes at two points separated by a known distance and by measuring the phase difference of the temperature wave between two points separated by a known distance.

However producing a sinusoidal heating is difficult. We can produce a periodic heating at the end x = 0 by supplying a known current to a heater in a periodic fashion as shown below.

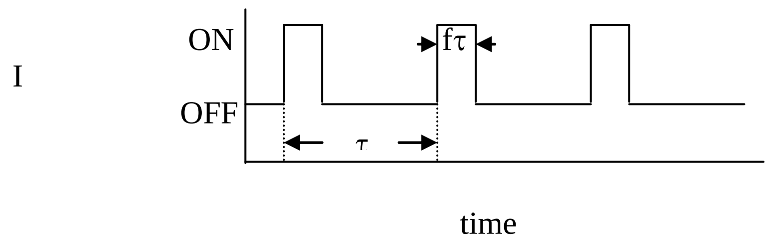


Figure II.2.5.2 Shows periodic heating at x = 0.

The current is switched on for times $n\tau \le t < (n+f)\tau$; n is integral and $0<f<1$
The current is switched off for times $(n+f)\tau<t<(n+1)\tau$

(i.e.) $I = I_0$ for $n\tau \le t < (n+f)\tau$; and 0 for $(n+f)\tau<t<(n+1)\tau$. (II.2.5.29)

τ is the period of current variation. So the heat input Q varies as

$$Q = H = I^2R \qquad \text{for} \quad n\tau \le t < (n+f)\tau;$$

And

$$= 0 \qquad \text{for} \quad (n+f)\tau<t<(n+1)\tau \qquad \text{(II.2.5.30)}$$

Such a periodic function Q(t) can be expanded in a Fourier series

$$Q(t) = H_0 + \sum_{n=1}^{\infty} [H_n \exp(\iota n\omega_0 t) + H_{-n} \exp(-\iota n\omega_0 t)] \qquad \text{(II.2.5.31)}$$

$$H_n = H^*_{-n}\ ; \qquad \omega_0 = 2\pi/\tau$$

The coefficients H_0 and H_n are obtained from the equations

$$H_0 = (1/\tau)\int_0^{f\tau} H\, dt = Hf = I_0^2Rf \qquad \text{(II.2.5.32)}$$

$$H_{-n} = (1/\tau)\int_0^{f\tau} H \exp(-\iota n\omega_0 t)\, dt = -\iota H (1/2\pi n)[1-\exp(-\iota 2\pi nf)] \qquad \text{(II.2.5.33)}$$

The corresponding temperature distribution is given by

$$T(x,t) = T_0 + (Hf/\kappa A)(L-x) + \sum_{n=1}^{\infty} [B_n \exp(-(1+\iota)\sqrt{n}\,\eta x/\sqrt{2})\exp(-\iota n\omega_0 t) + B_{-n} \exp(-(1-\iota)\sqrt{n}\,\eta x/\sqrt{2})\exp(\iota n\omega_0 t)] \qquad \text{(II.2.5.34)}$$

with $B_{-n} = B^*_n$

and $B_n = [\, H_{-n} / (\, \kappa A\sqrt{n}\, \eta)]\, \exp(-\iota\pi/4)$ (II.2.5.35)

Thus the temperature will show some periodic variation as shown in figure.

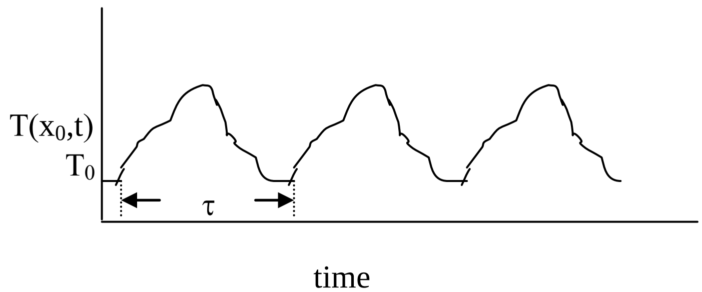


Figure II.2.5.3 shows the temperature variation with time at a point x_0 along the length of the rod.

We measure the temperature-time graph at two points. We Fourier transform the graphs to get the Fourier component at the frequency ω_0. This is done as follows. The complex amplitude B_1 of the Fourier component varying as $\exp(-\iota\omega_0 t)$ is found from the integral

$$B_1 \exp(-(1+\iota)\eta x_1/\sqrt{2}) = (1/\tau)\int_0^\tau T(x_1,t)\exp(\iota\omega_0 t)\, dt \qquad \text{(II.2.5.36)}$$

The RP of the integral on the right is

$$\text{RP of } I(x_1) = (1/\tau)\int_0^\tau T(x_1,t)\cos(\omega_0 t)\, dt \qquad \text{(II.2.5.37)}$$

And the imaginary part is

$$\text{IP of } I(x_1) = (1/\tau)\int_0^\tau T(x_1,t)\sin(\omega_0 t)\, dt \qquad \text{(II.2.5.38)}$$

We divide the time interval from 0 to τ into 20 equal parts, 0, $\tau/20, 2\tau/20, \ldots m\tau/20, \ldots \tau$. Take the temperature $T(x_1, m\tau/20)$ and multiply it by $\cos(2\pi m/20)$ and $\sin(2\pi m/20)$. We do this for each value of m. The real part of the integral is then given by

$$\begin{aligned}\text{RP of } I(x_1) = (1/\tau)(\tau/20)\,[\,&(1/2)\, T(x_1,0) \\ &+ \sum_{m=1}^{19} T(x_1, m\tau/20)\cos(2\pi m/20) \\ &+ (1/2)\, T(x_1,\tau)] \end{aligned} \qquad \text{(II.2.5.39)}$$

and the imaginary part of the integral is

$$\text{IP of } I(x_1) = (1/\tau)(\tau/20)\left[\,0 + \sum_{m=1}^{19} T(x_1, m\tau/20)\sin(2\pi m/20) + 0\,\right] \qquad \text{(II.2.5.40)}$$

This is the Simpson's rule for integration.

Once the RP and IP of $I(x_1)$ is found, the amplitude of the temperature varying at frequency ω_0 at x_1 is

$$\text{Amp of } \theta(x_1,\omega_0) = [\,\text{RP}^2 \text{ of } I(x_1) + \text{IP}^2 \text{ of } I(x_1)]^{1/2} \qquad \text{(II.2.5.41)}$$

And the phase $\phi(x_1)$ is

$$\text{Phase of } \theta(x_1,\omega_0) = \phi(x_1) = \tan^{-1}[\text{IP of } I(x_1)/\text{RP of } I(x_1)] \qquad \text{(II.2.5.42)}$$

We repeat this calculation for the temperature at the point x_2 to get Amplitude and phase of $\theta(x_2,\omega_0)$.

From the ratio of the amplitude of $\theta(x_1,\omega_0)$ to the amplitude $\theta(x_2,\omega_0)$, using the equation (II.2.5.27), one obtains α

$$\alpha = \ln(\,|\theta(x_1, \omega_0)|/|\theta(x_2,\omega_0)|)/(x_2 - x_1) \qquad \text{(II.2.5.43)}$$

One can also obtain β from the measurement of the phase difference using equation (II.2.5.28).

$$\beta = [\phi(x_2) - \phi(x_1)]/(x_2 - x_1) \qquad \text{(II.2.5.44)}$$

From this knowing τ one obtains the diffusivity $\kappa/\rho c$.

$$\begin{aligned} \alpha\beta &= (a^2+b^2)^{1/2}\cos(\phi/2)\sin(\phi/2) \\ &= (1/2)(a^2+b^2)^{1/2}\sin\phi = b/2 = (\omega\rho c/2\kappa) \qquad \text{(II.2.5.45)} \end{aligned}$$

So the diffusivity

$$D = \kappa/\rho c = \omega/(2\alpha\beta) \qquad \text{(II.2.5.46)}$$

The diffusivity can be calculated from a measurement of α and β.

2. APPARATUS REQUIRED

A DC power supply giving 15 V maximum and 2 A maximum, the thermal diffusivity box, DC differential amplifier, and a DMM reading in DC 200 mV range.

3. EXPERIMENTAL SET-UP

The schematic of the experimental set up is shown in Figure II.2.5.4. This experiment is designed to be performed on a brass rod. Take a brass rod (1) of 30 cm length and diameter about 5 mm (3/16"). A small heater (3) of about 10 Ohms is wound on the centre of the brass rod. Two copper-constantan thermocouples (4) and (5) are attached to the brass rod with superglue. The distance between the junctions is 3 cm. Symmetrically to the right of the heater two more junctions (4') and (5') are fixed with superglue to the brass rod. These junctions are not shown in the figure. The cold ends of the two couples (4) and (5) are embedded in a copper block so that they are at the same constant temperature. Similarly the cold ends of the thermocouple (4') and (5') are embedded in a copper block.

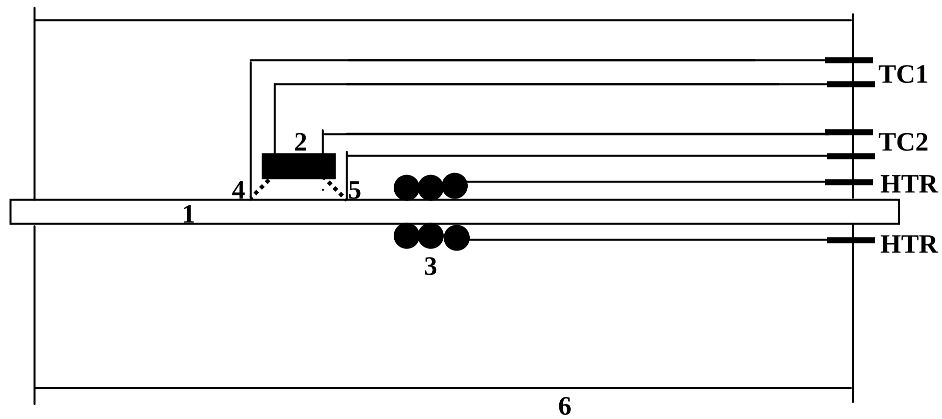


Figure:II.2.5.4 1 is brass tube; 2:Copper block in which cold junctions of the thermocouples 4 and 5 are embedded. Dotted line – constantan wire; continuous line Copper wire
3 Wire-wound heater
6 Box with lids at the ends The box is stuffed with cotton wool to prevent convection. The right lid carries terminals for leads. Two more thermocouples 4' and 5' are also provided to the right of the heater (not shown In Figure.)

The thermocouple leads are brought out to four pairs of banana terminals on the lid of the outer tube (6). Two pairs marked TC1 and TC2 correspond to the thermocouples (4) and (5). Those marked TC3 and TC4 correspond to the pair of thermocouples (5') and (4'). The heater leads are brought out to two big banana terminals. The rod is surrounded by a tightly wound layer of cotton. On this a layer of aluminium foil is wound. The box (6) is filled loosely with cotton to prevent convection currents.

The heater is connected to a power supply. A voltage of 5 Vis set on the power supply. The current through the heater coil will be approximately 0.5 amp. The current through the coil and the voltage across the coil gives the heat supplied. A switch in the source is toggled on for 5 minutes and off for five minutes regularly. This interrupts the current periodically. (An automatic electronic switch with adjustable period can also be used). One pair of thermocouples, either TC1 and TC2 or TC3 and TC4 is connected to the input terminals I1 and I2 on the DC differential amplifier. TC1 is farther away from the heater than TC2. So TC2 will indicate a higher temperature than TC1. The thermocouple voltages are amplified by a DC amplifier of amplification 100 and read on a milli-voltmeter.

For the first two cycles of heating no thermocouple readings are taken. In this time the heat will spread to the end of the rod and the temperature at the thermocouple junctions will start varying periodically with time.

We start counting the time from the instant the heater is switched on after two cycles. Call this instant zero. Then we start noting the DMM reading with the selector switch on the differential amplifier at I1. I1 is connected to TC1. **It is enough to note V_+ (i.e. positive reading) on DMM.** ***It is not necessary to note V_- by using the reversing switch. The offset, if any, will remain roughly constant. As we will find the amplitude and phase of the thermocouple signals at the period of heating by Fourier analysis, the nearly constant offset will make no contribution.*** Starting with 15 seconds after the heater is switched on (at time 1215 s) the amplified emf V_+ is noted every 30 seconds. ***One must not forget to switch off the heater at 1500 s and switch it on again at 1800s.*** **Thermocouple reading must be taken from 1215 s after switching on the heater to 1815 seconds. This will cover one period.**

After taking the reading at 1815 s, the selector switch is turned to I_2. This will now read TC2. **No readings are taken till the heater is switched on again at 2400s. .** Then readings of V_+ for the second thermocouple are taken for a full cycle.

Figure II.2.5.3 illustrates schematically the sequence in which readings are taken.

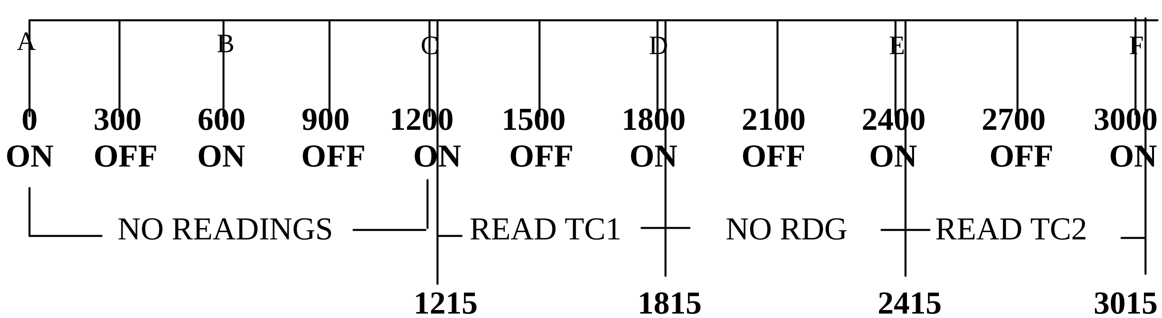


Figure II.2.5.5 Schematic showing how readings are to be taken
Numbers denote time in seconds. ON and OFF
Refer to switching heater on and off.

A sample set of readings is shown in Table II.2.5.1.

Table II.2.5.1
Power supply voltage 5 V

.

Period T	600	S						
Distance between thermocouple junctions				3	Cm			
time s	TC1 mV	TC1Cos	TC1Sin		Time s	TC2 mV	TC2Cos	TC2sin
1215	51.9	51.26098	8.119224		2415	58.3	57.58218	9.120438
1245	54.3	48.38126	24.65246		2445	73.6	65.57754	33.41476
1275	60.8	42.99094	42.99325		2475	86.8	61.37522	61.37852
1305	67.6	30.68749	60.23319		2505	98.3	44.62398	87.58762
1335	74.1	11.58826	73.18827		2535	107.4	16.79594	106.0785
1365	79.7	-12.4725	78.71802		2565	114.9	-17.981	113.4843
1395	85	-38.5945	75.73286		2595	121.6	-55.2128	108.3425
1425	89.3	-63.1497	63.13955		2625	127.4	-90.0927	90.07815
1455	93.1	-82.9566	42.25895		2655	132	-117.618	59.91601
1485	96.9	-95.7085	15.14874		2685	136.4	-134.723	21.32392
1515	100.3	-99.0634	-15.7015		2715	136.5	-134.817	-21.3685
1545	98.1	-87.4022	-44.5473		2745	121.4	-108.161	-55.1278
1575	91.8	-64.9037	-64.9211		2775	107.8	-76.2159	-76.2363
1605	84.9	-38.5328	-75.652		2805	97.3	-44.1607	-86.7013
1635	78.1	-12.2055	-77.1404		2835	88.2	-13.784	-87.1163
1665	73	11.43171	-72.0993		2865	81.1	12.70016	-80.0994
1695	68	30.88209	-60.583		2895	74.7	33.92488	-66.5522
1725	63.3	44.76827	-44.7515		2925	69.2	48.94098	-48.9226
1755	59	52.5747	-26.775		2955	64.3	57.29752	-29.1802
1785	55.5	54.81852	-8.67064		2985	60.1	59.36204	-9.38929
1815	52.1	51.45677	8.161559		3015	60.6	59.85183	9.4931
		-215.507	-6.63604			Sum	-333.451	130.2172
I1Cos	-10.77	I1sin	-0.33		I2cos	16.67	I2sin	6.51
	Amp1	10.78047				Amp2	17.89877	
	Phase1	3.172483				Phase2	2.769398	

Alpha 0.169

Beta 0.134

Diffusivity = 0.23 cm^2/s

The first column gives the time (measured from the time heater is switched on before readings are taken. Second column gives the readings of TC1 in mV from 1215 to 1815. In the third column are given values of (TC1 cos ($2\pi t/T$) where time t is from the first column and T is 600 s, the period of heating. Similarly the fourth column is TC1 Sin($2\pi t/T$). **The row marked sum gives the sum of all rows in the column 3 (or column 4) minus half the sum of the readings in the first and last rows.** The column value next to I1cos is the [(sum in column 3) x 30/600] where 30 seconds is the interval at which temperatures are measured. Similarly the value next to I1sin is the [(sum in column 4) x30/600]. Amp1 is $[(I1\cos)^2+(I1\sin)^2]^{1/2}$. Since both I_1cos and I_1sin are both negative, the angle is in the third quadrant. So phase 1 is Atan (I1sin/I1cos) + π.

A similar analysis of the readings TC2 gives Amp 2 and phase 2. The value of α is obtained from II.2.5.43 and the value of β from (II.2.5.44). The value of diffusivity is calculated from the values of α and β using (II.2.5.45). If the heater current is not switched off and on exactly with a period of 600 s and if the readings are not taken at exact intervals of30s,phase measurement will have a large error.

The aim of the experiment is (i) to illustrate Fourier analysis and (ii) to measure thermal diffusivity. Fourier analysis occurs whenever a function varies periodically either in space or in time or both. For example in a crystal the electron charge density varies periodically in space. X ray diffraction spots occur from different Fourier components of this charge density. Therefore it is necessary to understand how Fourier analysis is done.

Measurement of thermal diffusivity is very important for materials of poor thermal conductivity. Here the specimen will be thin and the frequency of periodic heating will be high. A whole branch of research called Photo-acoustic spectroscopy uses such periodic heating methods.

QUESTIONS:

1. I have an AC current, which is perfectly sinusoidal with a frequency f. If this signal is Fourier analyzed, at what frequencies will the Fourier components be non-zero?

2. White light is a mixture of colours. When it is passed through a prism the colours are separated. Does the prism Fourier analyze the white light?

3. Will heat diffuse faster in a poor conductor than in copper?

4. Indicate what uses photo-acoustic spectroscopy can be put to.

II.2.6 THERMOEMF ANALYSER

NEUTRAL TEMPERATURE OF
Fe-Cu THERMOCOUPLE

T G Ramesh and V Shubha
Materials Science Division
NAL, BANGALORE

National Aerospace Laboratories
Bangalore 560017
Council of Scientific & Industrial research

II.2.6 THERMO-EMF ANALYSER

1. General description

The Thermo-emf analyzer has been specially designed to study the temperature characteristics of standard and non-standard thermocouples. Several high temperature experiments in the curriculum of B.Sc and M.Sc (Physics) courses can be easily carried out using this precision equipment**.**

2. System

The block diagram of the system to measure thermo-emf of various thermocouples as a function of temperature is given in Figure II.2.6. 1. The system essentially consists of

1. High temperature furnace capable of reaching temperatures up to 800°C.

2. Thermo-emf analyzer unit consisting of a Precision DC Microvolt Amplifier for measuring low-level thermoelectric voltages. A two pole four way band switch provided in the front panel of the instrument (TC Selector Switch) is used to select various combinations of thermo- emf generated in a pair of dissimilar metals / alloys like Chromel-Alumel (Chr-Alu), Iron-Copper (Fe-Cu), Chromel-Iron (Chr-Fe) & Iron-Alumel (Fe-Alu).

3. A DC power amplifier to power the furnace

4. A specially designed Thermowell arrangement housing a reference or a standard thermocouple (Chromel-Alumel) and a sample thermocouple like Iron-Copper.

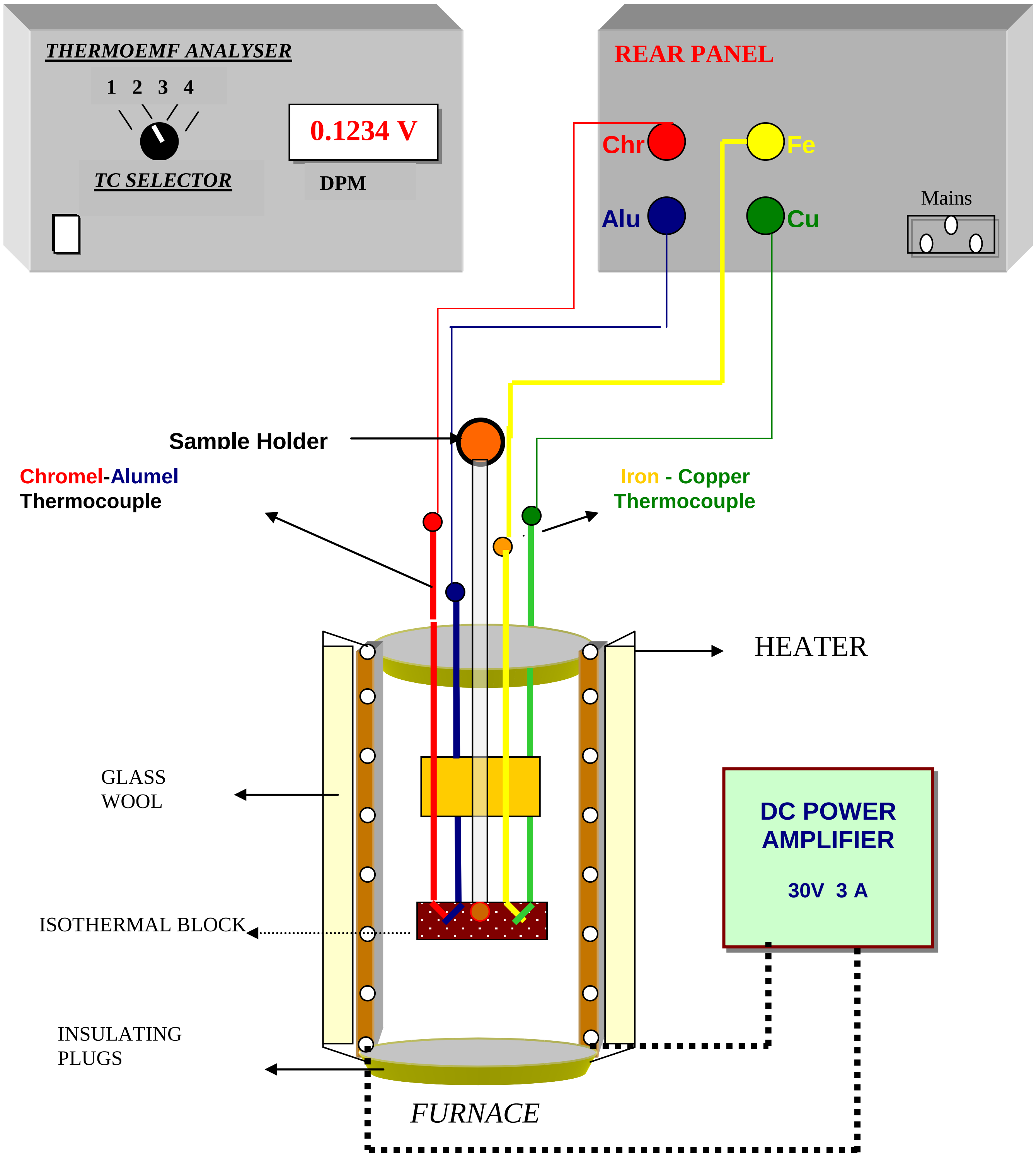


Figure II.2.6.1. BLOCK DIAGRAM OF THERMO EMF ANALYSER

3. Operating Instructions

Before turning on the equipment, make the following connections. The output at the rear panel of the DC Power amplifier unit is connected to the furnace using the moulded cable at the rear panel

1. through a three-pin socket and marked ‘To Furnace’. Use the supplied cable to connect the Power Selector to the Furnace. **Never connect the furnace socket directly to mains.**

2. The furnace assembly has a 4 –way terminal strip at the top cover for easy connection of thermocouples to the measuring system. The same arrangement can be used to verify the **Law of Intermediate Metals**. The following color code identifies the thermocouples coming out of the thermowell arrangement.

Reference thermocouple

Chromel ($+^{ve}$ leg) → **Red sleeves.**
Alumel ($-^{ve}$ leg) → **Blue sleeves.**

Sample thermocouple

Iron ($+^{ve}$ leg) → **Yellow sleeve.**
Copper ($-^{ve}$ leg) → **Green sleeve**.

The thermocouple wires are connected to the one side of the 4 –way terminal.

1. Make the appropriate connections between the thermocouple wires and the measuring instrument using the supplied four coloured inter connecting cable with lugs. The red wire should be connected to the terminal where the chromel wire with the red sleeve is connected in the 4-way terminal block of the furnace. The other end of the red wire is connected to the +ve binding post (Red connector at the back panel of the Thermo-emf Analyser). Similarly the blue wire is connected to Alumel wire with blue sleeve on the terminal end and to the –ve binding post (Blue connector at the back panel of the Thermo-emf Analyser).

2. The yellow wire is connected to the iron wire with the yellow sleeve on the four way terminal end and to the +ve binding post (Yellow connector at the back panel of the instrument). The green wire is connected to the Copper wire with green sleeve on the 4-way terminal end and to the –ve binding post (Green connector at the back panel of the instrument).

3. Turn on the mains of the equipment. Allow for a warm up period of 5 minutes before starting the measurements. Note the zero readings in all the four channels of the TC Selector. Typically the readings would be within ± 0.02 mV.

4. EXPERIMENTS WITH THERMOCOUPLES

Experiment Number 1:

Aim: Measurement of thermoemf of Iron-Copper (Fe-Cu) thermocouple as a function of temperature.

Method:

Chromel Alumel and Iron Copper thermocouples are kept in close proximity in a cell and the Thermowell assembly (Fig II.2.6.2) is then inserted into the center of the furnace. The heating coil has a nominal resistance of 6Ω and typical 40 to 50 watts is sufficient to attain temperatures around 500ºC at the centre of the furnace. The thermoemf developed across Chromel-Alumel (which is a standard thermocouple) is amplified using an instrumentation amplifier (Gain=100). Temperature can be measured by keeping the TC Selector switch in the position marked **'1'** and reading the voltage in the 4 ½ digit DPM provided in the unit.

Normally thermocouple voltages are measured with reference junction held at 0ºC. These values are tabulated in the NBS tables and used to read out the temperature. In the present experimental arrangement the concept of a single junction thermocouple is used where it is understood that the reference junction is at room temperature. Hence to use the NBS tables a voltage value corresponding to the room temperature has to be added to the measured voltage from the single junction thermocouple. This can be carried out by measuring the room temperature from a thermometer and the thermoemf corresponding to this temperature can be read off from the tables.

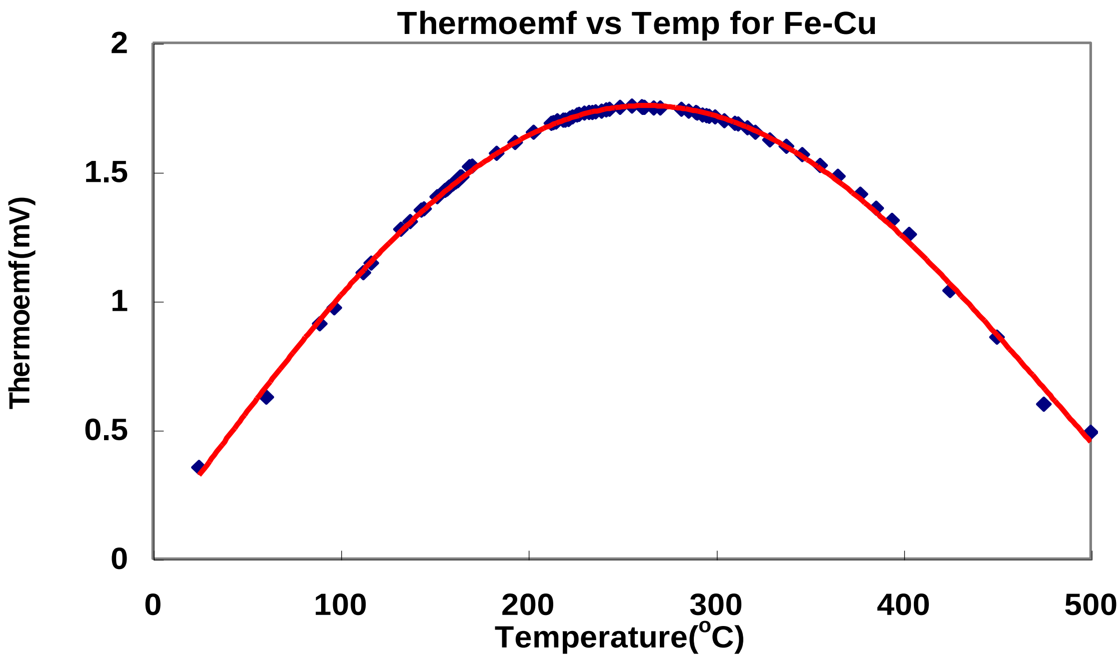


Figure II.2.6.2: Thermoemf vs Temperature data for

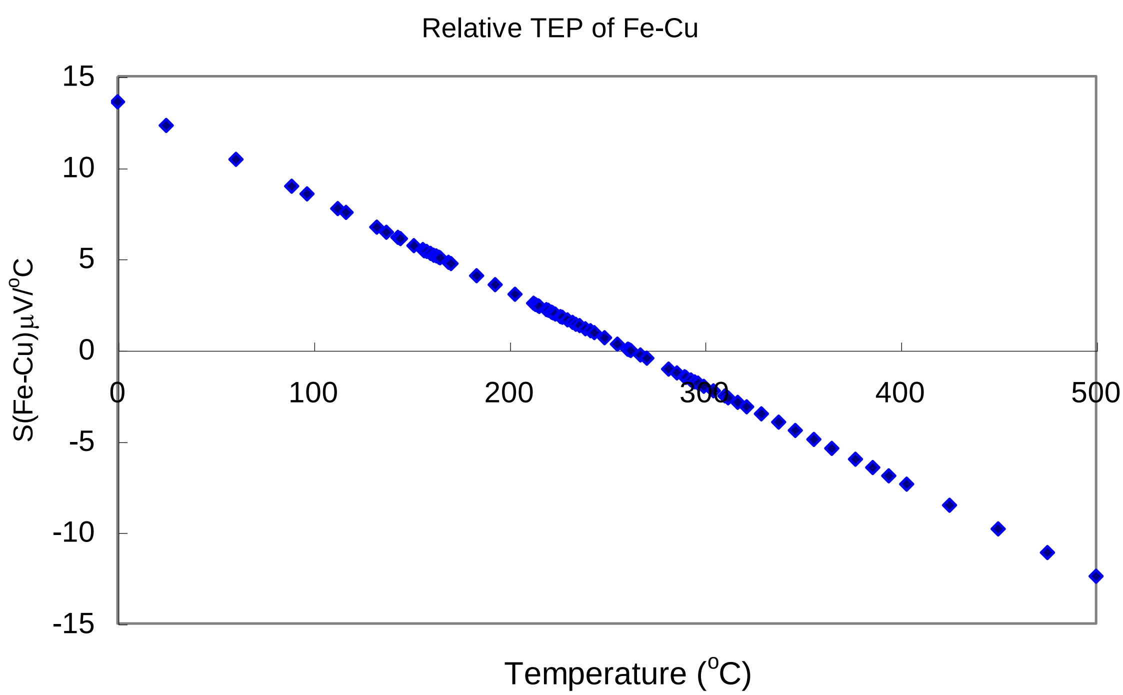


Figure II.2.6.3: Relative thermopower of Fe-Cu vs temperature

Typically for Chromel-Alumel thermocouple, room temperature of 25ºC corresponds to 1.0mV. For example, a 7mV read from single junction Chromel-Alumel thermocouple corresponds to 8mV (Reference junction at 0ºC) and the temperature as read by NBS tables is 196.58ºC.

The thermo-emf from Iron-Copper thermocouple is again amplified through same precision DC amplifier (Gain=100) and read through a DPM by keeping the TC Selector switch in position **'2'**. For this thermocouple, which has a sensitivity of $\approx$ 13.6 µV/ºC near room temperature a value of the order of 0.34mV ($13.6\times 25=340$ µV=0.34 mV) has to be added to the measured value from the single junction Fe-Cu thermocouple.

The entire experiment up to 500ºC can be carried out in about 50 minutes. It is also worth mentioning that the special Thermowell arrangement (Figure 1) facilities fast equilibration and reading can be taken during the heating run without waiting for the steady state.

5. Results:

Thermoemf of Fe-Cu is plotted against the temperature (through the NBS tables for Chromel-Alumel thermocouple). The typical plot of this data is given in the Figure II.6.2.2. An equation of the type,

$\mathbf{E_{Fe\text{-}Cu} = At + Bt^2}$ where A and B are the constants fits this data very well.

$A = 13.6\mu V/^{o}C \qquad B = -0.026\mu V/^{o}C$

The full curve is a fit of the experimental data points with the above constants for A and B.

The slope of the thermo-emf versus temperature curve gives the relative thermopower of Fe-Cu as a function of temperature.
The variation of $S_{Fe\text{-}Cu}$ as a function of temperature is given in Figure II.2.6.3.

$$dE_{Fe\text{-}Cu} / dt = A + 2Bt = S_{Fe\text{-}Cu}$$

$$\mathbf{S_{Fe-Cu} = S_{Fe} - S_{Cu}}$$

where S_{Fe} and S_{Cu} are the absolute thermopowers of Fe and Cu respectively. Knowing S_{Cu} (through Thomson effect studies), which is given by the relation

S_{Cu} = Ct + D   C = 4.989×10^{-3} μV/ºC   D = 1.7 μV/ºC , S_{Fe} can be determined.

The absolute TEP of Fe and Cu as a function of temperature are given in Figure II.2.6. 4. The neutral temperature corresponds to the case where the curves cross, i.e., $S_{Fe} = S_{Cu}$.

The relative TEP of standard thermocouple Type K (Chromel- Alumel) and the absolute TEP of Chromel and Alumel as a function of temperature are given in Figure II.2.6.5. It may be noted that the relative TEP of Type K has negligible variation with temperature over the entire temperature range up to 1000°C in contrast to the behaviour of Fe-Cu thermocouple (Figure 3). Further the curves depicting the behaviour absolute TEP of Chromel and Alumel never cross and run nearly parallel. Thus standard thermocouples doesn't exhibit a neutral temperature.

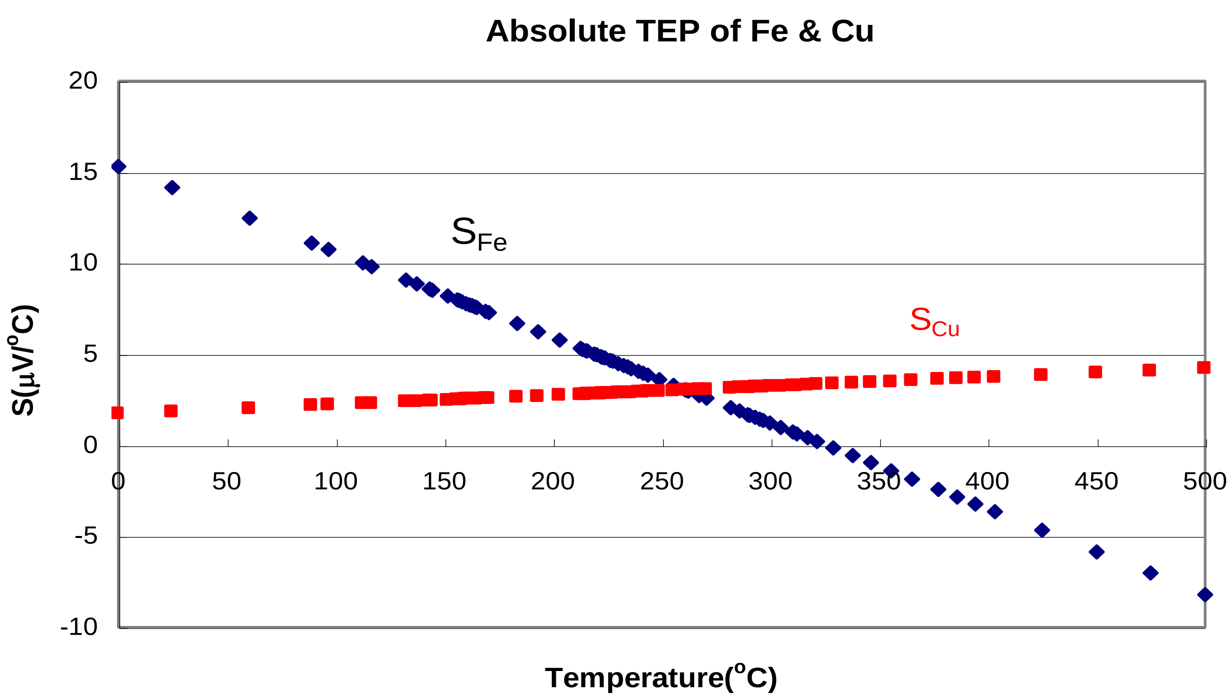
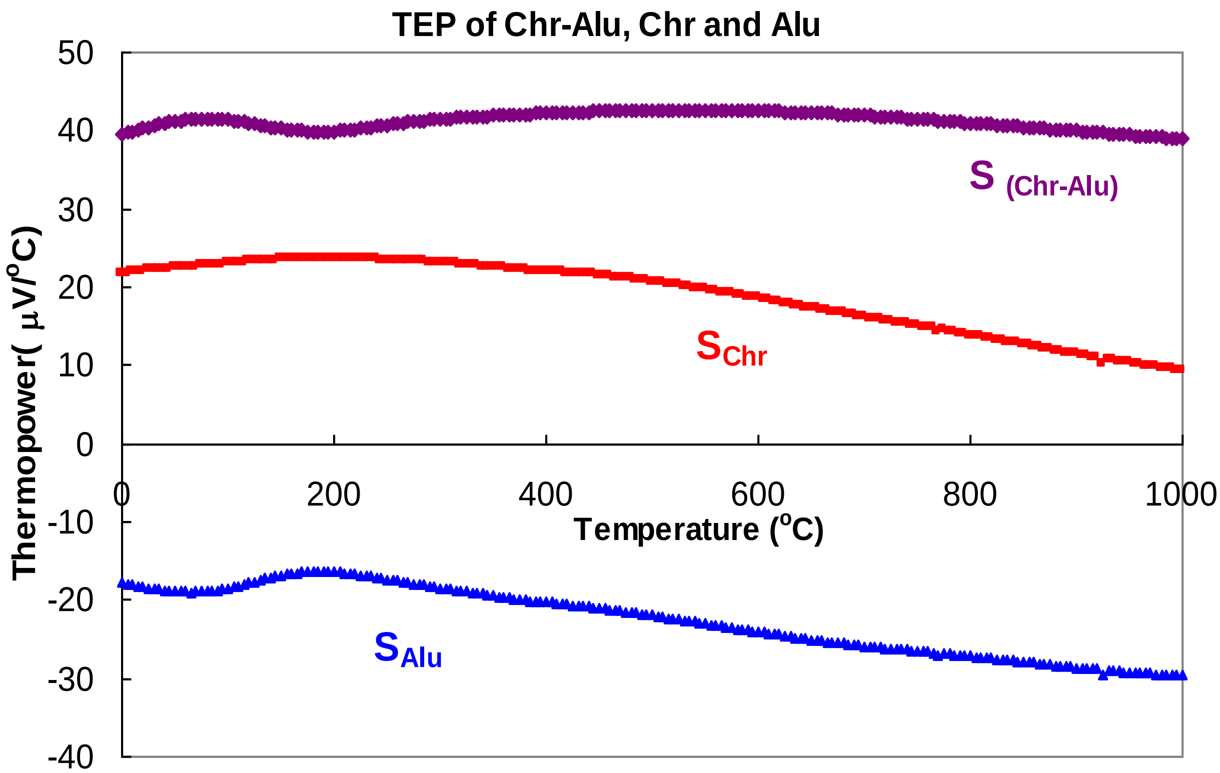


Figure II.2.6.4. Absolute TEP of Fe and Cu vs Temparature



FigureII.2.6. 5: Absolute TEP of Chromel, Alumel and Relative TEP of Chromel-Alumel vs Temperature

Significance of the experiments:

1. The coefficient B is negative (for Fe-Cu) and has a rather large magnitude leading to a neutral temperature and an inversion temperature.

2. Historically, this type of data led Thompson to predict another thermoelectric effect, now known as the Thompson effect, which can be observed in a single conductor subjected to a temperature gradient.

3. Graphically if the absolute thermopower versus temperature for the two constituents of the thermocouple **cross** each other, then there would be a **neutral temperature and subsequently an inversion point.**

4. Another system which exhibits a 'Neutral temperature' and 'Inversion point is an Iron-Silver (Fe-Ag) thermocouple. The behavior of absolute thermopower of Silver is very similar to that of Copper.

5. Another deduction from the fit of the quadratic expression is that the relative thermopower of Fe-Cu thermocouple (slope of the thermoemf versus temperature plot), is **linearly decreasing** with temperature (Figure 3).

6. **The existence of Neutral temperature and Inversion point is not universal. Only some combinations of metals and alloys exhibit this phenomenon.**

Experiment Number 2:

Aim: Verification of the Law of Intermediate Metals :

Principle: According to the Law of Intermediate Metals, the thermoemf generated in a thermocouple A-B operating between temperatures T_2 and T_1 is equal to the Algebraic sum of the thermoemf generated in couples A-C and C-B operating between the same temperatures. Thus

$$\mathbf{E_{AB}(T_2, T_1) = E_{AC}(T_2, T_1) + E_{CB}(T_2, T_1)}$$

Method:

The thermowell assembly, as explained earlier, ensures that the thermocouple junctions of different combinations like Chr-Alu, Chr-Fe, and Fe-Alu are essentially at the same temperature T_2, while the other junction not made (but hypothetical) is at the room temperature T_1. Internal connections have been made in the unit to verify the law of intermediate metals for the combination Chr-Alu (Position 1 in TC Selector Switch at the front panel of the unit), Chr-Fe (Position 3 in TC Selector Switch) and Fe-Alu. (Position 4 in TC Selector Switch). Thus

$$\mathbf{E_{Chr\text{-}Alu}(T_2, T_1) = E_{Chr\text{-}Fe}(T_2, T_1) + E_{Fe\text{-}Alu}(T_2, T_1)}$$

Data collection for this experiment is done essentially in the same way as in experiment no.1. The TC selector is used to select the thermocouple voltages of combinations like Chr-Alu, Chr-Fe & Fe-Alu. It is possible to combine both experiment 1 & 2 by collecting data for all the positions of the EMF selector.

A typical set of experimental data obtained using this apparatus is given in Table II.6.2.1. Excellent agreement between the values in column 1 with that in column 5 (within the resolution of ± 0.03 mV) proves the vadility of the Law of Intermediate Metals.

$E_{Chr-Alu}$ Position 1 of TC Selector	E_{Fe-Cu} Position 2 of TC Selector	E_{Chr-Fe} Position 3 of TC Selector	E_{Fe-Alu} Position 4 of TC Selector	Sum of columns (3+4)
0.52	0.14	0.11	0.42	0.53
1.0	0.27	0.21	0.81	1.02
3.0	0.76	0.74	2.31	3.05
4.03	0.96	1.05	3.00	4.05
5.32	1.19	1.51	3.85	5.36
7.02	1.39	2.20	4.84	7.04
10.02	1.5	3.63	6.41	10.04
11.82	1.41	4.58	7.26	11.84
13.5	1.23	5.52	7.99	13.51
14.5	1.08	6.1	8.42	14.52
15.5	0.91	6.69	8.80	15.49
16.0	0.81	6.99	9.00	15.99
16.5	0.71	7.3	9.2	16.5
17.0	0.6	7.61	9.39	17
17.5	0.48	7.92	9.58	17.5
18.0	0.36	8.22	9.77	17.99
19.04	0.16	8.82	10.22	19.04

TableII.6.2.1. Typical experimental data using Thermoemf Analyser

NBS TABLE FOR THERMOPOWER OF CHROMEL-ALUMEL THERMOCOUPLE IS GIVEN IN THE APPENDIX AT THE END OF THE MANUAL.

SECTION II.3

D C EXPERIMENTS WITH THE REGULATED POWER SUPPLY AND THE CONSTANT CURRENT SOURCE

II.3.1 HIGH RESISTANCE BY LEAKAGE

1. INTRODUCTION:

The tantalum capacitors used in the circuit are expected to have an internal resistance R' of the order of a few hundred megohms. If an external resistance R is connected across the capacitor the effective resistance through which the capacitor discharges is given by

$$1/R'' = 1/R + 1/R' \tag{II.3.1.1}$$

This experiment aims to measure a high resistance of the order of a few tens of megohms or higher. When a capacitor, charged to a DC voltage V_0, is discharged through a high resistance R”, the voltage decreases exponentially with time as

$$V(t) = V_0 \exp(-t/\tau'') \tag{II.3.1.2}$$

where

$$1/\tau'' = 1/CR'' \tag{II.3.1.3}$$

Please refer to the discussion of the High Resistance circuit in Section I.II. When the first toggle switch is in position O, R is not connected and so 1/R” = 1/R' and the time constant for decay of voltage is

$$1/\tau' = 1/CR' \tag{II.3.1.4}$$

A plot of ln(V) against t in this case will give a slope of $-1/\tau'$.

When the first switch is in position S, the resistance R is connected parallel to the internal resistance of the capacitor and so a plot of ln(V) against t in this case will have a slope given by $-1/\tau''$, where

$$1/\tau'' = (1/C) \times (1/R'+1/R) \tag{II.3.1.5}$$

The difference in slopes is

$$(1/\tau'') - (1/\tau') = 1/CR \tag{II.3.1.6}$$

This difference will be large if R'>>R. So one can measure by this technique resistances with a value R which is a small fraction of R'. Since the tantalum capacitors are expected to have an internal resistance of a few hundred megohms, one may measure resistances of the order of 100 megohms or less by this apparatus.

2. APPARATUS REQUIRED

High resistance by leakage circuit and a DMM which can measure less than 2V DC to 3 decimal places.

3. EXPERIMENTAL PROCEDURE:

The circuit for High resistance by leakage described in I.11 is used for the measurement. The circuit is switched on. For measuring the resistance connected internally one puts the toggle switch 3 at the position marked **INT.** The selector switch is put in the position 10 **μf.** A DMM in the DC 2 Volts range is connected to the banana terminals marked **DMM.** The toggle switch 2 is put in position marked **CH**. When the push switch is pressed the DMM will indicate approximately 2 volts. This voltage is noted. Toggle switch 1 is put in position O. Then the toggle switch 2 is put in position **DIS.** The condenser of 10 μf will now start discharging through its internal resistance R'. Every five minutes the push switch is pressed and the voltage reading on the DMM noted.

Then the condenser is charged again by putting the toggle switch 2 in the position CH. The switch 1 is put in position S and the switch 2 is put in position DIS. Then the time versus voltage is recorded as before **but now at intervals of two minutes**. The condenser will discharge faster because now the resistance R is connected in parallel to the internal resistance R'.

A plot of ln (V) is made against time t in seconds and a straight line is fitted to it for each set of readings. If α' is the slope of the line when the switch 1 is in position O and α'' is the slope when the switch 1 is in position S, then

$$1/\tau' = -\alpha' \qquad \text{(II.3.1.7)}$$

and

$$1/\tau'' = -\alpha'' \qquad \text{(II.1.3.8)}$$

The value of the high resistance R is then found from

$$R = (1/C)\,(\alpha' - \alpha'') \qquad \text{(II.1.3.9)}$$

A sample set of data is given below.

Table II.3.1.1
High resistance by leakage
C = 10 μfd

Switch 1 in position O				Switch 1 in position S		
Time s	V	ln(V)		Time s	V	ln(V)
0	1.994	0.690143		0	1.88	0.631272
300	1.969	0.677526		120	1.706	0.534151
600	1.947	0.666290		240	1.551	0.438900
900	1.926	0.655445		360	1.411	0.344299
1200	1.906	0.645007		480	1.285	0.250759
1500	1.887	0.634988		600	1.172	0.158712
1800	1.866	0.623797		720	1.071	0.068593
2100	1.85	0.615186		840	0.979	-0.021220
2400	1.834	0.606499				

A plot of ln (V) against time is shown in Figure II.3.1.1 below.

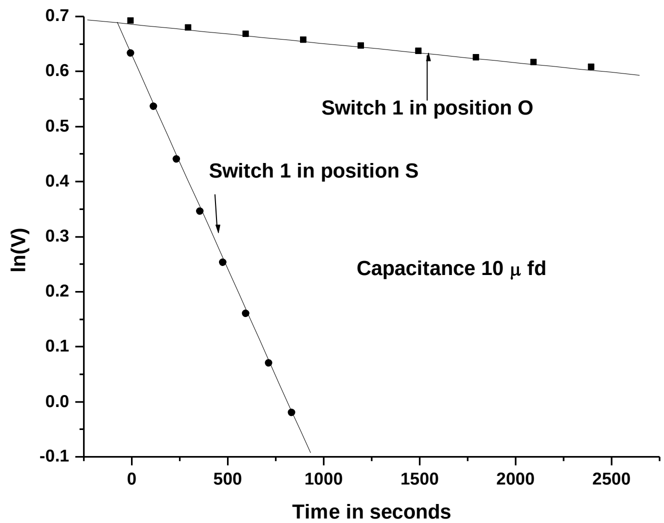


Figure II.3.1.1`: Plot of ln (V) across the capacitance as a function of time

We see that the plots are straight lines with slopes $\alpha' = -3.48\text{x}10^{-5}$ /s and $\alpha'' = -7.77\text{x}10^{-4}$/s. The respective standard deviations in the slopes are $6.38\text{x}10^{-7}$ and $4.26\text{x}10^{-6}$/s. From this using

$$1/\tau' = -\alpha' = 1/CR'$$

we get

$$R' = -1/C\alpha'$$

This is the internal resistance of the 10 microfarad capacitance. Substituting for α' we get

$$R' = 2865 \text{ Megohms.}$$

The high resistance R is

$$R = (1/(C [\alpha'-\alpha'']))$$

The value of R comes out to be 135 Megohms.

The error in R is calculated from

$$\text{Error in R} = \text{Rx (sum of the standard deviations)}/ (\alpha''-\alpha')$$

This comes out to be 0.9 Meg.

Similar measurements were made with C = 47 microfarad and C = 100 microfarad capacitors. The results are shown in Tables II.3.1.2 and II.3.1.3. Plots of ln (V) vs. time for these data are shown in Figures II.3.1.2 and II.3.1.3 respectively.

Table II.3.1.2
C = 47 microfarad

Switch 1 in position O				Switch 1 in position S	
Time secs	V	ln(V)	Time secs	V	ln(V)
0	1.85	0.615186	0	1.833	0.605954
120	1.794	0.584448	120	1.754	0.561899
240	1.745	0.556755	240	1.68	0.518794
360	1.701	0.531216	360	1.612	0.477476
480	1.659	0.506215	480	1.548	0.436964
600	1.62	0.482426	600	1.485	0.395415
720	1.582	0.458690	720	1.427	0.355574
840	1.547	0.436318	840	1.371	0.315540
960	1.512	0.413433	960	1.318	0.276115

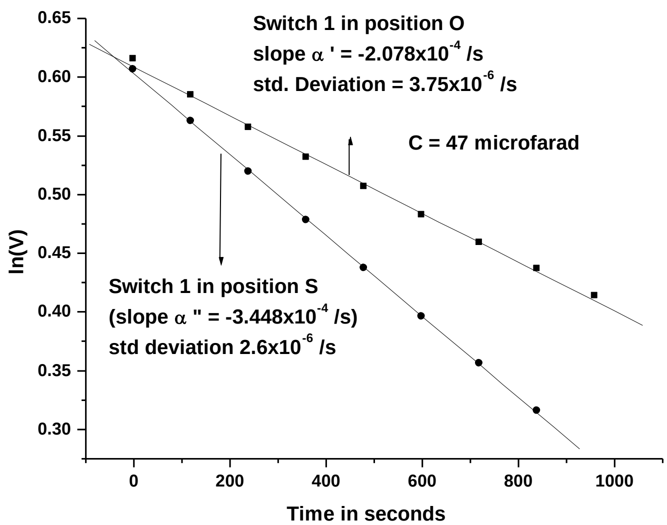


Figure II.3.1.2

Table II.3.1.3

C = 100 microfarad

Switch 1 in Position O			Switch 1 in position S		
Time secs	V	lnV	Time secs	V	ln(V)
0	1.995	0.690644	0	1.943	0.66423
600	1.987	0.686626	300	1.885	0.63393
1200	1.978	0.682086	600	1.836	0.60759
1800	1.97	0.678034	900	1.795	0.58501
2400	1.961	0.673455	1200	1.751	0.56019
3000	1.953	0.669367	1500	1.709	0.53591
			1800	1.669	0.51222
			2100	1.629	0.48797
			2400	1.591	0.46436

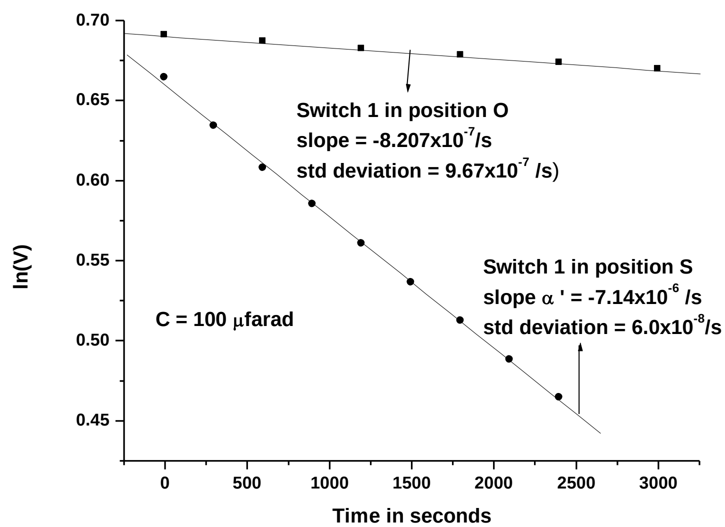


Figure II.3.1.3

From the values of α' and α" in the two figures the values of R' and R were calculated. From the sum of the standard deviations the error in R is calculated. These are collected in Table II.3.1.4.

Table III.3.1.4

Collected values of R’, R and error in R

Capacitance		R' in Meg	R in Meg	Error in R
100	μfd	1401	133	1.8
47	μfd	102	158	7
10	μfd	2865	135	0.9

We see from this table that both for the 100 and 10 microfarad capacitors the internal resistance is in the 1000 megohm range. So they both have given an accurate value for the resistance R which is in the 100 meg range. On the other hand the 47 microfarad capacitor has a lower internal resistance than the resistance R. The values of are close to each other. The value of R determined from the difference between α’ and α” has a larger error.

In our experience 10 μfd Tantalum capacitors usually have a resistance in Gigohms while 47 and 100 μfd capacitors have lower resistances. So to measure high resistance in the 100 Meg range 10 μfd capacitors are suitable. But for measuring resistance in the 10 to 100 Megohm range 47 and 100 μfd capacitors are quite OK.

One may connect an external high resistance to the terminals marked EXT and putting the switch 3 to position EXT.

II.3.2 LOAD REGULATION OF A CONSTANT CURRENT SOURCE

1. INTRODUCTION:

The constant current source described in Section I.4 will produce a constant current in a *varying load over a range of load values*. This is verified in this experiment.

2. APPARATUS REQUIRED:

Constant current source, a variable resistance provided by Ajay Sensors for low currents and a resistance box or rheostat 1 to 1000 Ohms which can carry a current of 0.5 A , a DMM reading DC Volts and another DMM reading in milli-amperes up to 200 mA.

3. PROCEDURE:

LOW CURRENT:

A variable load resistance box supplied by Ajay Sensors is used for this purpose. The top view of the box is shown in Figure II.3.2.1 below:

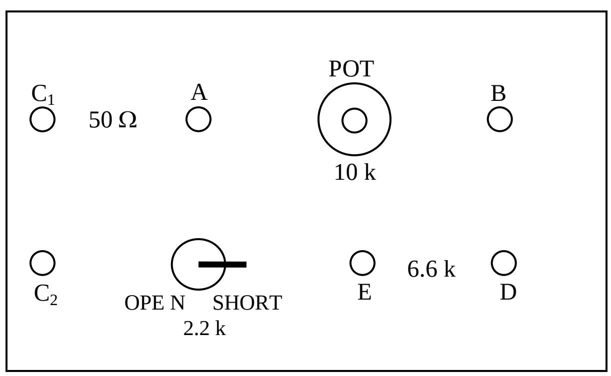


Figure II.3.2.1: Top view of the variable load
Resistance box

The figure is self explanatory. Connect the constant current source through a DMM reading up to 20 mA to the two terminals marked C_1 and C_2. Connect terminal B to terminal E with a wire so that the 6.6 k between E and D is not included in the circuit. Put the toggle switch in the short position. The total load in the circuit is the resistance of 50 Ohms between C_1 and A and the resistance of the pot which can be varied from 0 to 10 k. Adjust the pot to its minimum resistance position. Put the switches on the constant current source in the low current position. Turn the low current potentiometer on the constant current source to the left extreme. **Then switch on the source.** Set the current on the DMM to nearly 5 mA. Connect a DMM to read DC voltage across the terminals C_1 and C_2. Adjust the potentiometer on the variable resistance box till the DMM reads a voltage of approximately 2 V. Note the current and the voltage. The load resistance is Voltage divided by current. Then increase the voltage on the DMM in steps of 2 Volts by turning the pot on the variable resistance box and note the voltage and the current. If the DMM Voltage reaches twenty volts change the setting on the DMM to a higher range.

A sample set of readings are given below.

TABLE II.3.2.1

Load Regulation in the low current range

Voltage V in Volts	Current I in mA	Load R = V/I in Ohms	Voltage V in Volts	Current I in mA	Load R = V/I in Ohms
2.01	5.14	391	16.1	5.14	3132
4.04	5.14	786	18.06	5.14	3514
6.03	5.14	1173	20	5.13	3899
8.05	5.14	1566	20	4.99	4008
10.03	5.14	1951	20	4.78	4184
12.06	5.14	2346	20	4.52	4425
14.02	5.14	2728	20	4.21	4751

In Figure II.3.2.1 (a) the current from the source is plotted as a function of the load resistance. We see that the current remains constant until the load reaches a value of about 3900 Ohms. For the range of load from 0 to 3900 Ohms, the output voltage of the constant current source increases proportional to the load to keep the current constant. At the maximum load of 3900 Ohms the output voltage is 20 V. For a further increase in load the voltage remains constant and the current falls sharply. In Figure II.3.2.1 (b) the voltage output of the constant current source is plotted against the load resistance. Figure II.3.2.1 shows the variation of current with load.

The experiment is repeated for a current of about 10 mA till the load reaches about 2. kilohms.

The constant current source can regulate current till its voltage output reaches 20 V.
So the maximum load to which a current of 10 mA can be kept constant is R_{mx}, equal to $(20/(10x10^{-3}))$ 2 k.

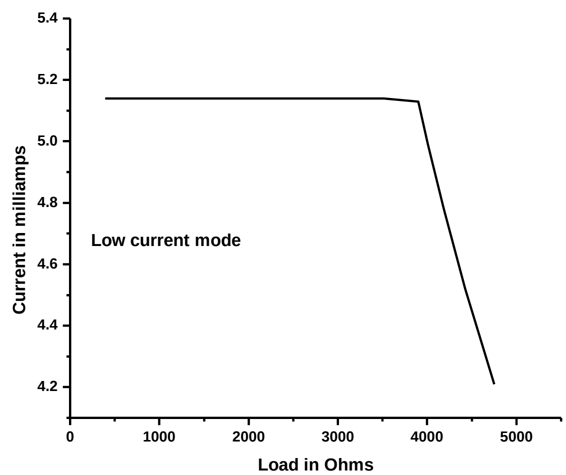


Figure II.1.2.1a: Load regulation of current at low currents
Current is 5.14 mA.

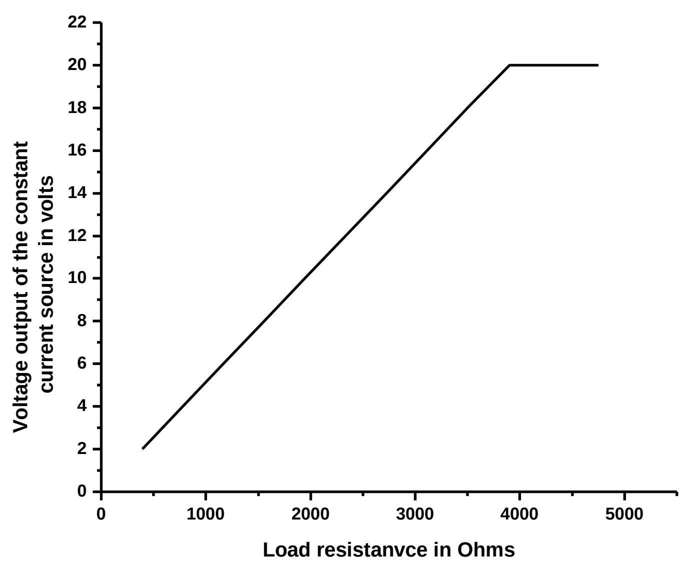


Figure II.3.2.1 (b) Voltage output of the constant
current source as a function
of the load resistance

HIGH CURRENT :

First turn the two control knobs at the top of the front panel of the constant current source (CCS) to the left and put the two range switches on the front panel to HIGH. Connect a decade resistance box (1 to 1000 Ohms, 200 mA maximum current) to the constant current source through a DMM in the DC 200 mA range. (In a decade resistance box it is safe to limit the current to this maximum value. A higher value may damage the resistances. If one uses a 100 Ohm rheostat which can carry a current of 1 amp one may carry out the experiment at a higher current.) Set the constant current source to give a current of approximately 100 mA. Pull out the plugs in steps of 20 Ohms and note the current. One may also connect a DMM in the 20 V range across the load to monitor the voltage across the load. For each load note the current and the voltage.

A sample set of readings are given in Table II.3.2.2.

TABLE II.3.2.2
Load Regulation in the high current range

Load Ohms	Current Amps	Voltage Volts	Load Ohms	Current Amps	Voltage Volts
20	0.102	2.03	160	0.1	16.16
40	0.102	4.05	180	0.1	18.09
60	0.102	6.08	200	0.099	20
80	0.101	8.09	220	0.098	21.7
100	0.101	10.1	240	0.098	23.6
120	0.101	12.08	260	0.097	25.3
140	0.1	14.1	280	0.09	25.3

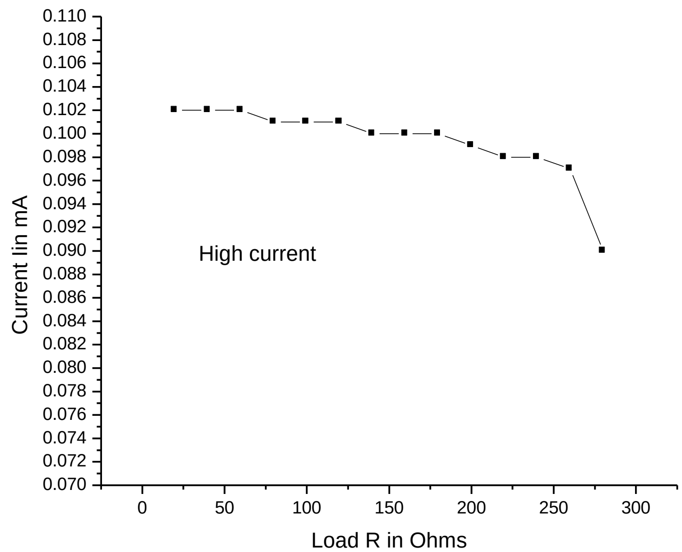


Figure II.3.2.3 (a): Load regulation for current of 0.100 amps.

Figure II.3.2.3 (b) shows how the voltage output from the current source varies in the high current range.

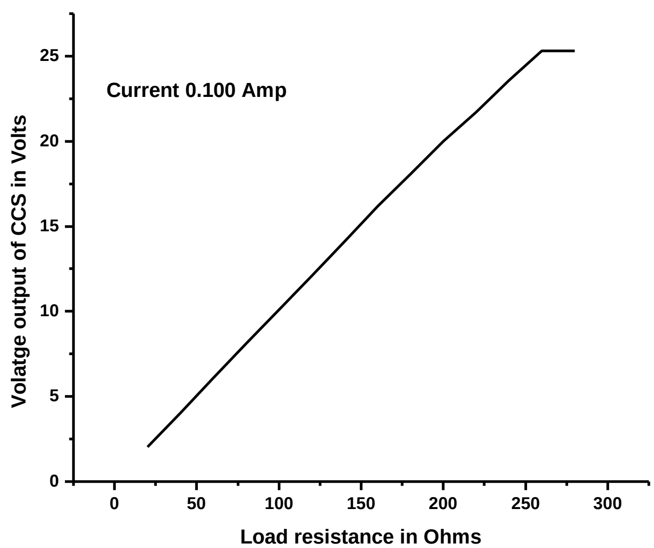


Figure II.3.2.3 (b) Variation of output voltage
as a function of load

We see that the regulation in the high current mode is only within 3% till the voltage reaches about 25 V. Thereafter the current decreases rapidly.

II.3.3 TEMPERATURE COEFFICIENT OF RESISTANCE OF COPPER

1. INTRODUCTION:

The electrical conductivity for a metal and a semiconductor is given by the expression

$$\sigma = ne\mu \qquad \text{(II.3.3.1)}$$

Here n is the number of charge carriers per unit volume of the material, e is the magnitude of the charge and μ is a quantity called the mobility of the charge carrier. The mobility is the drift velocity acquired by the charge carrier per unit applied electric field. The mobility is determined by the rate at which the charge carriers are scattered by impurities, defects in the material and by vibrations of the atoms. At high temperature and in pure materials, the dominant scattering process is the collision of the charge carrier with the vibrating atoms. As the temperature increases the atoms vibrate more vigorously and so the mobility decreases.

In a metal, the valence band is completely filled while the conduction band is partially filled with electrons. ***A completely filled band will not contribute to electrical conduction.*** The electrons in the conduction band of a metal are responsible for electrical conduction in metals. The number of electrons per unit volume in a metal (of the order of $10^{23}/cm^3$) is not dependent on temperature. So the temperature variation of conductivity σ arises only through the variation of mobility with temperature. **Since the mobility decreases as the temperature increases,** ***the conductivity of a metal comes down as it is heated.*** Conductivity σ is the reciprocal of specific resistance s. ***So as the temperature increases resistance of a metal increases. This is characteristic of metallic behavior.***

If the resistance of a length of metallic wire is R (T_0) at some temperature T_0 and changes to R (T) at a higher temperature T, then the mean temperature coefficient of resistance is defined by

$$\alpha = (1/R(T_0))\,[R(T) - R(T_0)]/[T - T_0] \qquad \text{(II.3.3.2)}$$

This implies that in this temperature range we may write

$$R(T) = R(T_0)\,[1 + \alpha(T - T_0)] \qquad \text{(II.3.3.3)}$$

(This resistance can vary from insert to insert).

2. APPARATUS REQUIRED

Regulated DC power supply, temperature controller, furnace, insert for measuring TCR of Copper, Constant current source and a DMM that can measure DC in millivolts.

3. EXPERIMENTAL PROCEDURE

For this experiment fine enamel coated copper wire (gauge 44) is used. It has a room temperature resistance of about 3.3 Ω/m. Approximately 6 meter long wire is wound on an insulating tube and fitted into a hole on a rectangular aluminium block. The block carries a Pt 100 temperature sensor. A four terminal arrangement is used to measure the resistance of the copper wire. The two ends of the copper wire are soldered to two current leads. Two other leads are soldered to the copper wire at two points lying between the soldered current lead joints. These leads are used for measuring the voltage drop across the copper wire when a current is passed through it. The total resistance of the sample is about 20 ohms at room temperature. A current of 5 mA will give rise to a voltage of about 100 mV. Such a voltage can be easily read on a digital multi-meter. Furthermore, the temperature coefficient of resistance for copper is 0.0043/^{o}C (referred to the resistance value at 0 C). This means that a rise in temperature of 10^{o}C would produce about 6 mV change in voltage. This change is also measurable on a digital multi-meter.

The aluminum block is suspended inside a furnace. The furnace is heated by passing a current through it from a regulated power supply. The voltage of the regulated supply is adjusted so that the temperature at the center of the furnace increased slowly, at a rate of about 1^{0} C per minute. The required voltage for our furnace is about 7.5 V. The two open ends of the ceramic tube of the furnace should be lightly plugged with wads of cotton to prevent convection currents. **The voltage is gradually increased as the furnace temperature rises to maintain approximately a uniform heating rate**. At a temperature of 100 C, this voltage will not exceed 12 Volts.

A constant current source is connected to the current lead terminals of the copper coil. on the terminal block. The current is set at 5 mA as read on the panel meter of the constant current source. The voltage lead terminals of the copper coil are connected to a DMM in the 200 mV range. The temperature of the furnace is read on the temperature indicator with the platinum thermometer. For every ten degree change in temperature the digital multi-meter reading is taken.

A sample set of readings is given in Table II.3.3.1. Column 1 of the table indicates the temperature measured by Pt 100 thermometer. Column 2 gives the voltage in mV read on the DMM. Since the source supplies a constant current through the range of measurement, the voltage across the coil is proportional to its resistance. This measured voltage is plotted against temperature in Fig. II.3.3.1. A straight line fit to the curve is also shown.

Table II.3.3.1
Current through coil = 5.0 mA

Temp C	V mV Meas.	V mV calc	Error^2
29.7	156.2	156	4.00E-02
40.5	162.4	162.9	2.50E-01
50	168.7	168.9	4.00E-02
60	175.1	175.3	4.00E-02
70	181.7	181.6	1.00E-02
80	187.7	188	9.00E-02
90	194.9	194.3	3.60E-01
100	201.2	200.7	2.50E-01
110	207.5	207.1	1.60E-01
120	214	213.4	3.60E-01
130	219.1	219.8	4.90E-01
140	225.7	226.1	1.60E-01
150	232.3	232.5	4.00E-02
		Variance	1.91E-01
		Std Dev	4.37E-01

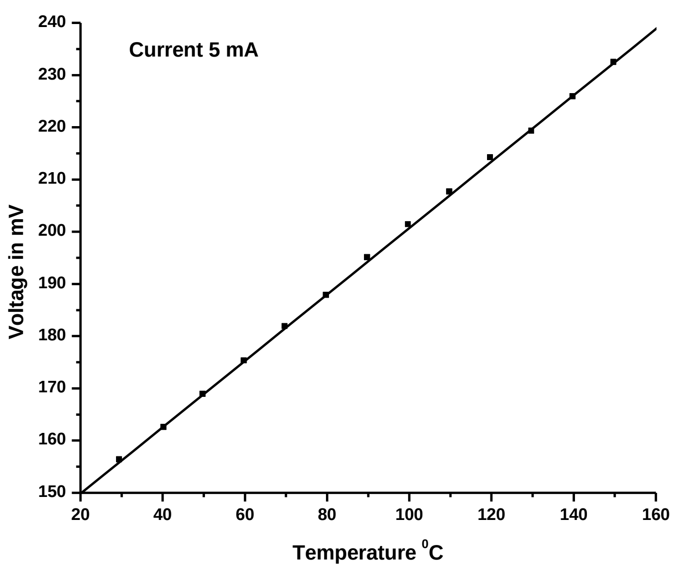


Figure II.3.3.1 Plot of voltage across the coil as a function of temperature

The slope of the graph is 0.636 mV/^{0}C and the intercept V (0) (Voltage across the coil at 0^0C for a current of 5 mA through the coil) is 137.1 mV. Since the resistance is proportional to the voltage we may calculate the temperature coefficient of resistance α from equation II.3.3.1 by replacing T_0 by zero, R (T_0) by V (0) and [(R (T)-R (T_0))/ (T-T_0)] by the slope of the voltage vs. temperature graph.

α comes out to be

$$\alpha = \text{slope/intercept} = 0.636/137.1 = 4.64\text{x}10^{-3}/\text{C}$$

To see how well the straight line fits the observed values in Fig. II.3.3.1, we calculate the voltage from the equation

$$V_{calc}\ (T) = A+BT$$

where B is the slope and A is the intercept at the temperatures recorded in the first column of this table. These values are given in the third column headed $V_{mV\ cal}$. By error we mean the difference between the measured and calculated values of the voltage. Column 4 shows the square of error at each temperature.

The variance is

$$\text{Variance} = \Box\xi^2 /(N-1) \qquad \text{(II.3.3.4)}$$

where N is the number of measurements and

$$\xi^2 = \sum_{\iota=1}^{N} (V_{i\ meas} - V_{i\ calc})^2 \qquad \text{(II.3.3.5)}$$

and the standard deviation σ is

$$\sigma = (\text{Variance})^{0.5} \qquad \text{(II.3.3.6)}$$

The standard deviation is 0.43 mV and shows the mean square deviation of the measured points from the straight line.

The value of α for pure copper is 4.3x10^{-3}/C.

Questions:

1. What is the unit of mobility?
2. Why does the resistance of a metal increase with temperature?
3. Can the change in resistance of a metal be used to measure temperature?

4. Two identical resistances R_1 and R_2 of the same material are connected in two arms of a Wheatstone bridge. In the other two arms equal resistances from a resistance box are connected. The bridge is balanced at room temperature. The resistance R_1 is put in a furnace and heated. Will the balance of the bridge be changed? If so can you think of a temperature controller for the furnace using this phenomenon?

II.3.4 ENERGY BAND GAP OF A SEMICONDUCTOR

1. INTRODUCTION:

In a crystal, in which the atoms are arranged in a periodic array, the potential energy of an electron varies periodically. The energy levels of an electron moving in such a periodic potential is split into allowed and forbidden bands of energy. In a metal the conduction band is partially filled while all energy bands with a lower energy are completely filled. Since closely spaced unoccupied energy levels are present in a metal a small electric field causes the electrons to move to higher energy levels and this leads to good electrical conductivity of a metal. In a semiconductor, the highest occupied band is completely filled and the next higher band is completely empty at absolute zero of temperature. The completely filled band is called the valence band and the unfilled next higher band is called the conduction band. At a finite temperature thermal energy raises some of the electrons near the top of the valence band to the bottom of the conduction band. The energy separation between the top of the valence band and the bottom of the conduction band is called the band gap E_g of a semiconductor. The number of electrons excited to the conduction band (when $kT << E_F$ (Fermi Level) that is equal to $E_g/2$ for a lightly doped semiconductor) is proportional to $\exp(-E_g/2kT)$. The electrons accelerated by the electrical field acquire a drift velocity in the direction of the electric field proportional to the field. The constant of proportionality is called the mobility μ. The electrical conductivity σ of a semiconductor is

$$\sigma = ne\mu$$

where n is the number of electrons per unit volume excited to the conduction band at temperature T and e is the magnitude of the electronic charge.

The temperature dependence of σ arises from the temperature dependence of n and μ. At room temperature and above, the mobility, in lightly doped semiconductors, decreases with temperature as $T^{-\nu}$. A simple calculation shows that ν is close to 3/2. So the electrical conductivity σ shows a temperature dependence $T^{-\nu} \exp(-E_g/2kT)$. The electrical resistance of the semiconductor sample will be inversely proportional to σ and will show a temperature dependence $T^{\nu}\exp(E_g/2kT)$. We use this fact to determine the band gap of the semiconductor.

2. APPARATUS REQUIRED:

A regulated DC power supply, temperature indicator, furnace, insert for band gap of a semiconductor, 9 V cell, and a DC multimeter reading in the 200 mV range.

3. EXPERIMENTAL PROCEDURE

A small sliver of silicon is mounted on one of the faces of an aluminium block of square section. Electrical contacts to the silicon sample are made by pasting two thin enamel coated copper wires to the surface of the silicon chip with silver paint. At about 30 ^{0}C the resistance between the two contacts on the silicon chip is about 50to 100 megohms. Because of this high resistance a two probe procedure is used to measure the resistance.

The electrical circuit for measuring resistance is shown in Figure II.3.4.1.

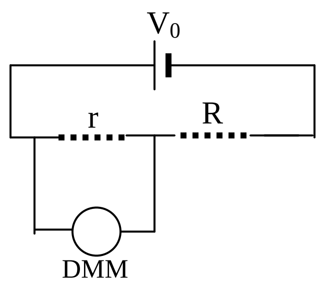


Figure II.3.4.1: Circuit for measuring the resistance R of the Silicon sample. r is a resistance of 470 kΩ.

A 9 V cell is connected to the electrical leads pasted on the silicon sample through a resistance of 470 kΩ. The current through the silicon chip is

$$I = V_0/(R+r) \qquad \text{(II.3.4.1)}$$

and the voltage V across r is

$$V = V_0\ r/(R+r) \qquad \text{(II.3.4.2)}$$

As the temperature increases R will decrease drastically, while r, which is at room temperature, does not vary. So V will increase as the temperature of the silicon chip is increased. Knowing V, V_0, and r, one can calculate R from the relation

$$R = [(V_0/V) - 1]r \qquad \text{(II.3.4.3)}$$

Thus one measures the resistance of the silicon chip as a function of temperature.

The aluminum block on which the silicon chip is mounted also carries a Pt 100 thermometer. This aluminum block is inserted in a furnace that is connected to a regulated power supply. The Pt 100 thermometer is connected to the corresponding terminals of the temperature indicator. The voltage on the regulated power supply is kept around 7.5 V to start with so that the temperature of the furnace rises at approximately 1 to 1.5^0 C per minute. The two open ends of the ceramic tube of the furnace must be loosely closed with wads of cotton. This prevents convection air currents. The voltage on the DMM connected to r is measured at intervals of 5^0 C. As the temperature of the furnace increases the voltage output of the regulated power supply is gradually increased to maintain the rate of heating within 1 to 1.5 degrees per minute. At 100 C, the voltage will not exceed 12 V.

A sample set of readings is shown below in Table II.3.4.1. Column 1 gives the temperature in centigrade as read on the temperature indicator, column 2 gives the temperature T in the absolute scale and column 3 gives 1/T. In column 4 the measured values of V in mV across r are given. Column 5 gives the value of R calculated from V_0, r and the measured value of V. Column 6 gives ln(R).

Table II.3.4.1
Resistance R of silicon chip as it is heated
$V_0 = 8.660$ V
R = 0.47 Megohm

Temp C t	Temp K T	1/T K^{-1}	V in mV	R Ohms	lnR	1.5lnT	lnR-1.5lnT
35	308.1	0.00325	94	4.29E+07	17.574	8.596	8.978
40	313.1	0.00319	120	3.35E+07	17.326	8.62	8.706
45	318.1	0.00314	162	2.47E+07	17.022	8.644	8.379
50	323.1	0.0031	203	1.95E+07	16.788	8.667	8.121
55	328.1	0.00305	266	1.48E+07	16.512	8.69	7.822
60	333.1	0.003	339	1.15E+07	16.261	8.713	7.548
65	338.1	0.00296	402	9.66E+06	16.084	8.735	7.349
70	343.1	0.00291	496	7.74E+06	15.862	8.757	7.105
75	348.1	0.00287	600	6.31E+06	15.658	8.779	6.88
80	353.1	0.00283	736	5.06E+06	15.438	8.8	6.637
85	358.1	0.00279	939	3.86E+06	15.167	8.821	6.346
90	363.1	0.00275	1137	3.11E+06	14.95	8.842	6.108
95	368.1	0.00272	1362	2.52E+06	14.739	8.863	5.877
100	373.1	0.00268	1612	2.06E+06	14.536	8.883	5.653
105	378.1	0.00264	1930	1.64E+06	14.309	8.903	5.407
110	383.1	0.00261	2290	1.31E+06	14.083	8.922	5.161

From the table we see that the resistance of the silicon chip drops from 42.9 Megohm at 35 C to 1.31 Megohm at 110 C.

The resistance R of the semiconductor is expected to vary as $T^{\nu} \exp(E_g/2kT)$

So $$\ln(R) - \nu\ln(T) = A + E_g/2kT \qquad \text{(II.3.4.4)}$$

Theory indicates that ν should be close to 1.5. So Column 7 gives 1.5 lnT. Column 8 gives ln(R) – 1.5ln(T).

Figure II.3.4.1 shows a plot of lnR-1.5lnT against 1/T. A straight line is fitted to the points. The slope of the line is found to be 5911 ± 67. The value of k in electron volt/K is 8.66×10^{-5} eV/k. Multiplying the slope of the curve in Figure II.3.4.1 with 2 k gives the band gap E_g in electron volt. From the slope the band gap comes out to be 1.02±0.01 eV.

Figure II.3.4.1: Plot of lnR-1.5lnT versus 1/T where R is resistance of the silicon chip at an Absolute temperature T.

Questions:

1. What is the meaning of doping? If the sample is heavily doped what happens to the energy band picture?

2. What is the meaning of Fermi energy?

II.3.5: DETERMINATION OF k/e USING A TRANSISTOR

1. INTRODUCTION:

In a transistor, the collector current I_{CE} increases exponentially as the base to emitter voltage V_{BE} increases according to the relation

$$I_{CE} = I_f \exp(eV_{BE}/kT) \qquad \text{(II.3.5.1)}$$

Here e is the magnitude of electronic charge, k is the Boltzmann constant, T is the absolute range to the banana terminals marked I_{CE}. If we measure I_{CE} as a function of V_{BE} at room temperature and plot V_{BE} against ln (I_{CE}) we will get a straight line. The slope α of this line is given by

$$\alpha = (k/e)\,T \quad \text{or } k/e = \alpha/T \qquad \text{(II.3.5.2)}$$

This provides a simple method to measure k/e.

2. APPARATUS REQUIRED

The k/e box described in I.15,aDMM to measure current in μA/mA range, and a DMM to measure in the DC 2 V range.

3. PROCEDURE:

Connect the DMM to measure DC 2 V to the banana terminals marked V_{BE}. Connect the DMM to measure current in the μA/mA ranges to the banana terminals marked I_{CE}. Switch on the power. Set the pot so that the DMM reads a V_{BE} of 0.5 V. Note the current I_{CE}. It will be a few μA. Increase V_{BE} in steps of 0.025 V and note I_{CE}. If I_{CE} goes beyond range turn the DMM to mA range. Take readings till V_{BE} reaches 0.725 V.

A sample set of readings is given in Table II.3.5.1.

Table II.3.5.1.

V_{BE} V	I_{CE} μ A	$\ln(I_{CE})$
0.5	1.60E-06	-13.3455069
0.525	4.10E-06	-12.4045236
0.55	1.07E-05	-11.4452668
0.575	2.95E-05	-10.4311203
0.6	8.16E-05	-9.4136813
0.625	2.24E-04	-8.40386451
0.65	6.75E-04	-7.30079787
0.675	2.31E-03	-6.07050775
0.7	6.10E-03	-5.09946651
0.725	1.09E-02	-4.52174858

The plot of V_{BE} against ln (I_{CE}) is shown in Figure II.3.5.3'.

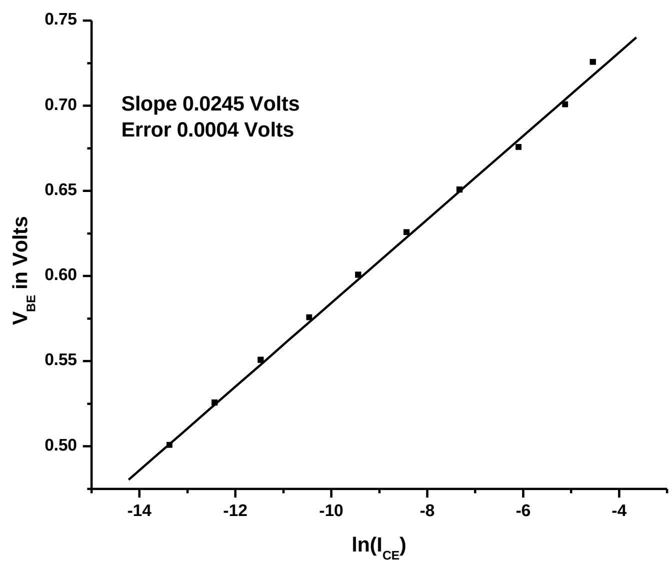


Figure II.3.5.3. Plot of V_{BE} vs. ln (I_{CE})

The slope α of the graph is 0.0245±0.0004 V. Note the room temperature Tin absolute scale. This was 297 K. The value of k/e is

$$k/e = \alpha/T = 0.0245/297 = 82.6\ \mu V/K$$

A repeat of the experiment gave a slope of 0.0250±0.0002 Volts. This gives a value of 84.1 μV/K for k/e. The experiment is reproducible. The actual value of(k/e) is 86.2 μV/K.

II.4 A C EXPERIMENTS WITH THE SIGNAL GENERATOR

II.4.1 INTRODUCTION

1. INTRODUCTION:

A current varying periodically with time is called an alternating current. When a current I passes through a coil of self inductance L, it generates a magnetic field B. The magnetic flux LI due to this current is linked with the coil. The flux linked with the coil changes with time, when the current changes with time, and induces an EMF $-LdI/dt$ in the coil. If I is a sinusoidal current with a frequency f,

$$I = I_0 \operatorname{Sin}(2\pi ft) \qquad \text{(II.4.1.1)}$$

the induced EMF is

$$V_{ind} = -\,2\pi fL\, I_0 \cos(2\pi ft) \qquad \text{(II.4.1.2)}$$

The voltage applied across the coil should be equal and opposite to this induced voltage to maintain the oscillating current. The applied voltage is therefore

$$V_{app} = -\,V_{ind} = \ 2\pi fL\, I_0 \cos(2\pi ft) \qquad \text{(II.4.1.3)}$$

Figure II.4.1.1 shows I and V_{app} as a function of ωt ($\omega = 2\pi f$). Note that when the current goes through zero, the voltage is either a maximum or a minimum. Since

$$\operatorname{Cos}(\omega t) = \operatorname{Sin}(\omega t+\pi/2) \qquad \text{(II.4.1.4)}$$

the phase (the argument of the trigonometric function) of the voltage is always more than the phase of the current by $\pi/2$. We say that the current lags behind the voltage by $\pi/2$. This is shown in Figure II.4.1.1.

A coil has an inductance and a resistance. The voltage necessary to drive a current through a resistance is

$$V = I\,R = I_0 R \sin(\omega t) \qquad \text{(II.4.1.5)}$$

The voltage across a resistance is always in phase with the current.

Since a coil has both resistance R and inductance L, the voltage across the coil when an alternating sinusoidal current passes through it is the sum of two parts

$$V_{app} = I_0(R \sin(\omega t) + \omega L \cos(\omega t)) \qquad \text{(II.4.1.6)}$$

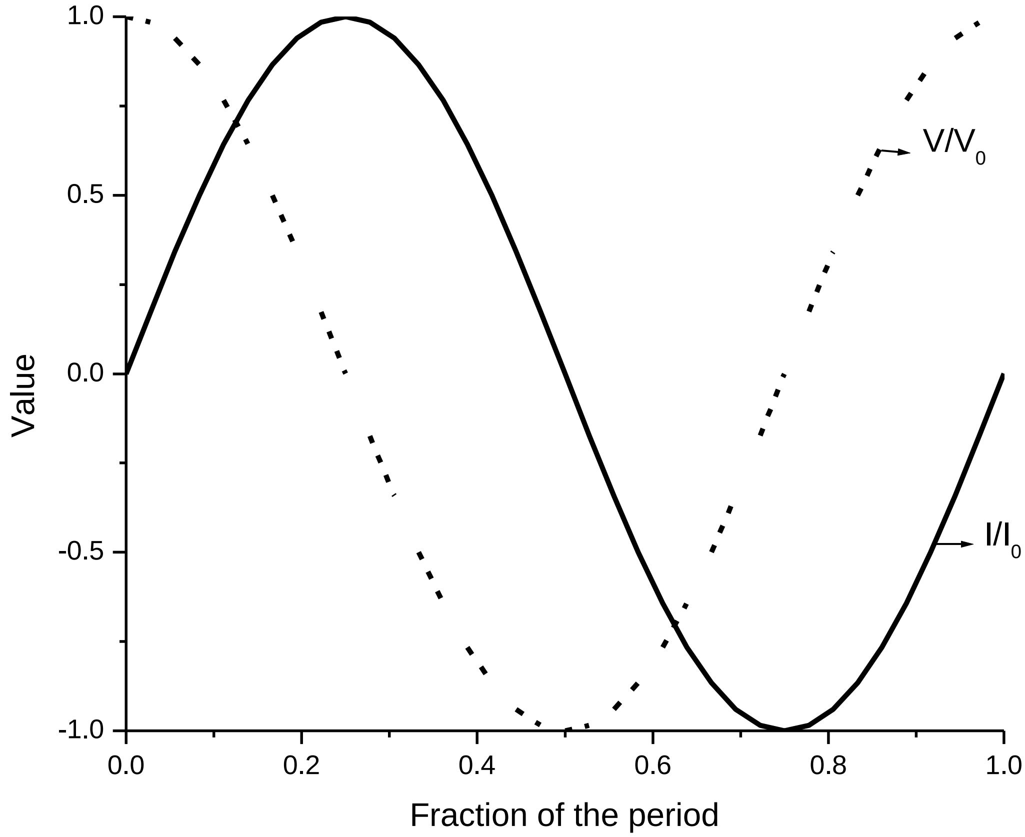


Figure II.4.1.1: Plot of I/I_0 (continuous curve) and $V_{app}/\omega I_0$ (dashed curve) as a function of fraction of a period (t/T) for a pure inductance.

If we put $R = Z \cos \phi$ and $\omega L = Z \sin \phi$ in equation (II.4.1.6) we get

$$V_{app} = Z\, I_0\, \mathrm{Sin}\,(\omega t + \phi) \qquad \text{(II.4.1.7)}$$

where

$$Z = (R^2 + \omega^2 L^2)^{1/2} \qquad \text{(II.4.1.8)}$$

is called the impedance of the coil.

The applied voltage now leads the current by the phase angle ϕ given by

$$\phi = \mathrm{Atan}\,(\omega L/R) \qquad \text{(II.4.1.9)}$$

The impedance of a coil increases as the frequency of the current is increased and the phase angle increases from 0 and approaches $\pi/2$ as the frequency becomes very high. These will be verified in one of the experiments.

When a capacitance C is charged to a voltage V, a charge

$$q = CV \qquad \text{(II.4.1.10)}$$

is stored on the positive capacitor plate and an equal charge but of opposite polarity is stored on the negative capacitor plate.

If the voltage varies sinusoidally as

$$V = V_0 \text{ Sin } (\omega t) \qquad \text{(II.4.1.11)}$$

the charge on the capacitor q varies sinusoidally as

$$q = CV_0 \text{ Sin } (\omega t) \qquad \text{(II.4.1.12)}$$

A time varying charge corresponds to a current

$$I = dq/dt = C\omega V_0 \cos(\omega t) \qquad \text{(II.4.1.13)}$$

If we write

$$I_0 = C\omega V_0 \text{ or } V_0 = I_0/\omega C \qquad \text{(II.4.1.14)}$$

then the current through the capacitor and the voltage across the capacitor may be written in terms of I_0 as

$$I = I_0 \cos(\omega t) \qquad \text{(II.4.1.15)}$$

and $$V = (I_0/\omega C) \sin(\omega t) = (I_0/\omega C) \cos(\omega t - \pi/2) \qquad \text{(II.4.1.16)}$$

The impedance of the capacitor

$$Z = V_0 / I_0$$

It has the magnitude $1/\omega C$.

Note that, unlike with an inductance, the voltage across a capacitor lags behind the current through the capacitor by $\pi/2$.

We may represent the voltages across a series combination of resistance, inductance and/or capacitance in relation to the current through the combination in an **Argand** diagram. In this diagram a vector along the X-axis represents the voltage in phase with the current. The voltage leading the current in phase by an angle ϕ is represented by a vector, which makes an angle ϕ with the X-axis in the first quadrant. Similarly a voltage lagging behind the current by a phase ϕ is represented by a vector in the fourth quadrant making an angle ϕ with the X-axis.

For example, when a current of amplitude I and angular frequency ω passes through a coil that has a resistance $R = R_L$ and an inductance L, the following Argand diagram represents the voltage across the coil in relation to the current through it.

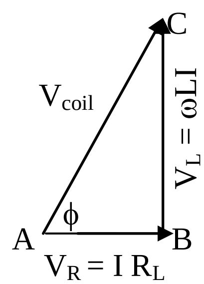


Fig. II.4.1.2 Argand diagram for the voltage applied to a coil of resistance R_L and inductance L.

The vector AB represents the voltage drop across R_L that is in phase with the current. The vector BC represents the voltage to overcome the induced EMF due to the inductance L. The vector AC represents the vector sum of the above two voltages and is the voltage applied to the coil. It is equal to ZI and leads the current by the phase angle ϕ.

Suppose we have a resistance R in series with the above coil and an AC current I at the frequency ω flows through them. The Argand diagram for this case is shown in Figure II.4.1.3.

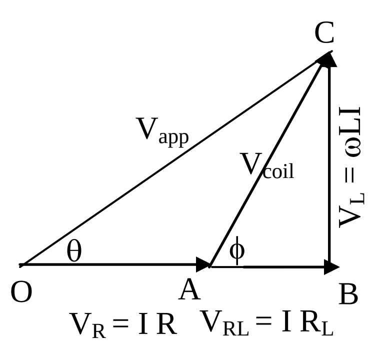


Figure II.4.1.3: Argand diagram for a resistance R in Series with a coil of resistance R_L and Inductance L.

The total applied voltage **V_{app} (vector OC)** ***is less than the sum of the voltages V_R*** **(vector OA)** ***across the resistance R and V_{coil}*** **(vector AC)** ***across the coil.*** This is because the voltage across the coil leads the current by an angle ϕ. If we measure V_{app}, V_R and V_{coil} they should be related by

$$V_{app}^2 = V_R^2 + V_{coil}^2 + 2V_R V_{coil} \cos\phi \qquad \text{(II.4.1.17)}$$

From this ϕ can be obtained at any frequency ω. The variation of ϕ with frequency can be studied.

Similarly if we have a resistance R in series with a capacitance C we may construct the Argand diagram as follows

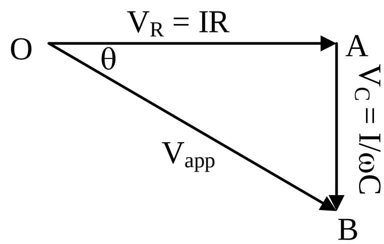


Figure II.4.1.4: Argand diagram for a resistance in series with a capacitor.

We can thus make an Argand diagram for any combination of resistances, inductances and capacitances.

The following four experiments can be conducted with a signal generator and a simple circuit shown below built in a box. The signal generator is described in Section I.

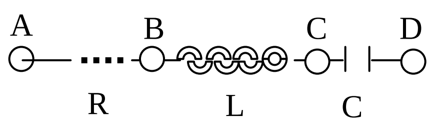


Figure II.4.1.5 R-L-C Box for AC experiments

A, B, C, and D are banana terminals. Between A and B is connected a resistance R of 1 kΩ, Between B and C is connected a coil L of inductance 100 mH. Between C and D is connected a capacitance C of 0.033 μf. One may choose other values depending on the availability.

It is a series R-L circuit if the signal generator is connected between A and C. If B and C are externally shorted by a wire and the signal generator is connected between A and D, it becomes a series R-C circuit. If the signal generator is connected between A and D, without shorting the terminals B and C, it becomes a series R-L-C circuit, L and C being in series with R. If D is connected to B externally and the signal generator is connected between A and C, it becomes R with L and C in parallel.

With this simple circuit the four experiments to be described in the following sections can be carried out with a signal generator.

II.4.2 MEASUREMENT OF SELF INDUCTANCE OF A COIL

1. **AIM:** To show that the impedance of a coil of resistance R_L and self inductance L varies with frequency as

$$Z_{coil} = (R_L^2 + 4\pi^2 f^2 L^2)^{1/2} \qquad \text{(II.4.2.1)}$$

and to measure the self-inductance of the coil.

2. **PRINCIPLE:** A coil with a self-inductance L and resistance R_L has an impedance $Z_{coil}(\omega)$ given by

$$Z_{coil}(\omega) = R_L + j\omega L \qquad \text{(II.4.2.2)}$$

where $j = \sqrt{(-1)}$ and $\omega = 2\pi f$, f being the frequency of the AC supply. The magnitude of the impedance is $(R_L^2 + \omega^2 L^2)^{1/2}$. If we connect an AC source across a resistance R in series with the inductor, then the rms voltage across R and across the coil will be in the ratio

$$V_{coil}/V_R = |Z_{coil}|/R = (R_L^2 + \omega^2 L^2)^{1/2}/R \qquad \text{(II.4.2.3)}$$

If we measure V_{coil}/V_R at different frequencies, a plot of $(V_{coil}/V_R)^2$ vs. f^2 will give a straight line, the slope of which is given by $(2\pi L/R)^2$. From the slope one can determine L knowing R.

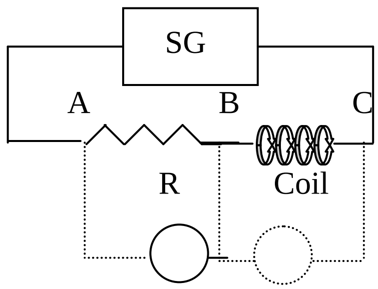
It is not necessary to keep the amplitude of the signal constant as one varies the frequency because we are only taking the ratio V_{coil}/V_R.

3. **APPARATUS REQUIRED**

Signal generator, R-L-C box, and a DMM to measure both AC voltage in the range 2V to three decimal places, and frequency.

4. PROCEDURE:

The experiment is performed as shown below.



AC
Voltmeter
Figure II.4.2.1 Connection diagram

The two terminals of the signal generator are connected to the terminals A and C on the R-L-C box. The signal generator voltage is applied across the resistance and the coil in series. The output of the signal generator is kept at around 1 Volt. A DMM in AC 2 V range connected between A and B measures the rms voltage drop V_R across the resistance. The same DMM connected between B and C measures the rms voltage drop V_{coil} aross the coil. Connected between A and C the DMM measures V_{app}. The frequency of the signal is varied between 200 and 2000 Hz in steps of 200 Hz and V_{coil}, V_R and V_{app} are measured. A sample set of readings is shown in Table II.3.2.2.1.

Table II.4.2.1 : Measurement of self inductance

Self-inductance of a coil							
Resistance R =		1000	Ohms				
f in Hz	f^2 in $(kHz)^2$	V_{coil} Volts	V_R Volts	V_{app} Volts	$(V_{coil}/V_R)^2$	Cos(ϕ)	tan(ϕ)
200	0.04	0.200	0.832	1.004	0.058	0.829	0.675
400	0.16	0.260	0.820	1.000	0.101	0.610	1.300
600	0.36	0.342	0.796	1.005	0.185	0.477	1.845
800	0.64	0.420	0.774	0.996	0.294	0.333	2.831
1000	1.00	0.492	0.738	0.997	0.444	0.285	3.357
1200	1.44	0.548	0.710	1.000	0.596	0.251	3.851
1400	1.96	0.605	0.670	0.986	0.815	0.194	5.057
1600	2.56	0.660	0.645	0.995	1.047	0.163	6.070
1800	3.24	0.702	0.612	0.993	1.316	0.138	7.169
2000	4.00	0.740	0.575	0.992	1.656	0.124	7.978

Figure II.4.2.2. shows a plot of $(V_{coil}/V_R)^2$ against f^2.

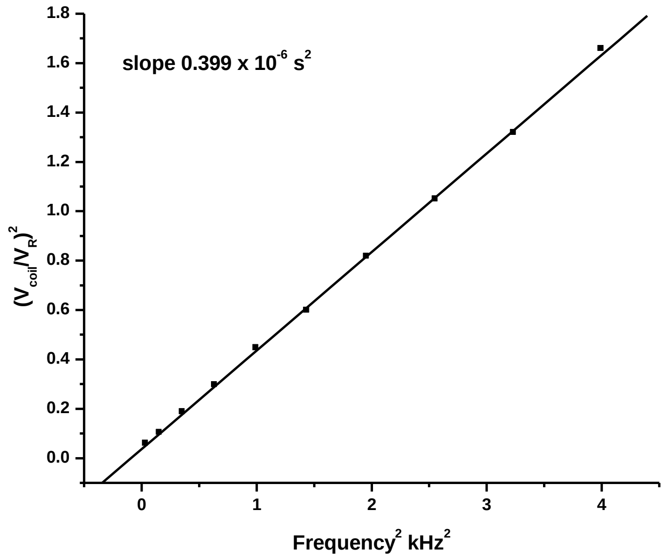


Figure II.4.2.2: Plot of $(V_{coil}/V_R)^2$ against (frequency)^2 for the coil

The linear fit to the points is shown by the continuous line. The slope α of the line is $3.99\text{x}10^{-7}$ $s^{2.}$ The self inductance is given by

$$L = \alpha^{0.5} R/2\pi \qquad \text{(II.4.2.4)}$$

In the present case R is 1 kΩ. So the coil inductance comes out to be 99 milli-Henries.

Note that the intercept according to theory should be $(R_L/R_0)^2$. The resistance of the coil is small compared to the resistance R of 1 kΩ. The ratio $(R_L/R_0)^2$ is of the order of 0.01. This is of the same order as the errors in calculating $(V_L/V_R)^2$. So one cannot get the intercept accurately.

4. PHASE CALCULATION:

The voltage across the resistance R is in phase with the current while the voltage across the inductance leads the current by a phase angle ϕ given by

$$\tan \phi = 2\pi f L/R_L \qquad \text{(II.4.2.5)}$$

The Argand diagram for this situation is shown in Figure II.4.1.3 of the previous chapter. From this diagram we see that

$$V_{app}^2 = V_R^2 + V_{coil}^2 + 2V_R V_{coil} \cos\phi \qquad \text{(II.4.2.6)}$$

The total applied voltage is less than the sum of the measured voltages across the resistance R and the coil.

Using the above formula we may calculate cos ϕ from the measured values of V_{app}, V_R and V_{coil}. From this we may obtain tan ϕ. The values of cosϕ and tanϕ are shown in the last two columns of Table II.4.2.1. A plot of tan ϕ against the frequency f can be fitted to a straight line to give the ratio L/R_L. Since L has been measured earlier one may obtain R_L, the resistance of the coil from such a plot.

A plot of the values of tan ϕ against the frequency is shown in Figure II.4.2.3.

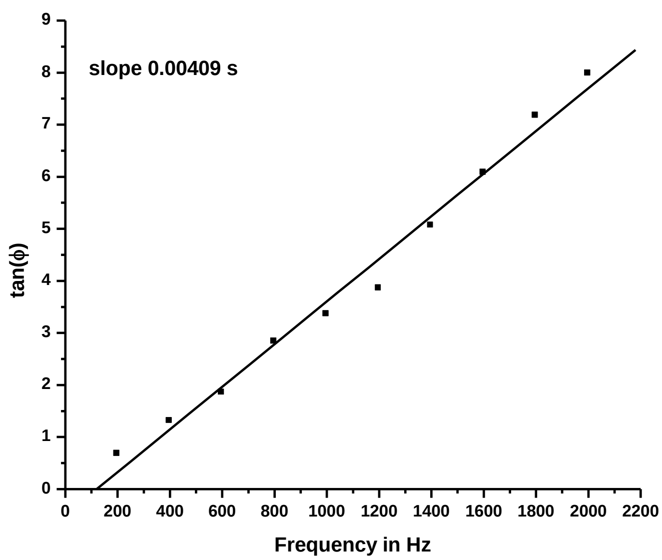


Figure II.4.2.3: A plot of tanϕ against frequency

A linear fit to the data is shown in the figure. The slope β of the graph is 0.00409 s. This should be equated to $2\pi L/R_L$. Taking L to be 99 mH, the value of R_L calculated is 152 Ohms.

Note that tan(ϕ) increases with f. At high frequency ϕ will approach 90 degrees. Tan(ϕ) will increase rapidly with ϕ as ϕ approaches 90 degrees. That is the reason why the measurements are restricted to frequencies below 2 kHz

II.4.3 MEASUREMENT OF CAPACITANCE

1. AIM:

To verify that the impedance of a capacitance C varies as

$$Z_C = 1/2\pi fC \qquad \text{(II.4.3.1)}$$

and to measure the value of the capacitance.

2. PRINCIPLE:

The impedance of a capacitance C at a frequency **f** is

$$Z_C = (1/j)\,(1/\,(2\pi fC)) \qquad \text{(II.4.3.2)}$$

The leakage resistance of a capacitor has a very high value and comes in parallel with the capacitance. Its effect on the total impedance can be neglected.

If a resistance R and a capacitance C are connected in series and an AC RMS voltage V_{app} at frequency f is applied across the combination, the ratio of the RMS voltage across R and C is given by

$$V_R/V_C = R/|Z_C| = 2\pi fCR \qquad \text{(II.4.3.3)}$$

If this ratio is measured at different frequencies and plotted as a function of frequency, one should get a straight line with a slope $2\pi CR$. One can determine C from the slope of the curve knowing R.

3. APPARATUS REQUIRED

A signal generator, R-L-C box and a DMM which can measure frequency up to 10 kHz and AC voltage up to three decimal places in the range of 2V.

4. PROCEDURE:

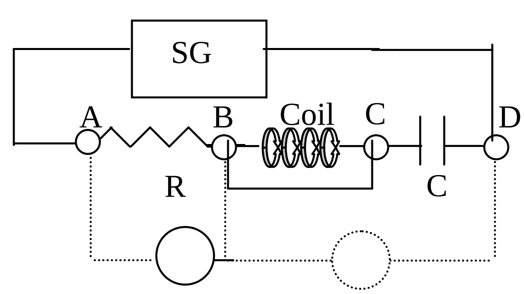


Figure II.4.3.1: Connection Diagram

Short B and C on the R-L-C box externally with a wire so that the coil is shorted out and we have a RC circuit. Connect a signal generator output to the two terminals A and D and set the frequency at 1 kHz. Set the RMS voltage of the output around 1 Volt. With a multi-meter in the AC 2 Volts range measure the voltage V_R across the resistance R and V_C across the capacitance C. Repeat the experiment changing the frequency of the signal generator from 1 to 10 kHz in steps of 1 kHz. It is not necessary to keep V_{app} constant as the frequency is changed, since we are measuring only the ratio V_R/V_C. Plot this ratio against f. Fit the data to a straight line and obtain the slope α. Then C can be calculated.

A representative set of readings is given below.

Table II.4.3.1: Measurement of Capacitance

f in kHz	V_R Volts	V_C Volts	V_R/V_C	V_{app} Volts
1	0.19	0.92	0.2065	0.94
2	0.37	0.87	0.4253	0.946
3	0.508	0.8	0.635	0.949
4	0.616	0.726	0.8485	0.955
5	0.7	0.654	1.0703	0.962
6	0.76	0.591	1.286	0.969
7	0.805	0.535	1.5047	0.976
8	0.841	0.483	1.7412	0.983
9	0.837	0.425	1.9694	0.958
10	0.857	0.384	2.2318	0.959

A plot of V_R/V_C against the frequency in kHz is shown in Figure II.4.3.2 below.

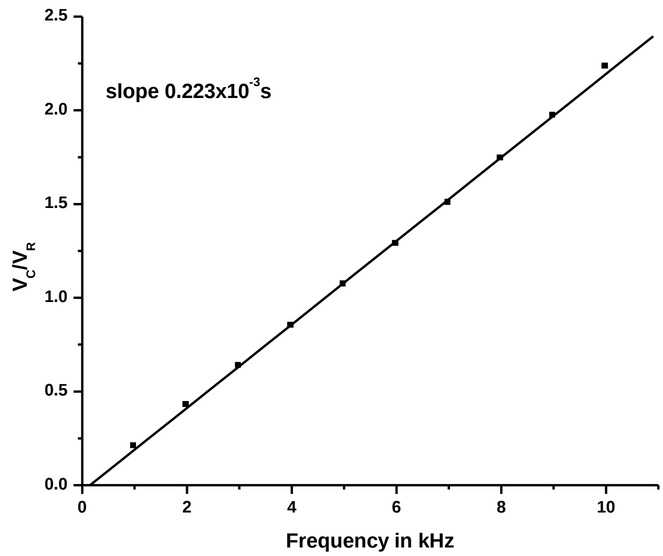


Figure II.4.3.2: Plot of (V_R/V_C) against frequency in kHz

The slope α of the curve is $2.23\text{x}10^{-4}$ s. We can calculate the unknown capacitance value from the equation

$$C = \alpha / (2\pi R)$$

In the present case R = 1 kilo-Ohm. From this formula C comes out to be 0.0354 μfarad.

NOTE: Unlike with the inductor, we are working with the capacitor in the range of frequencies 1 to 10 kHz. With the capacitance value in the R-L-C box, the impedance of the capacitance in this range of frequencies is comparable to the resistance R. If we had worked with frequencies below 1 kHz the impedance of the capacitance would have been high compared to the resistance R and so V_R would become smaller and smaller as the frequency is decreased.

For a capacitance the leakage resistance is very high. So the voltage across the capacitance will lag behind the current (i.e. lag behind V_R) by almost exactly 90^0 at all frequencies. This implies that at all frequencies

$$V_{app} = (V_R^2 + V_C^2)^{0.5}$$

This is checked below for the measured values of V_R, V_C and V_{app} in Table II.4.3 2.

Table II.4.3.2
Verification of relation between V_{app}, V_R and V_C

f in kHz	V_R Volts	V_C Volts	V_{app} Volts	$(Vr^2+Vc^{\wedge 2})^{\wedge}0.5$
1	0.190	0.920	0.940	0.939
2	0.370	0.870	0.946	0.945
3	0.508	0.800	0.949	0.948
4	0.616	0.726	0.955	0.952
5	0.700	0.654	0.962	0.958
6	0.760	0.591	0.969	0.963
7	0.805	0.535	0.976	0.967
8	0.841	0.483	0.983	0.970
9	0.837	0.425	0.958	0.939
10	0.857	0.384	0.959	0.939

If we put the coil and the capacitance in a circuit, the circuit has a resonance frequency f_{res}. By definition, this is the frequency at which the magnitude of the impedance of the inductance of the coil is equal to the magnitude of the impedance of the capacitor. f_{res} is given by

$$f_{res} = (1/\ 2\pi)\ (1/LC)^{\ 0.5}$$

Using L = 99 mH and C = 0.0354 μfd, the resonance frequency has the value 2.68 kHz.

II.4.4 SERIES AND PARALLEL RESONANT CIRCUITS

1. AIM:

To show that the impedance of a circuit, in which an inductance L and a capacitance C are in series, is a minimum at the resonance frequency f_{res}, and the impedance of a circuit, in which L and C are in parallel, is a maximum at f_{res}.

2. PRINCIPLE:

At the resonance frequency $f_{res} = (1/2\pi)\,(1/LC)^{1/2}$, $\omega L = 1/\omega C$. The impedance of an inductance is $Z_L = R_L + j\omega L$ and of a capacitance is $Z_C = -j/\omega C$. When the inductance and capacitance are in series the total inductance is Z_L+Z_C. This has a minimum value R_L at the resonance frequency. At any other frequency the impedance is more than R_L.

If the coil and capacitance are connected in parallel, the effective impedance of the combination is

$$Z = Z_L Z_C / (Z_L+Z_C) \qquad \text{(II.4.4.1)}$$

At resonance frequency Z_L+Z_C is a minimum. If R_L were zero, then $Z_L Z_C$ would be independent of frequency while Z_L+Z_C would be zero at the resonance frequency, making the effective impedance infinite at the resonance frequency. However R_L is non-zero and small for the coil. This makes the impedance to go through a finite, but maximum, value at a frequency close to the resonant frequency.

We verify the above results with the following experiment.

3. APPARATUS REQUIRED

Signal generator, R-L-C box and a DMM which can measure frequency up to 10 kHz and AC volts to three decimal places in the range 2 V.

4. PROCEDURE

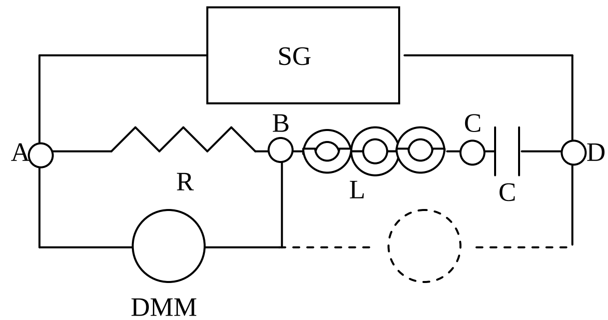


Figure II.4.4.1: Connection diagram for Series Resonant Circuit

Connect the signal generator output to the two terminals A and D. Set the frequency of the signal generator at 1 kHz. Set the RMS amplitude of the output around 1 Volt. Measure the voltage across A and B terminals with a multi-meter in the AC 2 Volts range. This gives the voltage $V_{R.}$ *With the same multi-meter measure the voltage across the terminals B and D.* This voltage V_Z is the voltage across the series combination of L and C. Repeat the experiment at different frequencies from 1 to 7 kHz. The resonance frequency of the above coil and capacitor has a value of about 2.68 kHz. **So in the range 2 to 4 kHz take readings at intervals of 200 Hz.** Plot the graph of V_Z/V_R against the frequency f. This ratio is |Z|/R, where Z is the impedance of the series L and C combination. The graph will show minimum impedance at a frequency f_{min}. Compare this with the value of $f_{res} = (1/2\pi)\,(1/LC)^{0.5}$.

A representative set of readings is shown in Table II.4.4.1 below for the series resonant circuit.

A plot of V_Z/V_R against the frequency in kHz is shown in figure II.4.4.2 below. The magnitude of the impedance of the coil and capacitance in series is (V_Z/V_R) R and since R is 1 k-Ohm, (V_Z/V_R) gives the impedance of the series resonant circuit in k-ohms. From the figure we see that this impedance goes through a minimum between 2.6 and 2.8 kHz. This should be compared with the resonance frequency of the coil and capacitance that is 2.68 kHz.

Table II.4.4.1
Series Resonant Circuit

f in kHz	V_R in volts	V_Z in Volts	V_Z/V_R	V_{app} in Volts
1	0.219	0.911	4.160	0.944
2	0.582	0.649	1.115	0.933
2.2	0.671	0.512	0.763	0.936
2.4	0.740	0.351	0.474	0.923
2.6	0.782	0.178	0.228	0.921
2.8	0.786	0.143	0.182	0.922
3	0.756	0.288	0.381	0.927
3.2	0.712	0.419	0.589	0.933
3.4	0.660	0.528	0.800	0.939
3.6	0.615	0.617	1.003	0.932
3.8	0.570	0.678	1.190	0.937
4	0.528	0.726	1.375	0.942
4.5	0.442	0.809	1.830	0.951
5	0.372	0.859	2.309	0.974
5.5	0.332	0.889	2.678	0.965
6	0.295	0.910	3.085	0.970
6.5	0.265	0.927	3.498	0.974
7	0.240	0.939	3.913	0.978

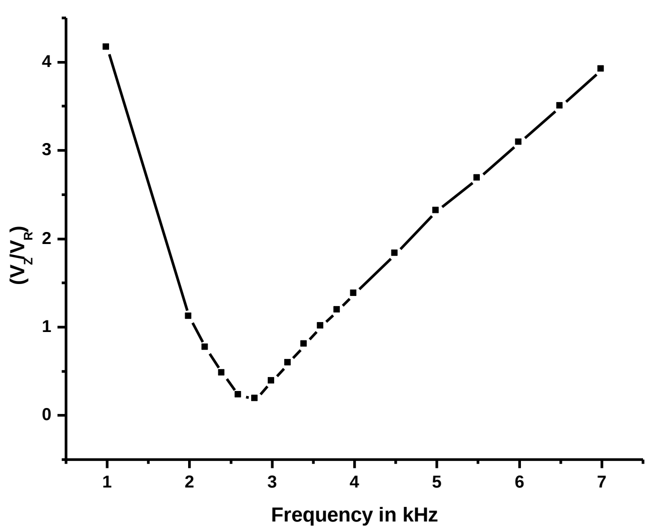


Figure II.4.4.2: A plot of V_z/V_R against frequency in kHz for the series resonant circuit.

Next connect the terminals B and D together by an external wire and connect the signal generator between A and C. This is shown below.

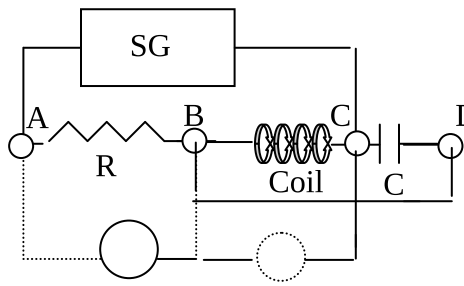


Figure II.4.4.3 Connection diagram for the Parallel resonance circuit

As before set the signal generator frequency at 1 kHz and the RMS output at around 1 Volt. Measure the voltage between A and B on a multi-meter in the AC 2 Volts range. This is V_R, the voltage across the resistance R. With the same multi-meter measure the voltage V_Z between B and C. This is the voltage across the impedance due to L and C connected in parallel. Vary the frequency from 1 to 7 kHz. **From 2 to 4 kHz vary it in intervals of 200 Hz.** The graph of V_Z/V_R shows a sharp maximum at a frequency $f_{max.}$ Verify that f_{max} is nearly equal to $f_{Res} = (1/2\pi)\,(1/LC)^{0.5}$.

A representative set of data for the parallel resonant circuit is shown in the following Table II.4.4.2.

Since R is 1 k-Ohm, (V_Z/V_R) is the impedance of the parallel resonant circuit in kilo-ohms. Figure II.4.4.4 shows the plot of (V_Z/V_R) against the frequency in kilo-Hz for the parallel resonant circuit. We see that the impedance of the parallel resonant circuit goes through a maximum at a frequency around 2.8 kHz again close to the resonant frequency of 2.68 kHz. The peak is much sharper than the minimum for a series resonant circuit.

Table II.34.4.2
Parallel Resonant Circuit

f in kHz	V_R in Volts	V_Z in Volts	V_Z/V_R	V_{app} in Volts
1	0.654	0.492	0.752	0.913
1.2	0.597	0.572	0.958	0.917
1.4	0.531	0.651	1.226	0.921
1.6	0.454	0.714	1.573	0.932
1.8	0.372	0.772	2.075	0.936
2	0.296	0.821	2.774	0.935
2.2	0.207	0.856	4.135	0.945
2.4	0.138	0.880	6.377	0.941
2.6	0.063	0.896	14.222	0.944
2.8	0.043	0.903	21.000	0.945
3	0.148	0.906	6.122	0.957
3.2	0.165	0.899	5.449	0.948
3.4	0.195	0.895	4.590	0.948
3.6	0.263	0.881	3.350	0.949
3.8	0.329	0.863	2.623	0.949
4	0.377	0.845	2.241	0.950
4.5	0.473	0.798	1.687	0.948
5	0.549	0.751	1.368	0.950
5.5	0.611	0.703	1.151	0.951
6	0.656	0.660	1.006	0.953
6.5	0.725	0.632	0.872	0.974
7	0.758	0.596	0.786	0.978

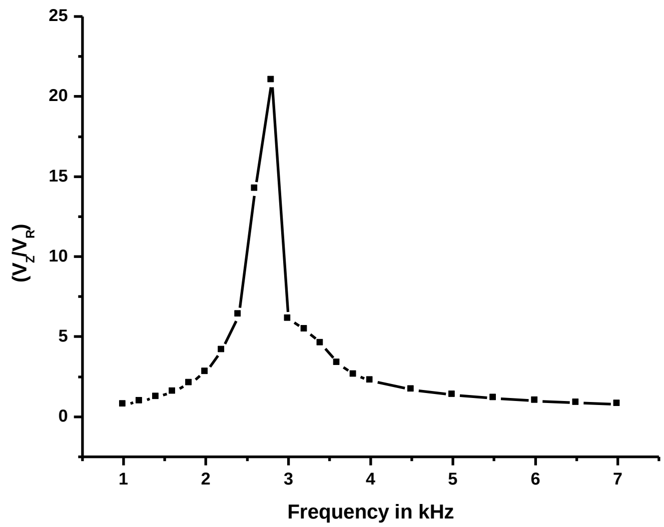


Figure II.4.4.4: (V_Z/V_R) against frequency in kHz for a parallel resonant circuit

Questions:

1. A coil is operated at a frequency of 1 kHz and 10 kHz. At which frequency will it have larger impedance? The coil has N turns. How will its impedance change if half the number of turns is removed?

2. A parallel plate capacitance has a capacitance of 50 pf. What is its impedance at 1 kHz and 100 kHz? If the plates of the capacitance are brought closer to one another will its impedance decrease or increase?

3. A coil of resistance 100 Ohms and inductance 100 mH is connected in series with a capacitance of 0.1 μf. What is the approximate resonance frequency of the circuit? What is the impedance of the circuit at this appropriate resonant frequency?

4. If the coil and capacitance are connected in parallel what will be the frequency at which the impedance will be a maximum?

II.4.5 PASSIVE FILTERS

1. INTRODUCTION:

A filter selectively transmits signals over a certain frequency range and blocks signals of other frequencies. The transmission of an ideal (a) low pass (b) high pass and (c) band pass filter is shown as a function of frequency in Figure II.4.5.1 (a), (b) and (c) respectively.

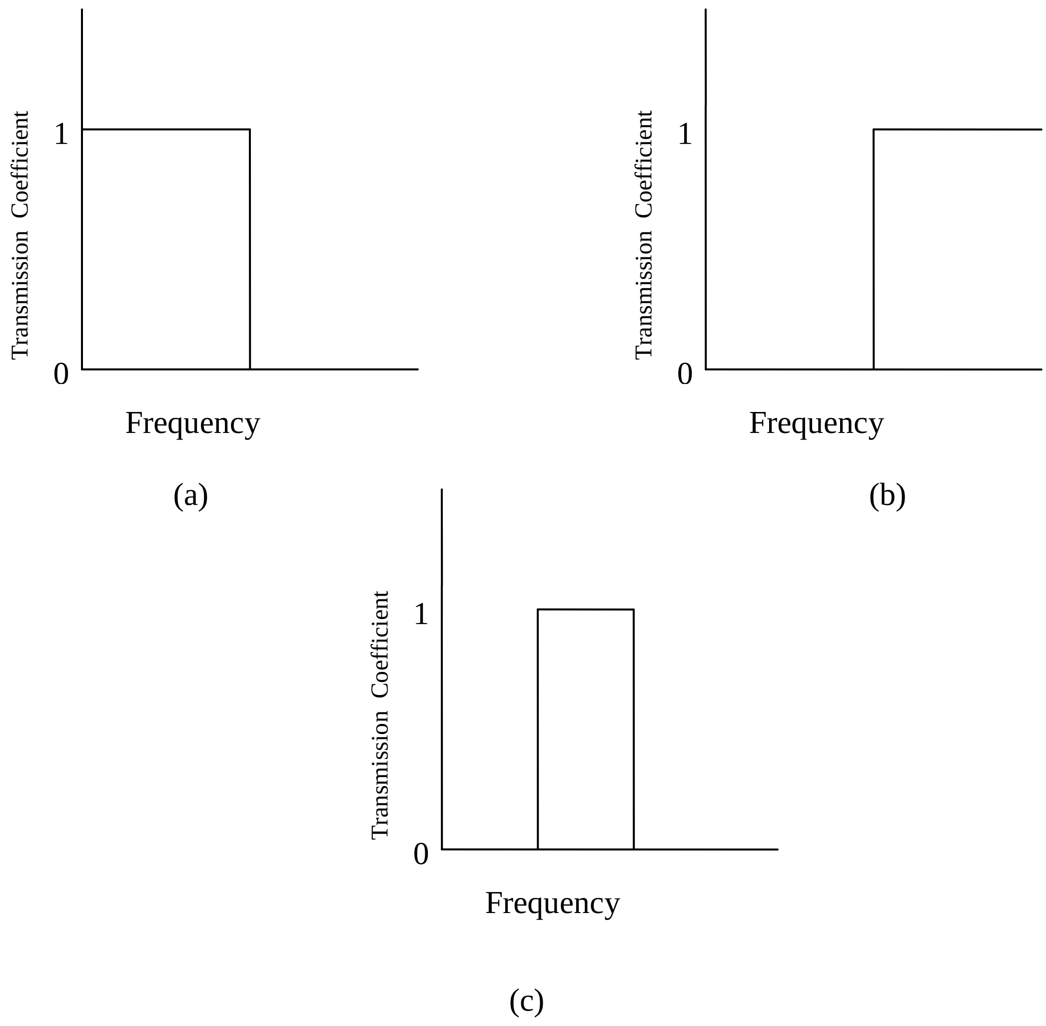


Figure II.4.5.1: Transmission characteristics of an ideal (a) low pass (b) high pass and (c) band pass filter

An ideal low pass filter will transmit all frequencies from zero to f_{max} without loss and block all frequencies above f_{max}. An ideal high pass filter will block all frequencies below f_{min} and will transmit all frequencies above f_{min} without any loss.. A band pass filter will transmit all frequencies between f_{min} and f_{max} without any loss and blocks all other frequencies. An actual filter will not have such abrupt cut off of transmission.

We shall consider passive filters shown in Figure II.4.5.2.

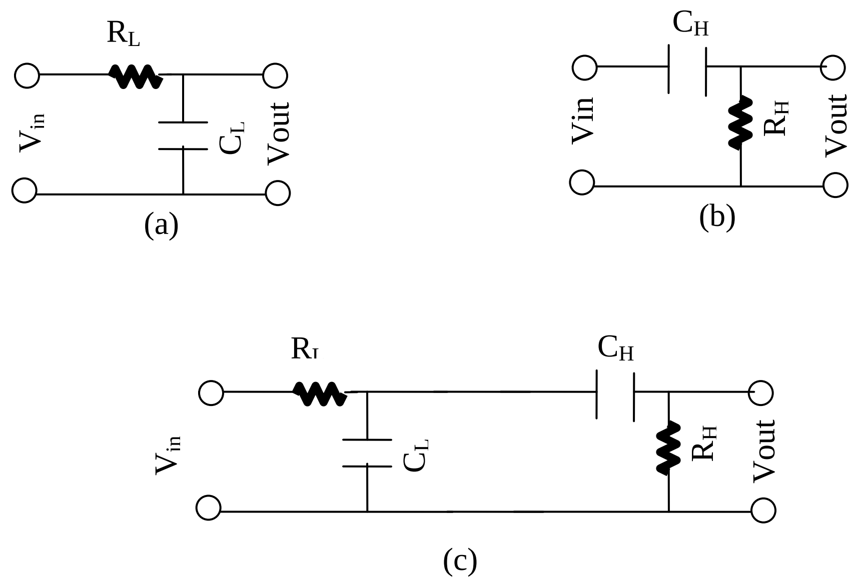


Figure II.4.5.2 Passive Filters (a) Low Pass, (b) High Pass and (c) Band Pass

The passive filters are constructed out of a resistance and capacitor in series. In the low pass filter one measures the output voltage across C_L. At very low frequencies the impedance of the capacitance is much .higher than the resistance R_L. Then almost the full input voltage V_{in} appears at the output terminals. When the frequency is very high, the impedance of the capacitance is very much smaller than the resistance. So the output voltage is very low. The transmission coefficient T defined as the ratio of the output signal to the input signal varies from nearly unity at very low frequencies to zero at very high frequencies. The roll off frequency f_{RLP} for the low pass filter is the frequency at which the impedance of the capacitor is equal to the resistance. This happens at a frequency

$$f_{RLP} = 1/(2\pi C_L R_L) \qquad \text{(II.4.5.1)}$$

The roll off frequency increases as R_L decreases. Since the voltage across the capacitance and the voltage across the resistance are $\pi/2$ out of phase, at the roll of frequency f_{RL}, V_{out} is $V_{in}/\sqrt{2} = 0.707\ V_{in}$. The transmission coefficient T = 0.707.

In the high-pass filter the output voltage is measured against the resistance R_H. At low frequencies the impedance of the capacitor C_H is very much more than the resistance R_H. So the output voltage across R_H is low. At very high frequencies the capacitance acts like a short. So the output voltage across R_H is almost equal to the input voltage. The roll on frequency f_{RHP} in this case is

$$f_{RHP} = 1/(2\pi\, C_H\, R_H) \qquad \text{(II.4.5.2)}$$

To decrease the roll on frequency we have to increase the value of R_H. For the same reason as with the Low Pass Filter, the transmission coefficient of a high pass filter at the roll on frequency f_{RHP} is 0.707.

For the low pass filter

$$T \sim 1 \quad \text{for } f \ll f_{RLP}$$

and

$$T \sim 0 \quad \text{for } f \gg f_{RLP}$$

For the high pass filter

$$T \sim 0 \quad \text{for } f \ll f_{RHP}$$

and

$$T \sim 1 \quad \text{for } f \gg f_{RHP}$$

The band pass filter is obtained by putting the low and high pass filters ***in series and adjusting f_{RHP} to be less than f_{RLP}.*** At any frequency f the band pass filter has a transmission coefficient which is the product of T_{LP} and T_{HP} at that frequency. Since T_{HP} decreases as the frequency decreases below f_{RHP} and T_{LP} decreases as the frequency increases beyond f_{RLP}, the transmission coefficient T_{BP} of the band pass filter reaches a maximum at a frequency between f_{RHP} and f_{RLP}. In fact the frequency f_{max} at which the band pass filter shows maximum transmission is related to f_{RHP} and f_{RLP} as

$$f_{max} = (f_{RHP}\, f_{RLP})^{1/2} \qquad \text{(II.4.5.3)}$$

The aim of the experiment is to verify these facts.

2. EXPERIMENTAL BOX

The low and high pass filters are built in a box. The top view of the box is shown in Figure II.4.5.3. The low pass filter is on the left and the high pass filter on the right of the box.

On the left are three banana terminals L_1, L_2 and L_3. The low pass filter is connected between these three terminals, the resistance between L_1 and L_2 and the capacitance between L_2 and L_3. The resistance can be adjusted by the pot P_{LP}. The knob of this pot is projecting to the left of the box. The pot P_{LP} is set at around 150 Ohms. This is the value of R_L. The capacitance C_L between the L_2 and L_3 terminals has a value of 0.22 μf. This gives a value of f_{RLP} of about 5 kHz.

The high pass filter is found on the right of the box. A capacitance of 0.033 μf is connected between the red H_1 and H_2 terminals on the right of the box. This sets the value of C_H. A pot P_{HP} of 10 k connected between H_2 and H_3 terminals sets the value of R_H. The value of R_H is chosen around 5 k so that the roll on frequency f_{RHP} is around 1 kHz.

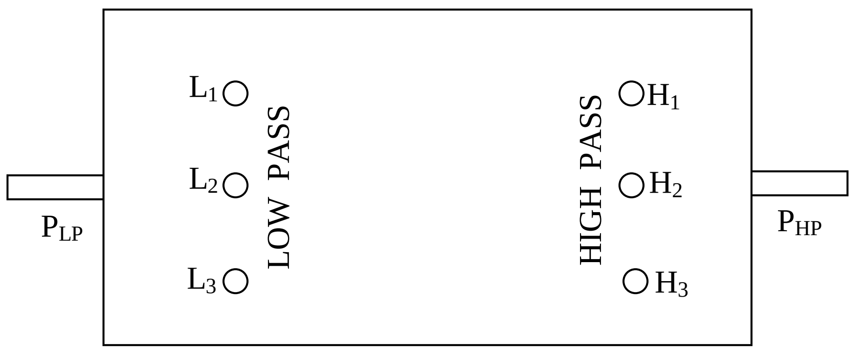


Figure II.4.5.3: Top view of the filter box

The two terminals L_3 and H_3 are connected together internally. If you find they are not connected by checking the continuity between them with a multimeter, connect a wire externally between the two terminals. The earth of the signal generator must be connected to one of these terminals.

4. APPARATUS REQUIRED

Signal generator, Filters box and a DMM measuring frequency up to 10 kHz and AC volts to three decimal places in the 2 V range.

5. LOW PASS FILTER

For studying the transmission characteristics of a low pass filter a signal generator is connected to the L_1 and L_3 terminals on the low pass side. The frequency of the signal generator is set at 200 Hz and the RMS voltage V_{input} is measured between the L_1 and L_3 terminals on the LP side with a DMM set in AC 2 V range. The output voltage V_{output} of the low pass filter is measured between L_2 and L_3 terminals on the LP side with the same voltmeter. The frequency of the signal generator is changed in steps up to 10 kHz and the ratio of V_{output}/V_{input} is measured at each frequency.

A sample set of data is shown in Table II.4.5.1 below.

Table II.4.5.1

Transmission coefficient of a low pass filter
R_L ~ 150 Ohms, C_L = 0.22 μfd
Calculated Roll off frequency f_{RLP} 4.82 kHz

f kHz	log(f)	V_{input} Volts	V_{output} Volts	T	f kHz	log(f)	V_{input} Volts	V_{output} Volts	T
0.2	-0.6990	1.001	0.999	0.998	5.0	0.6990	1.005	0.697	0.694
0.4	-0.3979	1.001	0.996	0.995	5.5	0.7404	1.000	0.654	0.654
0.6	-0.2218	1.000	0.993	0.993	6.0	0.7782	1.000	0.630	0.630
0.8	-0.0969	0.999	0.985	0.986	6.5	0.8129	0.989	0.590	0.597
1.0	0	1.000	0.981	0.981	7.0	0.8451	0.990	0.564	0.570
1.5	0.17609	1.002	0.960	0.958	7.5	0.8751	0.985	0.530	0.538
2.0	0.3010	1.004	0.920	0.916	8.0	0.9031	0.987	0.510	0.517
2.5	0.3979	1.000	0.888	0.888	8.5	0.9294	0.993	0.487	0.490
3.0	0.4771	0.996	0.850	0.853	9.0	0.9542	0.995	0.470	0.472
3.5	0.5441	0.996	0.808	0.811	9.5	0.9777	0.990	0.447	0.452
4.0	0.6021	1.000	0.773	0.773	10.0	1.0000	0.985	0.428	0.435
4.5	0.6532	1.002	0.732	0.731					

The transmission coefficient is plotted as a function of frequency in Figure II.4.5.4.

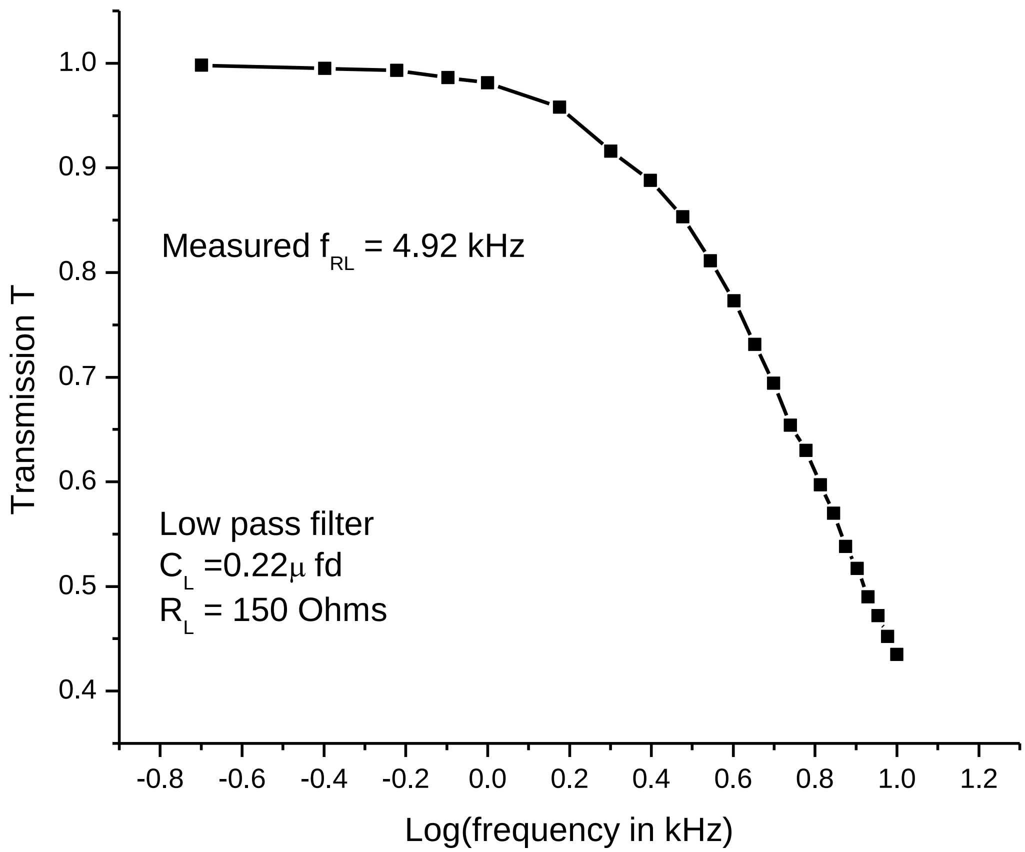


Figure II.4.5.4: Transmission coefficient as a function of frequency for the low pass filter.

The log of the roll off frequency when the transmission is 0.707 is read off from the curve and is 0.6902. So $f_{RLP} = 10^{0.6902} = 4.92$ kHz. This agrees well with the calculated value of 4.82 kHz.

6. HIGH PASS FILTER

For studying the characteristic of the high pass filter connect the signal generator between the H_1 and H_3 terminals on the high pass side . The input voltage is measured between H_1 and H_3 terminals while the output voltage is measured between H_2 and H_3 terminals. The procedure is the same as for low pass filter.

A sample set of readings are given in Table II.4.5.2.

Table II.4.5.2
Transmission coefficient of a high pass filter
R_H = 5 k Ohm; C_H = 0.033 μf
Roll-on frequency f_{RHP} = 0.965 kHz

f kHz	log(f)	V_{input} Volts	V_{output} Volts	T	f kHz	log(f)	V_{input} Volts	V_{output} Volts	T
0.10	-1.0000	1.002	0.102	0.102	2.40	0.3802	0.987	0.920	0.932
0.20	-0.6990	1.004	0.202	0.201	2.60	0.4150	0.985	0.930	0.944
0.30	-0.5229	1.002	0.300	0.299	2.80	0.4472	0.99	0.952	0.962
0.40	-0.3979	0.998	0.380	0.381	3.00	0.4771	0.984	0.950	0.965
0.50	-0.3010	1.005	0.467	0.465	3.50	0.5441	0.998	0.965	0.967
0.60	-0.2219	1.003	0.525	0.523	4.00	0.6021	1.002	0.975	0.973
0.70	-0.1549	1.000	0.590	0.590	4.50	0.6532	1.000	0.980	0.980
0.80	-0.0969	0.996	0.634	0.637	5.00	0.6990	1.005	0.986	0.981
0.90	-0.0458	0.994	0.679	0.683	5.50	0.7404	1.001	0.984	0.983
1.00	0	0.998	0.730	0.731	6.00	0.7782	0.993	0.984	0.991
1.20	0.0792	1.002	0.782	0.780	6.50	0.8129	0.992	0.990	0.998
1.40	0.1461	1.006	0.827	0.822	7.00	0.8451	0.989	0.985	0.996
1.60	0.2041	1.002	0.860	0.858	8.00	0.9031	0.984	0.98	0.996
1.80	0.2553	0.995	0.883	0.887	9.00	0.9542	0.986	0.982	0.996
2.00	0.3010	0.993	0.900	0.906	10.00	1.0000	0.982	0.981	0.999
2.20	0.3424	0.991	0.920	0.928					

In Figure II.4.5.5 we plot the transmission coefficient as a function of logarithm of the frequency. The roll-on frequency of the HP filter is measured by reading off the frequency from the graph at the point at which transmission is 0.707. This occurs at a frequency $10^{(-0.0148)}$ = 0.966 kHz. This agrees with the calculated value.

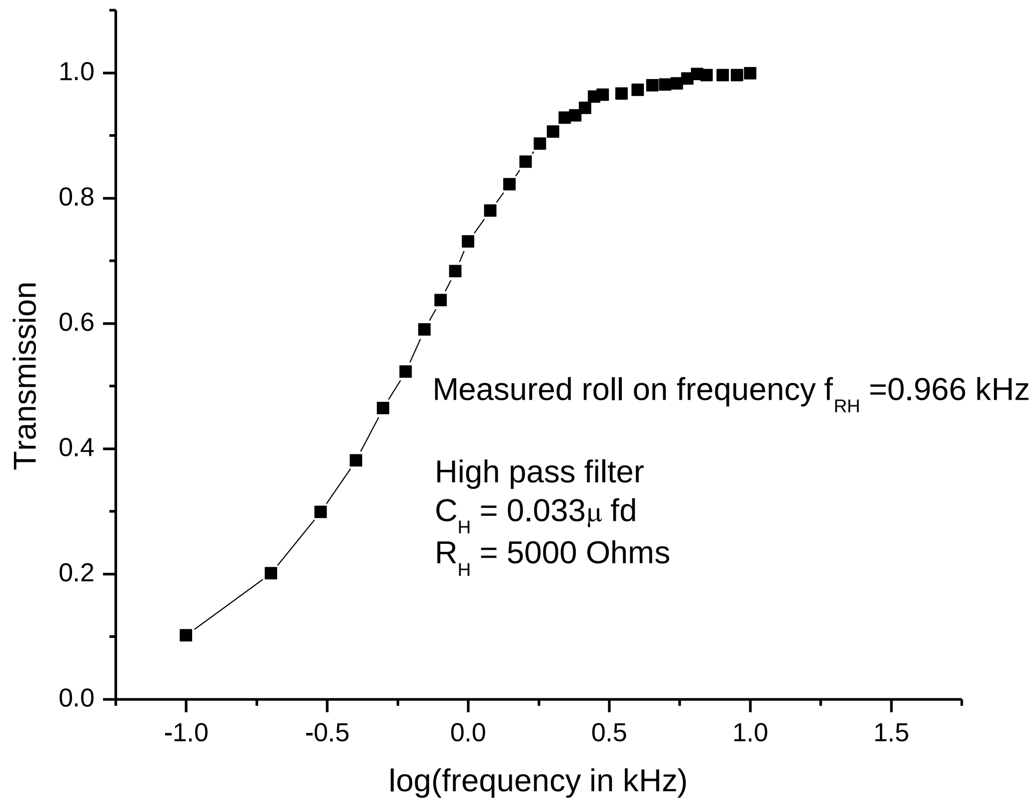


Figure II.4.5.5: Transmission coefficient as a function of log(frequency) for a high pass filter.

7. BANDPASS FILTER

To measure the transmission characteristic of the band pass filter connect the signal generator between the L_1 and L_3 terminals on the Low Pass side. Check if L_3 and H_3 are shorted. If not connect them together. Connect the L_2 terminal on the low pass side to the H_1 terminal on the high pass side. The output from the LP filter is now the input to the HP filter. V_{input} is measured between L_1 and L_3 terminals on the LOW PASS side while V_{output} is measured between H_2 and H_3 terminals on the HIGH PASS side. The rest of the procedure is the same as before.

A sample set of data is given in Table II.4.5.3.

Table II.4.5.3
Transmission coefficient of a band pass filter
f_{RHP} = 0.966 kHz; f_{RLP} = 4.92 kHz
f_{max} ~ 2.42 kHz

f kHz	log(f)	V_{input} Volts	V_{output} Volts	T	f kHz	log(f)	V_{input} Volts	V_{output} Volts	T
0.2	-0.6990	1.003	0.205	0.204	4.0	0.6021	0.994	0.748	0.753
0.4	-0.3979	1.005	0.390	0.388	4.5	0.6532	0.992	0.705	0.711
0.6	-0.2219	1.002	0.524	0.523	5.0	0.6990	0.996	0.675	0.678
0.8	-0.0969	0.998	0.624	0.625	5.5	0.7404	1.000	0.643	0.643
1.0	0	0.996	0.714	0.717	6.0	0.7782	1.002	0.620	0.619
1.5	0.1761	1.000	0.810	0.810	6.5	0.8129	0.995	0.585	0.588
2.0	0.3010	1.002	0.835	0.833	7.0	0.8451	1.001	0.575	0.574
2.5	0.3979	1.005	0.840	0.836	8.0	0.9031	0.993	0.510	0.514
3.0	0.4771	1.000	0.820	0.820	9.0	0.9542	0.995	0.467	0.469
3.5	0.5441	0.998	0.782	0.784	10.0	1.0000	0.986	0.420	0.426

The transmission coefficient is plotted as a function of log(frequency) in Figure II.3.2.5.6. Maximum transmission occurs at a frequency $f_m = 10^{0.384} = 2.42$ kHz. The value of $(f_{RLP}f_{RHP})^{0.5}$ is at 2.18 kHz. This is in rough agreement with f_m.

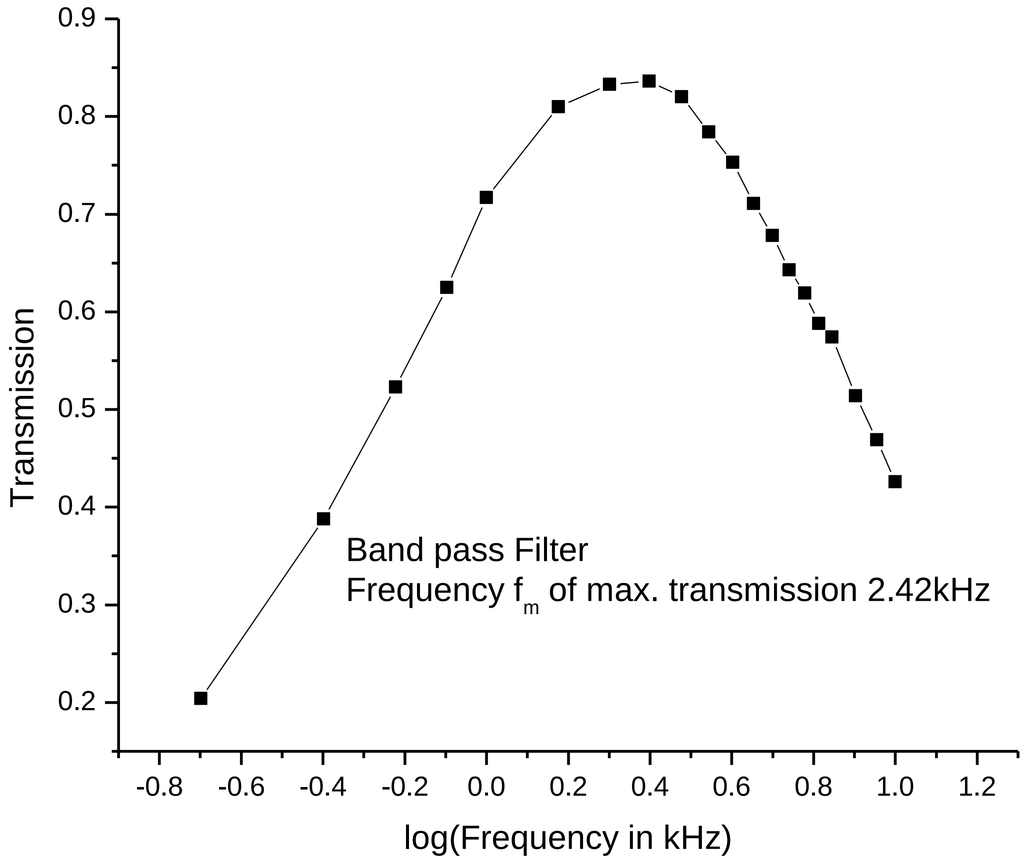


Figure II.4.5.6: Transmission coefficient of a band pass filter
As a function of log(frequency)

Questions:

1. To reduce the roll off frequency of a low pass filter should you increase the resistance or decrease the resistance keeping the capacitance fixed?

2. To decrease the roll on frequency of a high pass filter should you increase or decrease the resistance keeping the capacitance fixed?

3. If the roll on frequency of the high pass section of a band pass filter is more than the roll off frequency of the low pass section, sketch the transmission coefficient as a function of frequency. What would you call such a filter?.

II.5 A C BRIDGE EXPERIMENTS WITH THE SIGNAL GENERATOR AND LOW COST AC BRIDGE

II.5.1 AC WHEATSTONE BRIDGES

1. INTRODUCTION:

An AC Wheatstone bridge has impedances Z_1 to Z_4 connected in the four arms of the Wheatstone bridge as shown in Figure II.5.1.1. An AC signal is applied to the points A and C of the bridge. The AC voltage is measured between the pair of points B and D.

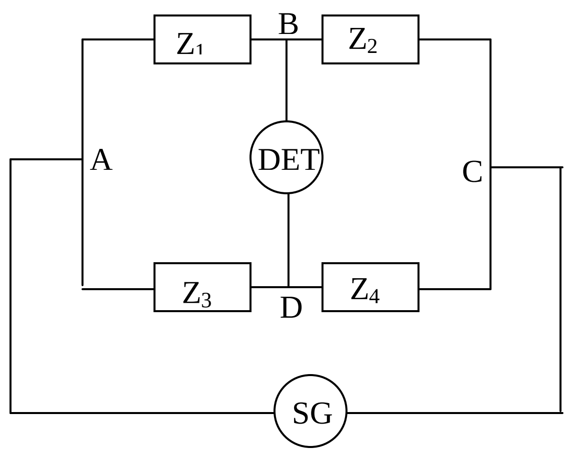


Figure II.5.1.1 AC Wheatstone Bridge

The condition that the bridge is balanced (ie no AC voltage appears at the detector) is

$$Z_1 / Z_2 = Z_3 / Z_4 \qquad \text{(II.5.1.1)}$$

Impedance Z is a complex number

$$Z = |Z| \exp(\iota\phi) \qquad \text{(II.5.1.2)}$$

Two conditions have to be satisfied simultaneously for perfect balance, namely

$$|Z_1|/|Z_2| = |Z_3|/|Z_4| \qquad \text{(II.5.1.3)}$$

and

$$\phi_1 - \phi_2 = \phi_3 - \phi_4 \qquad \text{(II.5.1.4)}$$

On the other hand in a DC Wheatstone bridge all impedance are resistances and only one condition needs to be satisfied namely

$$R_1/R_2 = R_3/R_4 \qquad \text{(II.5.1.5)}$$

The balance in AC bridges to be described in the following chapters will generally not be perfect and we look for a minimum and not zero in the detector voltage. Also since Z depends on the frequency the balance will be frequency dependent.

We consider the following three idealised bridges in which the frequency drops out of the balancing condition and the balance will be independent of the frequency. But even in these bridges the sensitivity of the balance will depend on the frequency. If we change one of the impedances away from the balancing condition, a voltage will appear across the detector. The sensitivity is the amount of change in the impedance to produce a given small amount of unbalanced voltage. The sensitivity will be a maximum if the magnitudes of the impedances in the four arms are nearly equal. One will not be able to choose impedances to satisfy this condition. But there will be one frequency near which the disparity of the impedances in the four arms will be a minimum. At this frequency the bridge will be most sensitive.

In the idealised Maxwell's bridge we take Z_1 and Z_3 to be ideal inductances (resistance $R_L = 0$) and Z_2 and Z_4 to be resistances. In the idealised DeSauty's bridge we take Z_1 and Z_3 to be ideal capacitors (with infinite leakage resistance) and Z_2 and Z_4 are resistors. In the Maxwell-Wien bridge one of the two impedances Z_1 and Z_4 is an ideal inductance and the other is an ideal capacitance. Z_2 and Z_3 are resistances.

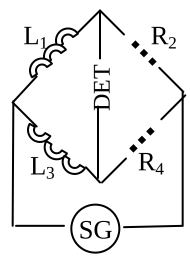
With the low cost AC bridge described in Section I all the three bridges can be realised quite easily by making external connections between the banana sockets. The detector can be a DMM in the AC mV range (if such a DMM is available) connected to the terminals marked D_1,D_2. The impedance Z_2 is a pot the resistance of which can be varied to achieve balance.

If one uses a multi-meter in the AC 200 mV range, the signal generator can be set to an amplitude of 0.5 to 1 V.

II.5.2 MAXWELL'S BRIDGE

1. INTRODUCTION:

In this bridge one compares the self-inductances of two coils L_1 and L_3. These coils are connected as shown in the figure below:



Balancing condition $L_1/L_3 = R_2/R_4$

Figure II.5.2.1: Maxwell's Bridge

2. APPARATUS REQUIRED

Signal generator, AC Bridge set up, a DMM to measure AC volts to three decimal places and a DMM to measure resistance up to 2 k-ohms.

3. EXPERIMENTAL PROCEDURE

This bridge can be realized by using the AC bridge circuit described in Section I. Connect the signal generator between the terminals marked SG_{in} and SG_{out} on the bridge. Adjust the frequency to be about 5 kHz and set an output voltage between 0.5 to 1 V. Connect the terminal A to L_1, terminal B to L_3 and terminal C to R_4. Put the switches S_1 and S_2 on. Connect a multimeter in the AC milli-volts range to the terminals DMM on the bridge. Change the potentiometer R_2 till the reading on the multimeter is a minimum. Then switch off S_1 and S_2 and measure the resistance R_2 between the terminals T_1 and T_2 with a different multimeter. **While balancing the bridge the multimeter to measure resistance should NOT be connected to $T_1$and T_2.**

Connect C to R'_4 and repeat the operations. Then connect B to L_3' and repeat the observations. Then connect A successively to L_1' and L_1'' and repeat the observations. A sample set of readings are given below:

Table II.5.2.1

Frequency: 5 kHz Amplitude 0.5 V

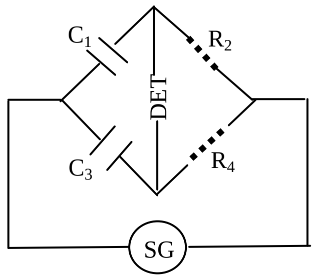
L3 mH	R4 Ohm	R2 Ohm	L1 mH	
L3 3.2 mH	R4 220	249	3.6 mH	
L3 3.2 mH	R4' 440	521	3.8 mH	coil L1
L3' 7.4 mH	R4 220	107	3.6 mH	3.6 mH
L3' 7.4 mH	R4' 440	211	3.5 mH	
			L1' mH	
L3 3.2 mH	R4 220	443	6.4 mH	
L3 3.2 mH	R4' 440	886	6.4 mH	coil L1'
L3' 7.4 mH	R4 220	210	7.1 mH	6.7mH
L3' 7.4 mH	R4' 440	409	6.9 mH	
			L1"mH	
L3 3.2 mH	R4 220	626	9.1 mH	
L3 3.2 mH	R4' 440	1345	9.6 mH	coil L1'
L3' 7.4 mH	R4 220	266	8.9 mH	9.3 mH
L3' 7.4 mH	R4' 440	557	9.4 mH	

NOTE: The coils sold in the market as having 3.6 mH inductance do not all have the same inductance. The inductances vary by as much as 20%. Please take the above readings as sample readings only.

II.5.3 De SAUTY'S BRIDGE

1. INTRODUCTION:

In this bridge one compares the capacitances of two condensers C_1 and C_3. These condensers are connected as shown in the figure below:



Balancing condition
$C_3/C_1 = R_2/R_4$

Figure II.5.3.1: DeSauty's Bridge

2. APPARATUS REQUIRED

Signal generator, AC Bridge set up, a DMM to measure AC volts to three decimal places and a DMM to measure resistance up to 2 k-ohms.

3. EXPERIMENTAL PROCEDURE:

This bridge can be realized by using the AC bridge circuit described in Section I. Connect the signal generator between the terminals marked SG_{in} and SG_{out} on the bridge. Adjust the frequency to be about 5 kHz and the amplitude between 0.5 to1 V. Connect the terminal A to C_1, terminal B to C_3 and terminal C to R_4. Put the switches S_1 and S_2 on. Connect a multimeter in the AC 200 milli-volts range to the terminals DMM in the bridge. Adjust the potentiometer R_2 till the reading on the multimeter is a minimum. Switch off S_1 and S_2 and measure the resistance R_2 between the terminals T_1 and T_2 with another multimeter. **When balancing the bridge the DMM to measure resistance should NOT be connected to terminals T_1 and T_2.**

Now connect C to R'_4 and repeat the operations. Then connect B to C_3' and repeat the observations. Then connect A successively to C_1' and C_1" and repeat the observations. A sample set of readings is given below:

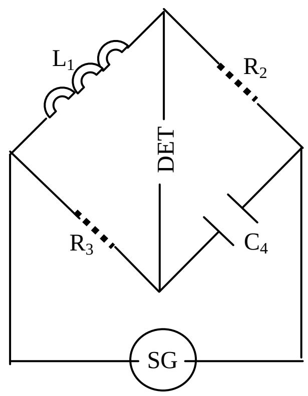
Table II.5.3.1
DeSauty's Bridge

C3 microfd	R4 Ohms	R2 Ohms	C1microfd	Cap C1
C3 0.1	220	222	0.103	0.102
C3 0.1	440	453	0.105	μfd
C3' 0.2	220	429	0.101	
C3' 0.2	440	882	0.100	
				Cap C1'
C3 0.1	220	102	0.210	0.215
C3 0.1	440	206	0.210	μfd
C3' 0.2	220	196	0.220	
C3' 0.2	440	396	0.218	
				Cap C1"
C3 0.1	220	49	0.441	0.446
C3 0.1	440	100	0.440	μfd
C3' 0.2	220	95	0.455	
C3' 0.2	440	192	0.450	

II.5.4 MAXWELL-WIEN BRIDGE

1. INTRODUCTION:

In this bridge one compares the self-inductances of a coil L_1 with the capacitance of a condenser C_4. The coil and condenser are connected as shown in the figure below:



Balancing condition
$L_1 = C_4R_2R_3$

Figure II.5.4.1 Maxwell-Wien Bridge

2. APPARATUS REQUIRED

Signal generator, AC Bridge set up, a DMM to measure AC volts to three decimal places and a DMM to measure resistance up to 2 k-ohms.

3. EXPERIMENTAL PROCEDURE:

This bridge can be realized by using the AC bridge circuit described in Section I. Connect the signal generator between the terminals marked SG_{in} and SG_{out} on the bridge. Adjust the frequency to be about 5 kHz and the amplitude between 0.5 and 1 Volt. Connect the terminal A to L_1, terminal B to R_3 and terminal C to C_4. Put the switches S_1 and S_2 on. Connect a multimeter in the AC 200 millivolts range to the terminals DMM on the bridge. Adjust the potentiometer R_2 till the reading on the multimeter is a minimum. Switch off S_1 and S_2 and measure the resistance R_2 between the terminals T_1 and T_2. Now connect B to R'_3 and repeat the operations. Then connect A successively to L_1' and L_1" and repeat the observations. Then connect C to C_4' and repeat the observations. A sample set of readings is given below:

Table: II.5.4.1

Maxwell-Wien Bridge

L1 mH	R3 Ohms	R2 Ohms	C4 μfd
L1 3.6 mH	R3 220	346	0.047
L1 3.6 mH	R3’ 320	209	0.054
L1’ 6.7mH	R3 220	668	0.046
L1’ 6.7mH	R3’ 320	447	0.047
L1” 9.3mH	R3 220	916	0.046
L1” 9.3mH	R3’ 320	601	0.048
L1 mH	R3 Ohms	R2 Ohms	C4' mfd
L1 3.6 mH	R3 220	166	0.098
L1 3.6 mH	R3’ 320	110	0.102
L1’ 6.7mH	R3 220	314	0.097
L1’ 6.7mH	R3’ 320	209	0.100
L1” 9.3mH	R3 220	458	0.093
L1” 9.3mH	R3’ 320	267	0.108

So C_4 is 0.048 microfarad and C_4’ is 0.100 microfarad.

Questions

1. If the frequency of the AC signal is changed will the balance of the above bridges be destroyed?

2. Is the sensitivity of the bridge dependent on frequency? When do you think the sensitivity of the bridge will be a maximum?

3. If an inductance is connected instead of a capacitance in the fourth arm of the Maxwell-Wien Bridge, can the bridge be balanced? Justify your answer.

4. Can I use a steady DC signal to balance any of the bridges discussed above?

II.6 EXPERIMENTS ON CAPACITANCE AND DIELECTRIC CONSTANT

II.6.1 COMPARISON OF CAPACITANCES AND VERIFICATION OF THE LAW OF ADDITION OF CAPACITANCES

1. INTRODUCTION

In a small box three capacitances with nominal values of 100 pfd, 47 pfd and 22 pfd are soldered respectively between a Red, Yellow, and Black terminal and a common (Green) terminal. Thus between green and black terminals one has 22 pfd, between green and yellow 47 pfd and between green and red 100 pfd. Between black and yellow terminals the 22 pfd and 47 pfd are in series. Between black and red, the 22 pfd and 100 pfd are in series and between yellow and red the 47 pfd and 100 pfd are in series. If black and yellow terminals are externally connected, and we measure between green and black (or yellow) terminals, we will have 22 pfd in parallel with 47 pfd. If red and black are externally connected, and we measure between green and black (or red) terminals, we will have 22 pfd and 100 pfd in parallel. When yellow and red terminals are externally connected, and we measure between green and yellow (or red) terminals we have 47 pfd in parallel with 100 pfd.

Figure II.6.1.1 Connections in the capacitance box

2. APPARATUS REQUIRED:

Capacitance measuring circuit, capacitance box and a DMM measuring DC 2 V to three decimal places.

3. PROCEDURE

Connect the terminals marked C on the capacitance circuit described in Section I to the black and red terminals on the box. Switch on the capacitance circuit and note the reading on a DMM in the DC 20 V range connected to the output terminals of the capacitance meter. Then change the connection from the red terminal to the yellow and black in succession and note the readings. These three readings will correspond to the nominal values of the capacitances 100, 47 and 22 pfd respectively.

To measure the capacitances in series connect the terminals red and yellow on the box to the capacitance circuit and measure the DC multimeter reading. This corresponds to 100 pfd in series with 47 pfd. Then change the connections on the box to the pair red and black and then yellow and black. This measures the capacitance of the series combination of 100 pfd and 22 pfd and the series combination of 47 pfd and 22 pfd respectively.

Short the yellow and red terminals on the box with a wire. Connect the green and red terminals on the box to the terminals marked C on the capacitance circuit. The DMM reading now corresponds to the parallel combination of 100 and 47 pfd capacitances. Short red and black terminals on the box and connect the red and green terminals to the terminals marked C on the capacitance circuit. This measures the capacitance of the parallel combination of 100 and 22 pfd capacitances. Then short the yellow and black terminals on the box and connect the green and black terminals to the terminals marked C on the capacitance circuit. This measures the capacitance of the parallel combination of 47 and 22 pfd capacitances.

Taking the 100 pfd capacitance as standard and assuming the output voltage is proportional to the capacitance, calculate the values of the capacitances for the various combinations. This is shown in Table II.6.1.1 below.

Table II.6.1.1: Comparison of capacitances

Assume capacitance between red and green is 100 pfd

		Calc Cap	Nominal
Connection	DC Volts	μf	Value μf
Green and black	0.45	20.5	22
Green & Yellow	1.10	50.0	47
Green & red	2.20	100	100
Black & Yellow	0.31	14.5	15.0
Black & Red	0.35	17.0	18.0
Yellow & red	0.70	33.3	32.0
Black & Yellow short			
Between black & Green	1.69	76.8	69
Red and black short			
Between black & green	2.83	128.6	122
Red and Yellow short			
Between yellow & green	3.45	156.8	147

We see that the results are in reasonable agreement with the nominal values. This shows that the output voltage of our capacitance circuit varies linearly with capacitance. We have also verified the law of addition of capacitances.

II.6.2 DIELECTRIC CONSTANT OF A NON-POLAR LIQUID

1. INTRODUCTION

A molecule in which the center of gravity (CG) of negative charge is displaced from the center of gravity of the positive charge is said to have a dipole moment. In a highly symmetric molecule, like Carbon tetrachloride CCl_4 and benzene C_6H_6, the CG of positive and negative charges will coincide (at the point marked x in Figure II.6.2.1 for benzene) and the molecule has no electrical dipole moment. In asymmetric molecules like $CHCl_3$ and acetone, there is a dipole moment per molecule.

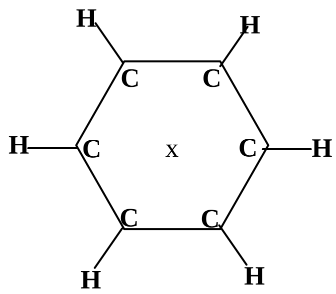


Figure II.6.2.1: Structure of the benzene molecule
C is carbon atom and H is hydrogen atom.
x marks the center of the molecule.

When an electric field is applied there can be a shift of the CG of negative charge relative to the CG of positive charge leading to a dipole moment induced in the direction of the field, even in molecules that do not have a permanent dipole moment, like a molecule of benzene. This leads to the relation

$$\mathbf{p}_{\text{induced}} = \varepsilon_o \alpha \mathbf{E}_{\text{local}} \qquad \text{(II.6.2.1)}$$

Here ε_0 is the electric permittivity of free space and has the value 8.854×10^{-12} Farad/m and $\mathbf{E}_{\text{local}}$ is the local electric field at the site of the molecule. $\mathbf{p}_{\text{induced}}$ is the induced dipole moment and α is called the electronic polarizability of the molecule. The local electric field is the sum of the applied electric field **E** and the electric field arising from the induced dipoles. In a gas the density of the molecules is very low. So the field due to the induced dipoles can be neglected in comparison to the applied field **E.** If there are N molecules per unit volume the electric polarization **P,** which is the dipole moment per unit volume, is

$$\mathbf{P} = N\mathbf{p}_{\text{ind}} = \varepsilon_0 N\alpha \mathbf{E} \qquad \text{(II.6.2.2)}$$

The electrical displacement **D** is

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \varepsilon_r \mathbf{E} \qquad \text{(II.6.2.3)}$$

ε_r is the dielectric constant of the material.

So for a gas $\varepsilon_r = 1 + N\alpha$ (II.6.2.4)

In a liquid the density is high and the local electric field is the sum of the applied electric field **E** and the electric field due to the polarization **P** in the medium.

$$\mathbf{E}_{local} = \mathbf{E} + \mathbf{P}/ (3\,\varepsilon_o) \qquad \text{(II.6.2.5)}$$

The polarization **P** is itself given by

$$\mathbf{P} = \varepsilon_0 N\alpha\ \mathbf{E}_{local} = \varepsilon_0 N\alpha\ (\mathbf{E} + \mathbf{P}/3\,\varepsilon_o) \qquad \text{(II.6.2.6)}$$

Solving for **P** in terms of **E** we have

$$\mathbf{P} = \varepsilon_0 N\alpha\ \mathbf{E} / (1 - N\alpha/3) \qquad \text{(II.6.2.7)}$$

and

$$\mathbf{D} = \varepsilon_o\mathbf{E} + \mathbf{P} = \varepsilon_o\mathbf{E}\ [1 + (N\alpha / (1 - N\alpha/3))] = \varepsilon_o\,\varepsilon_r\,\mathbf{E} \qquad \text{(II.6.2.8)}$$

From this we get

$$(\varepsilon_r - 1)/ (\varepsilon_r + 2) = N\alpha/3 \qquad \text{(II.6.2.9)}$$

This is called the Clausius-Mosotti relation.

Note that from a measurement of the dielectric constant and knowledge of the density of the liquid one can obtain the value of the polarizability α of the molecule. The polarizability has the dimensions of volume and can be written in terms of (nano-meter)^3.

2. APPARATUS REQUIRED:

Capacitance measuring circuit, cylindrical capacitance in a jar, a DMM measuring to three decimal places in the DC 2 V range and Analar grade benzene.

3. EXPERIMENTAL DETAILS

For this measurement a cylindrical capacitor of 15 cm is provided. The cylindrical capacitor is suspended in a tall cylindrical graduated plastic jar (of volume 100 ml). The cylindrical capacitor consists of two coaxial metal tubes of different radii insulated from each other. Two leads taken out of the tubes are connected to banana plugs on the insulating mount on top of the jar. Such a cylindrical capacitor will have a capacitance of a few tens of pico-farads.

Figure II.6.2.2 Measuring jar with a cylindrical capacitor inside

The capacitance of the leads may be a sizeable fraction of the capacitance of the cylindrical condenser if the connecting leads are large in diameter and are close to each other. The capacitance of the cylindrical capacitor is of the order of 10 to 15 pfd. To reduce the lead capacitance use thin wires for connection and separate the wires by a large distance. Connect the black terminal on the cylindrical capacitance to the ground terminal of the pair C on the capacitance circuit. Bring the wire from the other terminal of the pair C close to the red terminal of the cylindrical capacitor. Without connecting it to the red terminal note the DC reading on the DMM (in the DC 2 V range) connected to the output of the capacitance meter. Let it be V_0. Then connect the lead to the red terminal and measure the voltage. Let it be V_{air}. Then $(V_{air}-V_0)$ is proportional to the capacitance of the cylindrical condenser with air as the medium.

Analar grade (this means high purity) benzene (or carbon tetrachloride) is added in approximately 10 ml quantities into the jar through a tube provided in the cap of the capacitor. If the liquid spills out it may damage the plastic terminals. The liquid is now added in steps of 10 ml and the DC voltage is noted on the DMM after each such addition. Please wait for a minute or two after adding the liquid for the reading on the DMM to settle down to a steady value. Note the level of the liquid in milli-liters from the graduation on the jar for each addition of the liquid.

The jars purchased in the market may be of different cross-sections. So in some the top of the cylindrical capacitor may be above the 100ml mark on the jar. In the others the top of the cylindrical capacitor may be below the 100ml mark. Take readings of the output voltage till you fill the jar up to 80 ml. Then note the height of the jar, h, from 0 to 100 ml. Find the height of the top of the cylindrical capacitor from the bottom of the jar. Let it be h'. So h' would correspond to (h'/h)x100 = v milliliters of liquid. This is the volume of the liquid to fill the capacitor. In the case of the sample readings given below this corresponds to 90 ml.

A graph is plotted of the DC voltage against the volume of benzene. A straight line is fitted on the computer to the points and the equation of the straight line is noted as

$$V = V' + bm \qquad \text{(II.6.2.10)}$$

where V is the DC voltage with the liquid, V' is the intercept, b is the slope and m is the volume of the liquid in milliliters. The reading corresponding to the situation when the liquid is filled to reach the top of the cylindrical capacitor is obtained from equation (II.6.2.10) by putting m = v i.e.

$$V_{liq} = a + bv. \qquad \text{(II.6.2.11)}$$

V_{liq} includes the contribution V_0 of the leads. So the voltage due to cylindrical capacitance filled with liquid, omitting the capacitance of the leads, will be $(V_{liq}-V_0)$.

The dielectric constant of the liquid is

$$\varepsilon_r = \text{capacitance with liquid/capacitance with air}$$
$$= (V_{liq} - V_0)/(V_{air}-V_0) \qquad \text{(II.6.2.12)}$$

A sample set of readings is given below:

Table II.6.2.1

The voltage due to leads V_0 = 0.020 V
The voltage of capacitance in air with leads V_{air} = 0.412 V
When benzene is filled in the jar the following readings were obtained

Milli-liter	V in mV
0	412
10	459
20	510
30	555
40	606
50	650
60	705
70	750
80	800

Figure II.6.2.2 shows a plot of the readings in Table II.6.2.1. The linear fit to the points is shown. The intercept is 411.2 mV and the slope is 4.85 mV/ml.

The voltage V_{liq} when the capacitor is filled with liquid is

$$V_{liq} = 411.2+4.85\text{x}90 = 847.7 \text{ mV} = 0.848 \text{ V}.$$

The dielectric constant of benzene is

$$\varepsilon_r = (V_{liq} - V_0)/(V_{air}-V_0) = (0.848-0.020)/(0.412-0.020) = 2.11 \qquad \text{(II.6.2. 13)}$$

For benzene the value of the dielectric constant from tables is 2.25 at 300 K.

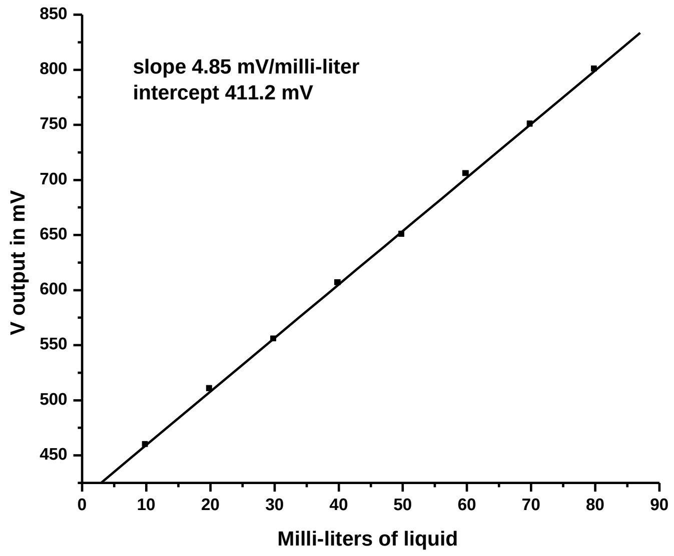


Figure II.6.2.3: Plot of DC Voltage against ml of benzene filled

Use the Clausius Mosotti relation

$$(\varepsilon-1)/(\varepsilon+2) = N_B\alpha/3$$

where N_B is the number of benzene molecules per unit volume and α is the electronic polarizability of benzene.

The molecular weight of benzene (C_6H_6) is 78 gm and its density, ρ_B, is 0.899 g/cc. So 78 grams of benzene contain the Avogadro number N_A ($6.022 x 10^{23}$) of molecules. 1cc of benzene has a mass 0.899 g. The number of molecules in 1 cc is

$$N_B = 6.022x10^{23}x0.899/78 = 6.94x10^{21} \text{ molecules/cc.} \qquad \text{(II.6.2.14)}$$

In the SI units the unit of volume is 1 $m^3 = 10^6$cc. So the number N_B of molecules in 1 m^3 is

$$N_B = 6.94x10^{21}x10^6 = 6.94x10^{27} /m^3 \qquad \text{(II.6.2.15)}$$

Substituting for ε and for N_B

α for benzene = $3x((2.10-1)/(2.10+2))/6.94x10^{27} = 0.116x10^{-27}$ m^3

$= 0.116$ (nano-meter)^3

Note that the electronic polarizability of a molecule has the dimensions of volume and roughly gives the size of the molecule.

II.6.3 DIPOLE MOMENT OF AN ORGANIC MOLECULE, ACETONE

1. INTRODUCTION:

A molecule in which the center of gravity of negative charge is displaced from the center of gravity of the positive charge is said to have an electric dipole moment. In a highly symmetric molecule like Carbon tetrachloride CCl_4 and benzene C_6H_6, the CG of positive and negative charges will coincide and the molecule has no dipole moment as mentioned earlier. In asymmetric molecules like $CHCl_3$ and acetone, there is a dipole moment per molecule.

Consider a molecule that has a permanent dipole moment **p** even in the absence of an applied electric field. The moments on different molecules in the material are oriented at random in the absence of the field. This random orientation arises due to thermal agitation. When an electric field is applied, it tries to align the moments in the direction of the field and this is opposed by thermal energy.

The energy of a dipole in an electric field is

$$-\,\mathbf{p}.\mathbf{E}_{local} = -pE_{local}\cos\theta \qquad \text{(II.6.3.1)}$$

where θ is the angle between **p** and $\mathbf{E}_{local}$.

The probability that a dipole is oriented at an angle θ to the electric field is proportional to the Boltzman factor exp $[-(-pE_{local}\cos\theta /kT)$ and the average dipole moment per molecule in the direction of the local electric field is

$$p_{av} = 2\pi\int_0^\pi p\cos\theta \exp(pE_{local}\cos\theta /kT)\sin\theta d\theta \,/\, 2\pi\int_0^\pi \exp(pE_{local}\cos\theta /kT)\sin\theta\, d\theta$$

When $pE_{local}/kT \ll 1$, the above integral is nearly equal to

$$= [p^2/ 3kT]\, E_{local} = \varepsilon_0\, \alpha_{orien}\, E_{local} \qquad \text{(II.6.3.2)}$$

α_{orien} is called the orientational contribution to the polarizability.

For a molecule with a permanent dipole moment p, the total polarizability α is the sum of the electronic and orientational polarisabilities.

$$\alpha = \alpha_{el} + \alpha_{orien} = \alpha_{el} + (p^2/\varepsilon_0)/ 3kT \qquad \text{(II.6.3.3)}$$

So

$$(\varepsilon_r - 1)/ (\varepsilon_r + 2) = N\alpha/3 = N\,[\alpha_{el} + (p^2/\varepsilon_0)/3kT]/3 \qquad \text{(II.6.3.3)}$$

where N is the number of molecules per unit volume.

The orientational contribution to the polarizability often outweighs the electronic contribution. The dielectric constant is then strongly dependent on temperature and increases as the temperature is reduced.

If we have a non-polar liquid G in which a polar liquid H is dissolved in small quantities, the dielectric constant of the mixture satisfies the relation

$$(\varepsilon_r - 1)/ (\varepsilon_r + 2) = (1/3)\,[N_G\,\alpha_{el\,G} + N_H\,(p_H^2\,/3\varepsilon_0 kT)] \qquad \text{(II.6.3.4)}$$

Here we have neglected the electronic contribution to the polarizability of the molecule H in relation to its orientational contribution. N_G and N_H are respectively the number of molecules per cubic meter of liquid G and liquid H in the mixture. By adding small quantities of liquid H to liquid G and varying the relative concentration of G and H one can plot the dielectric constant of the mixture as a function of the concentration N_H. For low concentrations of liquid H (i.e. a few percent) the variation will be linear. From this plot one can calculate the dipole moment p_H of molecule H. One can also measure the temperature variation of the dielectric constant for a mixture with a given concentration of H in G. A plot of the dielectric constant as a function of 1/T is used to get the dipole moment $p_{H.}$

The dielectric constant of any material is a function of frequency. The electronic motions are characterized by frequencies in the ultra-violet. The vibrational motions of the atoms in the molecule are in the infra-red region. The orientational motions of molecular groups are in the far infra red and microwave region. As the frequency is varied, the dielectric constant shows a variation as shown in Figure II.6.3.1.

Near the characteristic frequencies of orientation, vibration and electronic motions, the dielectric constant goes through a maximum and a minimum. The region of frequency between the maximum and the neighboring minimum (the region in which the dielectric constant decreases) is called the region of anomalous dispersion.

To calculate the dipole moment one should work at frequencies low compared to the orientation frequency. A frequency up to a few hundred kilo-hertz is normally OK though in some materials one may see a noticeable dispersion in this region.

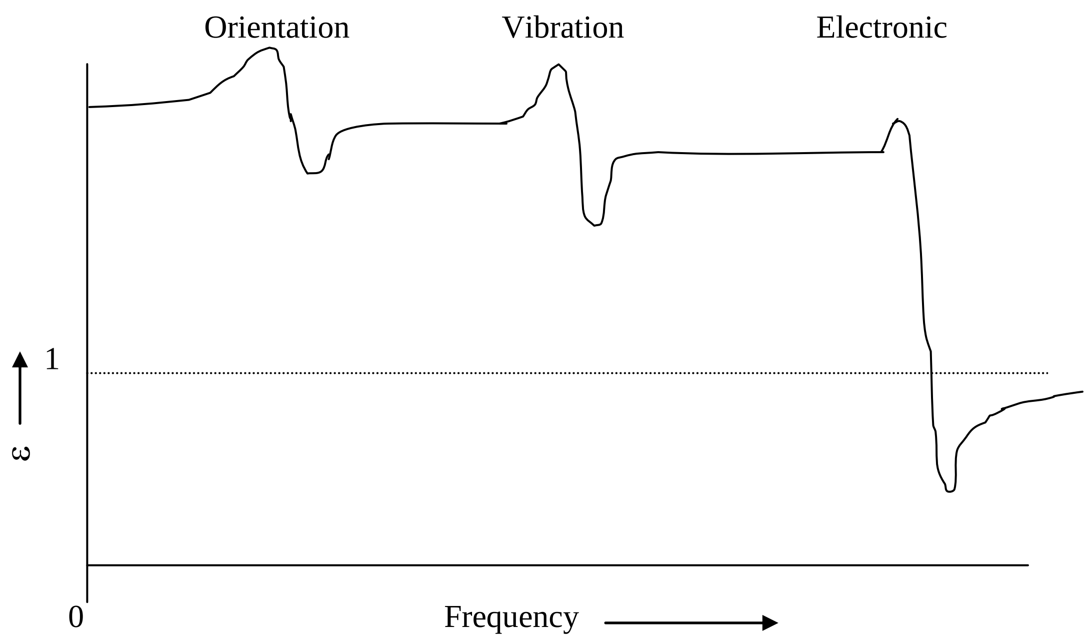


Figure II.6.3.1   Variation of dielectric constant of a material with frequency showing the orientational, vibrational and electronic contributions

2. APPARATUS REQUIRED

Capacitance measuring circuit, cylindrical capacitance in a graduated jar, a DMM to measure to three decimal places in the DC 2 V range and mixtures of different composition of acetone and benzene in wash bottles.

3. DIPOLE MOMENT OF ACETONE: EXPERIMENT

Acetone is a molecule that has a permanent dipole moment. Its polarizability at low frequencies arises mainly from the orientational contribution of the dipole. To measure the dipole moment of the molecule, mix 100 milliliters of Benzene and v_{AC} milliliters of acetone. For example we can prepare three mixtures, with v_{AC} =10,15 and 20 milliliters. **These mixtures have to be stored in an airtight bottle so that no water is adsorbed by the mixtures.**

The dielectric constant of each mixture is measured following the procedure described with benzene.

Let the jth mixture contain 100 milliliters of benzene and v_{jAC} milliliters of Acetone. Let the dielectric constant of this mixture be ε_{mj}. Then we calculate the value of

$$K_j = \{[(\varepsilon_{mj} - 1)/(\varepsilon_{mj} + 2)*(v_{100}+v_{jAC})] - [(\varepsilon_B - 1)/(\varepsilon_B + 2)*100]\}/ v_{jAC} \qquad \text{(II. 6.3.5)}$$

for this mixture j. Here ε_B is the dielectric constant of benzene, and ε_{mj} is the dielectric constant of the jth mixture. This value K_j should be the same for all the mixtures j. Take the average value K_{av}. Then

$$K_{av} = (p_{AC}^2/9\ \varepsilon_0\ kT)\ N_{AC} \qquad \text{(II.6.3.6)}$$

where N_{AC} is the number of molecules per meter^3 of pure Acetone, k is the Boltzman constant, T is the absolute temperature .

Sample readings are given below:

Table II.6.3.1

Dipole moment of Acetone

DC Voltage due to leads V_0 = 0.020 V
DC Voltage with the capacitor in air: V_{air} 0.412 V
Three mixtures were made by mixing 10, 15 and 20 ml of Acetone (AC) with 100 ml of Benzene (B) The DC Voltages for different fillings in ml are given.

	Mixture 1	Mixture 2	Mixture 3
	$100C_6H_6$+10 Acetone	$100C_6H_6$+15 Acetone	$100C_6H_6$+20 Acetone
	V= 427+8.32 m	V= 447+9.89m	V=444+13.46m
milliliters	V in mV	V in mV	V in mV
0	427	447	444
10	512	547	580
20	591	642	713
30	680	742	850
40	755	845	980
50	843	945	1120
60	928	1040	1250
70	1010	1135	1390
80	1088	1242	1520

As in the previous chapter a graph is drawn between the voltage and milliliters of liquid and a straight line is fitted. The fitted equation for the straight line is given in the third row of the table. From these equations the dielectric constant of the mixtures are evaluated as described in the previous chapter.

The dielectric constants of various mixtures and the dielectric constant of pure benzene are given in Table II.6.3.2 and the values of K are given in the last column.

Table II.6.3.2

Liquid	ε_r	$(\varepsilon_r-1)/(\varepsilon_r+2)$	K
Benzene	2.1	0.2683	
Mix1	2.95	0.3939	1.65
Mix2	3.36	0.4403	1.59
Mix3	4.17	0.5138	1.74
		Average K	1.66

The average value of K = 1.66.

The molecular weight of Acetone (C_3H_6O) is 58 g. Its density is 0.792 g/cc. So the number of acetone molecules N_{AC} per m^3 is

$$N_{AC} = (6.022\text{x}10^{23} \text{ x } 0.792/58)\text{x}10^{6} = 8.49\text{x}10^{27}/\text{m}^3 \qquad \text{(II.6.3.7)}$$

Dipole moment of acetone molecule is

$$p = [(K/N_{AC})\,(9\varepsilon_0\, kT)]^{1/2} \qquad \text{(II.6.3.8)}$$

Here $\varepsilon_0 = 8.85\text{x}10^{-12}$ fd/m
$k = 1.36\text{x}10^{-23}$ J/Kelvin
$T = 300$ K

Substituting for the various quantities we get

$$p = 8.0\text{x}10^{-30} \text{ coulomb meter.}$$

From the literature the electrical dipole moment of acetone molecule is 2.88 Debye (1 Debye = $3.34\text{x}10^{-30}$ coulomb meter) or $9.6\text{x}10^{-30}$ coulomb meter.

The experiment is intended to show the order of dipole moment values in molecules. The electronic charge is of the order of 10^{-19} Coulombs and the relative displacement of the CG of positive charge relative to CG of negative charge is of the order of a fraction of an Angstrom ie of the order of 10^{-11} m. So the dipole moment of a molecule is of the order of $10^{-19} \mathrm{x} 10^{-11} = 10^{-30}$ coulomb meter.

CAUTION: NEVER USE WATER WITH THE CYLINDRICAL CAPACITOR

Questions:

1. In a cylindrical capacitor will the capacitance increase if the ratio r_2/r_1 is increased? Here r_2 is the radius of the outer cylinder and r_1 the radius of the inner cylinder.

2. Can you have a liquid for which the dielectric constant at very low frequencies is less than 1?

3. Can you think of a way of controlling the level of a liquid using its dielectric constant?

4. Derive the formula involving p and K (Equation II.6.3.5).

5. Water has a dielectric constant of 80. Do you expect the water molecule to have a large electric dipole moment?

II.6.4 TEMPERATURE DEPENDENCE OF THE CAPACITANCE OF A CERAMIC CAPACITOR

VERIFICATION OF CURIE-WEISS LAW FOR THE ELECTRICAL SUSCEPTIBITY OF A FERROELECTRIC MATERIAL

1. INTRODUCTION

The electrical susceptibility χ of a material is the ratio of change of the electric polarization **P** in the material to the change in the applied electric field **E. For an isotropic material the vector P is in the direction of E and the susceptibility is a number**

$$\boldsymbol{\varepsilon}_0\, \chi = dP/dE \qquad \text{(II.6.4.1)}$$

For a linear dielectric material, P is proportional to E, and χ is independent of the field as long as the field is not very high. This behaviour is seen in all materials at high temperatures. Such behaviour is called paraelectric in analogy with paramagnetic behaviour. In these materials there is no electric polarization in the absence of an electric field.

There is a class of solid materials called ferroelectric materials. Below a temperature T_C characteristic of a ferroelectric material, the material exhibits a spontaneous electric polarization even in the absence of an electric field. Above T_C these materials show paraelectric behaviour. The variation of P with E above and below T_C in the para- and ferro- electric states are shown in figures II.6.4.1 (a) and 1(b). In the paraelectric state the material shows no hysteresis while in the ferroelectric state the material shows hysteresis. The P vs. E curve in a ferro-electric material is similar to the Magnetization vs magnetic field curve for a ferromagnetic material. Hence these materials are called ferroelectric. Examples of ferroelectric materials are Potassium di-hydrogen phosphate (T_C = -151^0 C), Triglycine sulphate (47^0 C), Rochelle salt (it is ferro-electric between -18^0 C to 24^0 C), and Barium titanate (T_C 128^0 C).

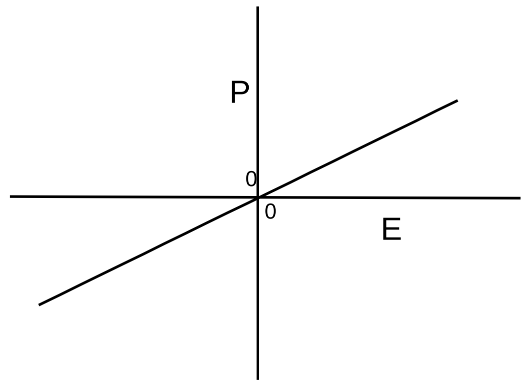


Figure II.6.4.1 (a) Variation of P with E in the paraelectric state of a material

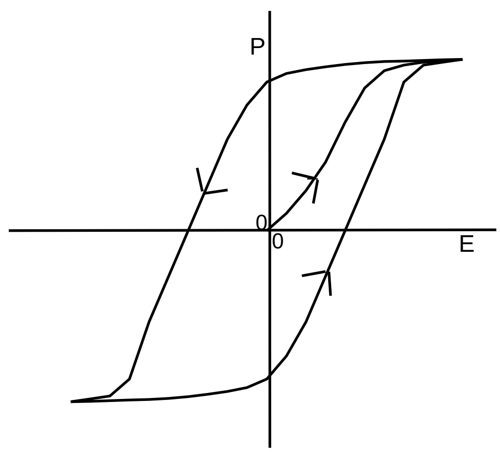


Figure II.6.4.1 (b) Variation of P with E in the ferroelectric state of a material

Above T_C (i.e.in the paraelectric state), the susceptibility of a ferroelectric material varies strongly with temperature following the Curie-Weiss law

$$\chi = C/ (T\text{-}\Theta) \tag{II.6.4.2}$$

Here C and Θ are constants characteristic of a given material. Θ has a value close to the transition temperature T_C. This law, derived in the mean field approximation of Landau, is valid at temperatures not too close to T_C (i.e. not closer than about 10^0 C). Near T_C fluctuations will play a dominant role. For a ferroelectric material with a T_C close to room temperature the susceptibility will have a very high value (of the order of hundreds and thousands). The dielectric constant

$$\varepsilon = 1+\chi = 1+C/\ (T-\Theta) \approx C/(T-\Theta) \qquad \text{(II.6.4.3)}$$

for a ferroelectric material. Because of the large dielectric constant a capacitance made of such a material will have a relatively small size.

The so called ceramic capacitances available in the market at low cost are made of materials with T_C around 10 to 20 C. These materials can be used to verify the Curie-Weiss law. The capacitance, being proportional to the dielectric constant, will show a strong dependence on temperature, the capacitance decreasing as the temperature increases.

On the other hand one can also get capacitances in the market with a polymer dielectric material. The dielectric constant of these polymer materials is not large and the dielectric constant shows a very weak dependence on temperature. Capacitances with such materials as dielectrics show a weak dependence on temperature.

2. AIM

To measure the capacitances of a ceramic capacitor and a polymer capacitor as a function of temperature in the range 40 to 130 C; to show that the capacitance of the ceramic capacitor shows a strong dependence on temperature while that of a polymeric capacitor shows a weak dependence and to verify the Curie-Weiss law for the ceramic capacitor.

3. APPARATUS REQUIRED

A regulated DC power supply, temperature controller, furnace, insert with ceramic and polymeric capacitors, a signal generator and a DMM to measure AC volts to three decimal places in the range of 2 Volts.

4. EXPERIMENTAL SET UP

A ceramic capacitor and a polymeric capacitor, each of a nominal **v**alue of 0.1 μfd, are pasted with Araldite on two faces of an aluminium block which also carries a Pt 100 sensor. Leads from the capacitor are brought to a circular disc carrying eight small banana terminals as shown in Figure II.6.4.2. The two central unmarked terminals are for the leads of Pt 100 thermometer. The ceramic capacitor leads are connected to the terminals R_1 (Red) and G_1 (Green) marked Ceramic. Similarly the terminals R_2 and G_2 are connected to the leads from the capacitance with polymeric dielectric. Between G_1 and B_1 (Black) is connected a 1 k resistor at the top disc. Similarly between G_2 and B_2 is connected a 1 k resistor at the disc. Terminals R_1 and R_2 are shorted at the disc (as shown by the dotted line). So also, terminals B_1 and B_2 are shorted. If a signal generator is connected between R_1 and B_1 the same AC voltage is applied to both the RC combinations as shown in Figure II.6.4.3.

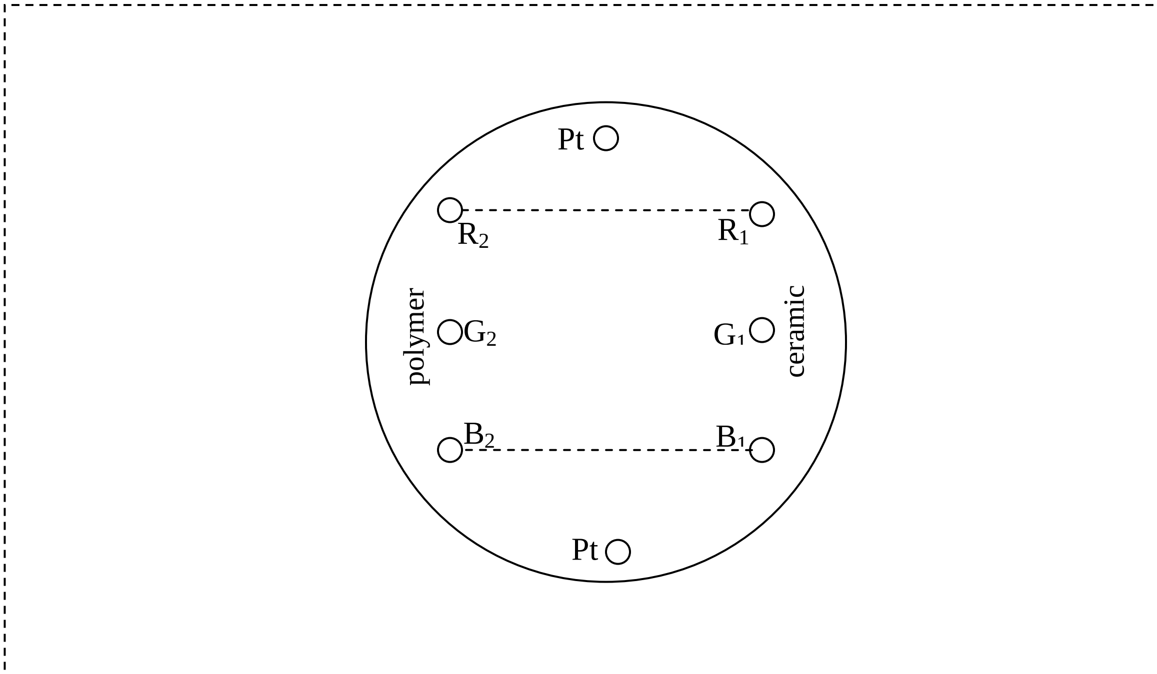


Figure II.6.4.2: Circular disc containing the banana terminals

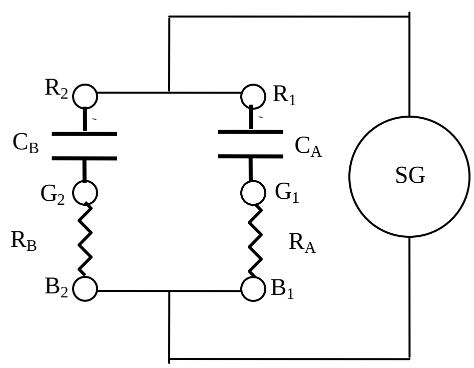


Figure II.6.4.3: C_A Ceramic capacitor, R_A = 1 k;
C_B Polymer capacitor, R_B = 1k
SG Signal generator

If we measure the voltage V_{RA} across R_1 and G_1 and the voltage V_{CA} across G_1 and B_1 then

$$V_{CA}/V_{RA} = Z_{CA}/R_A \qquad \text{(II.6.4.4)}$$

where Z_{CA} is the impedance of the capacitor C_A and is equal to

$$Z_{CA} = 1/(2\pi f C_A) \qquad (II.6.4.5)$$

So

$$C_A = 1/(2\pi f\ (V_{CA}/V_{RA})R_A) \qquad (II.6.4.6)$$

Knowing f the frequency of the signal generator, and R_A, one can calculate the capacitance C_A.

Similarly

$$C_B = 1/(2\pi f\ (V_{CB}/V_{RB})R_B) \qquad (II.6.4.7)$$

where V_{RB} is the voltage across G_2 and B_2 and V_{CB} is the voltage across R_2 and G_2.

Connect the furnace to the regulated power supply. Close the open ends of the ceramic tube of the furnace loosely with plugs of cotton to prevent convection currents. Start with a DC voltage of 7.5 V so that the furnace temperature rises at the rate of 1 to1.5 degrees/minute. As the temperature increases, increase the DC voltage gradually. At 120 C the DC voltage should not exceed 12 V. Connect the signal generator terminals to R_1 and B_1. Set the signal generator frequency to 1 kHz as measured at the terminals R_1 and B_1 with a DMM. Set the output voltage at 1 V. Connect the two Pt 100 terminals to the corresponding terminals on the temperature indicator.

At a given value of temperature as indicated on the temperature indicator, measure with a DMM, set in AC 2 Volts range, V_{RA}, V_{CA}, V_{RB} and V_{CB}. Carry out the measurements at intervals of 10 C from 40 to 110 C. A sample set of readings is given below.

Table II.6.4.1

Temperature dependence of capacitor
C_A is a ceramic capacitor and C_B is a polymeric capacitor
R_A and R_B are 1 kΩ resistors
Frequency of signal generator is 1 kHz

	Ceramic capacitance				Polymer capacitance			
Temp C	V_C	V_R	Z_c	C_A in μfd	V_C	V_R	Zc	C_B in μfd
40	0.887	0.409	2168.7	0.0734	0.814	0.533	1527.2	0.1042
50	0.875	0.284	3080.9	0.0517	0.773	0.507	1524.6	0.1044
60	0.89	0.232	3836.2	0.0415	0.771	0.508	1517.7	0.1049
70	0.897	0.191	4696.3	0.0339	0.766	0.507	1510.8	0.1053
80	0.9	0.164	5487.8	0.029	0.761	0.506	1503.9	0.1058
90	0.903	0.14	6450	0.0247	0.752	0.511	1471.6	0.1082
100	0.905	0.129	7015.5	0.0227	0.751	0.517	1452.6	0.1096
110	0.855	0.106	8066	0.0197	0.701	0.493	1421.9	0.1119

From the ratio V_C/V_R , the impedance Z_C is calculated using

$$Z_C = (V_C/V_R)*1000$$

From Z_C, the capacitance C is calculated from

$$C = 1/(2\pi f Z_C)$$

with f =1000 Hz.

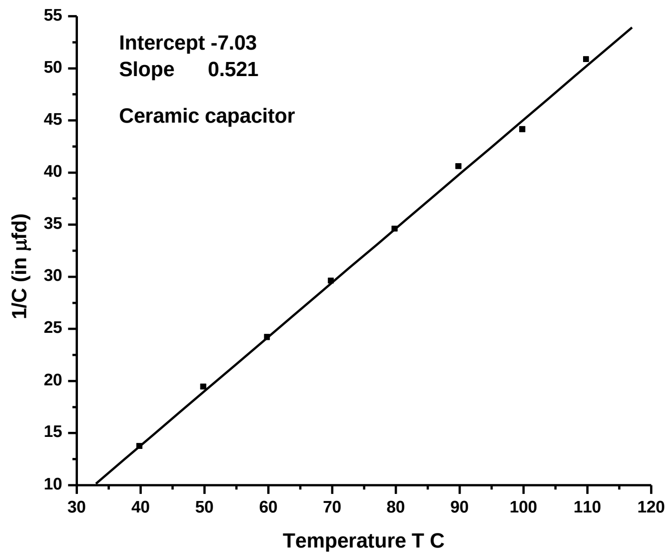


Figure II.6.4.4: A plot of $1/C_A$ vs. t.

Figure II.6.4.4 shows a plot of $1/C_A$ against temperature T in ^{0}C. The linear fit to the points is shown by the black line. The fit is very good. The intercept A = -7.03 and the slope B is 0.521. So the Curie temperature Θ is given by

$$\Theta = -A/ B = 13.4 \text{ C}.$$

The equation to the straight line is

$$1/C_A(\text{in } \mu\text{fd}) = 0.521(T-13.4) \qquad \text{(II.6.4.8)}$$

or

$$C_A \text{ (in } \mu\text{fd}) = 1.921/(T-13.4) \qquad \text{(II.6.4.9)}$$

The ceramic capacitance (and hence the dielectric constant of the ferroelectric material used in the capacitance) follows the Curie-Weiss law with Θ = 13.4 ^{0} C.

Figure II.6.4.5 shows a plot of C_A and C_B with temperature. The black curve represents the fit of C_A to equation (II.6.4.9).

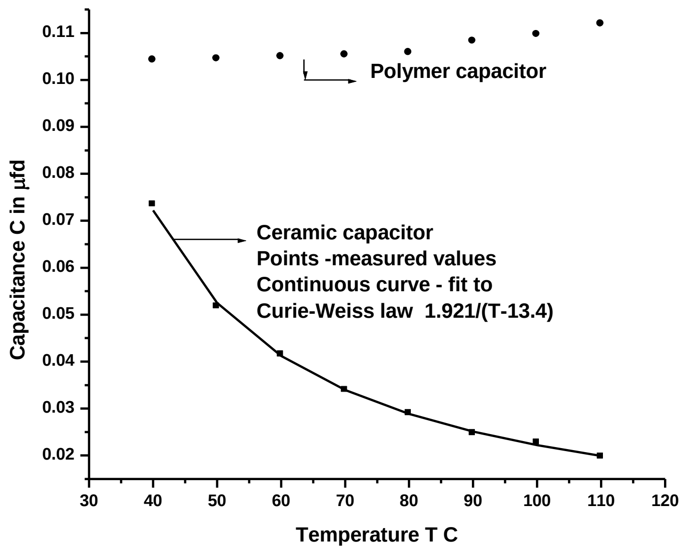


Figure II.6.4.5: Plot of the capacitance value C_A of the ceramic capacitor as a function of temperature T in ^{0}C. The continuous curve shows the fit to the experimental points for C_A from the Curie Weiss equation.

Thus this experiment demonstrates that

1. The capacitance of a ceramic capacitor decreases rapidly as temperature increases while the capacitance of a polymer capacitor increases slightly with increase of temperature.

2. The dielectric constant of the ferroelectric material used in the ceramic capacitor follows the Curie-Weiss law.

3. The dielectric constant of the material used in the polymer capacitor increases slightly as the temperature is increased.

Questions:

1. If we use a ceramic capacitor in the tank circuit of the oscillator how will the vary as the temperature changes, assuming the inductance in the circuit is independent of temperature.

2. Can you use a ferroelectric as a memory element to represent the binary states 0 and 1?

II.7 EXPERIMENTS IN MAGNETISM

II.7.1 B-H CURVE IN A FERROMAGNETIC MATERIAL

1. INTRODUCTION:

In a ferromagnetic material the magnetic induction field **B** is not a linear function of the magnetic field **H**. The magnetic induction field, for a given **H,** depends on the previous history of the specimen. The curve of **B** vs. **H** is shown in the following figure.

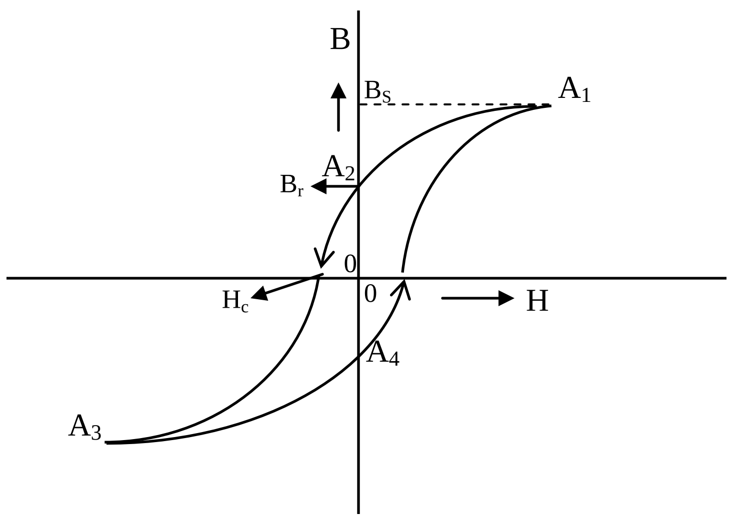


Figure II.7.1.1 B-H curve for a ferromagnetic material

When the magnetic field H is very large in the positive or negative direction, the magnetic induction field B saturates at a value $\pm B_S$, called the saturation magnetic field. At any given value of the magnetic field H, there are two values for B, one while the magnetic field H is decreasing and another while the magnetic field H is increasing. Thus B depends on the history and the phenomenon is called hysteresis. If the magnetic field is reduced from a maximum value to zero, the magnetic induction field does not go to zero. It has a value $\pm B_r$ depending on whether the magnetic field H is brought to zero from a positive value or from a negative value. The value of B_r is called the remanent magnetic induction field. To make the magnetic induction field B zero one has to apply a magnetic field $\pm$ Hc (+ when the magnetic field is increasing and – when it is decreasing). This field H_c is called the coercive field. The values of B_r and H_c are characteristics of a ferromagnetic material. A material with a small H_c is called a soft ferromagnetic material while one with a large H_c is called a hard ferromagnetic material. If the ferromagnetic material is subjected to an AC magnetic field H, the area enclosed by the B-H curve gives the amount of heat generated per cycle per unit volume in the material. So hysteresis leads to wastage of electrical energy as heat.

Examples of ferromagnetic materials are Fe, Ni and Co at room temperature and ceramic materials which are oxides of iron or Ni and other elements like Zn.

One can use materials with properties suited for a particular application. For example if one wants a ferromagnetic core material for winding a transformer, one should reduce the hysteresis loss i.e. one should have a soft magnetic material like soft iron with low coercive field. On the other hand if one would like to make a permanent magnet, the material must have a large remanent magnetization and a large coercive field. Such a material is hard iron. If one wants to use a ferrite material for computer memories then it should have a square hysteresis loop (i.e. B_r nearly equal to B_s) with a small coercive field. The state $+B_r$ will be called the state 1 and the state $-B_r$ will be called the state 0. A large variety of magnetic materials tailor-made for a number of applications are now commercially available.

2. SET UP FOR B-H CURVE:

One may use an AC method with an oscilloscope for observing the B-H loop of the material. But to illustrate various physical principles involved, we have preferred to use a DC method with an integrator and multi-meter to record the B-H loop.

The front panel of the B-H curve set up is shown in Figure II.7.1.2.

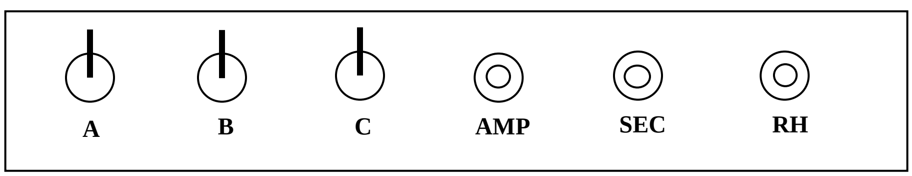


Figure II.7.1.2: Front panel of the B-H curve set-up box.

Inside the box there is a steel rod of about 6 mm diameter bent in the form of a torus of diameter about 10 cm. The two ends are welded together. Three layers of SWG 26 copper wire are wound round the torus with an average number of 13 turns per centimeter in each layer. A secondary coil of 100 turns is wound on the primary coil. The primary coil, connected in series with a nichrome wire of about 3 ohms resistance, is connected to banana terminals at the back of the box through a rheostat with a resistance of about ten ohms.

On the front panel there are three DPDT switches A,B and C. There are three RCA sockets marked AMP, SEC and RH respectively.

The primary coil circuit is shown in Figure II.7.1.3.

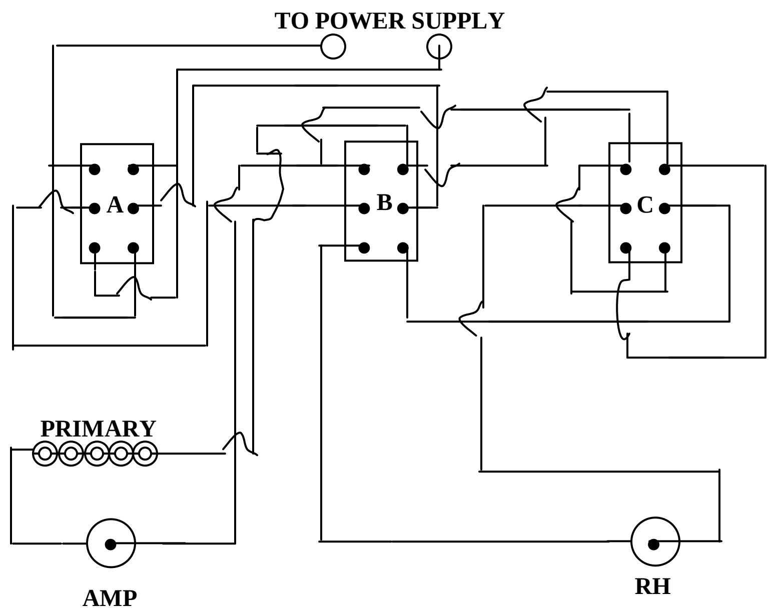


Figure II.7.1.3 Circuit diagram in the BH curve set-up

A regulated power supplies provides a DC voltage at the top terminals of the DPDT switch A. When the switch A is put up this voltage V appears at the central terminals of the switch A. When the switch A is toggled down the voltage at the central terminals of switch A is reversed. The voltage at the central terminals of A is connected to the central terminals of switch B. When switch B is up, this voltage is applied to the primary coil and sends a current through the primary coil in one direction. A DMM connected to the RCA socket AMP on the front panel measures the DC current through the primary coil. A plug pattern resistance box from 1 to 1000 Ohms is connected to the RCA socket marked RH on the front panel.. When switch B is put down the DC voltage is applied through the resistance box to the middle terminals of switch C. When C is up, the DC voltage sends a current through the primary coil in series with the resistance in the resistance box. The direction of the current through primary coil is the same as when the switch B is up. But the magnitude of the current is reduced because of the resistance in the box. When switch C is down the primary coil is connected in the reverse direction. So the current flows in the reverse direction.

The secondary coil is connected to the RCA socket marked SEC.

3. Principle:

When a current I passes through the primary coil a magnetic intensity field H = N I is generated in the rod. N is the total number of turns per meter on the primary coil.

$$N = nL$$

Here n is the number of turns/m in each layer and L is the number of layers in the primary coil. For the given primary coil on the torus n = 1300 /m and L = 3. So the magnetic intensity field H for a current I is 3900 I ampere turns/m. By reversing the current the magnetic field direction can be reversed.

This magnetic field H produces a magnetic induction field B in the torus. This induction field will be larger, the larger the magnetic permeability of the material of the torus. Since the magnetic material has no free ends there is no demagnetizing field due to free ends as one would have with a straight coil.

A secondary coil of n_s =100 turns of the same wire is wound on the primary coil on one part of the torus. For a magnetic induction field B the flux Φ linked with the secondary coil is given by

$$\Phi = n_s \, \phi = n_s \, BA \qquad \text{(II.8.1.1)}$$

Here ϕ is the flux linked with one turn of the secondary coil and A is the area of cross section of the secondary coil. B changes when the magnetic field H changes, and so the flux linked with the secondary coil changes. An emf is induced in the secondary coil given by

$$v = -d\Phi/dt = -n_s \, d\phi/dt \qquad \text{(II.7.1.2)}$$

To measure the flux change, when the magnetic field is changed from H_1 to H_2, we have to find the integral of v over the time needed for the change in magnetic field. The integrator described in Section I.12 is used to measure the change in flux. The secondary coil output is connected to the RCA socket marked **Input o**n the front panel of the integrator. The second toggle switch S_2 on the front panel is put in the appropriate position (This is described later). The Band switch on the integrator is put in the position marked **BH**.

The change in voltage of the capacitor C in the integrator when a pulse of voltage v(t) is applied has been derived in I.12 and is given by

$$V = 1/CR \int_0^{t_f} v(t)\, dt \qquad \text{(II.7.1.4)}$$

Putting v(t) = - dΦ/dt in the above equation we get

$$V = n_s\,(\phi_0 - \phi_1)/CR \qquad \text{(II.7.1.5)}$$

Here ϕ_0 is the flux linked with one turn of the secondary coil at the initial current I_{init} and ϕ_1 is the flux linked with one turn of the secondary coil at the final current I_{final}. Thus the peak voltage across the capacitor is proportional to the change in flux. In the integrator R = 220 Ohms and C = 47 μf.

The output voltage is amplified twice in the integrator. Each stage provides an amplification of 2.2. This amplification is enough for BH curve.

The peak amplified voltage V_p is recorded on the multi-meter connected to the banana terminals marked **DMM** on the front panel of the integrator, if it has the proper sign.

If H is increased in the primary coil, the sign of the input voltage pulse to the integrator is negative. So no voltage will be recorded on the meter as the diode in the peak detector will conduct only in the forward direction. By putting the toggle switch S_2 in the position REV, the input signal to the integrator is reversed and Vp becomes positive and will go through the diode to charge the capacitor.

4. APPARATUS REQUIRED

A DC power supply going up to 20 V and delivering a maximum current of 2 A, B-H curve set up**,** integrator, two resistance boxes 1 to 1000 Ohms which can carry 1 A current, A DMM to measure current in DC 10A range, and a DMM to measure DC volts to 2 decimal places in the DC 20 Volts range.**,**

5. PROCEDURE

The connection diagram for the B-H curve experiment is shown in Figure II.7.1.4.

Connect a DC power supply to the black and red banana terminals at the back of the box of the BH Curve set up through one of the resistance boxes and set a resistance of 10 Ohms in the box.. Connect DMM A in the DC 10 amperes range to the RCA socket marked **AMP** on the front panel of the BH curve set up. Connect a resistance box (plug pattern from 1 to 1000 Ohms) to the RCA socket marked **RH** on the front panel of the BH curve set up. Put switches A, B and C in the up position. Adjust the voltage output of the power supply to give a current of 0.8 A on DMM A.

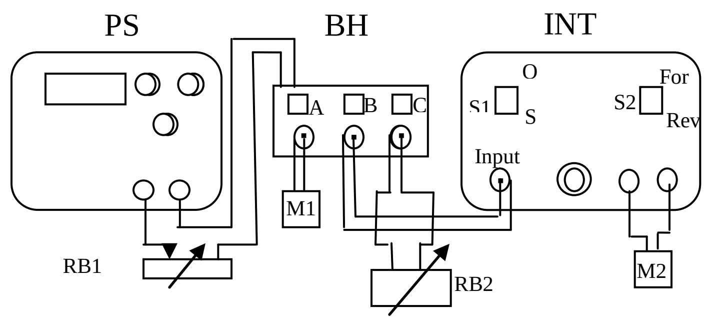


Figure II.7.1.4 Connection diagram for B-H curve Experiment

Legends:

PS: Regulated Power Supply   BH: BH curve Set Up   INT: Integrator

□oggle switches   B◯nana Terminals   R⊙A sockets   P◎

M1 DMM 10 A DC   RB Plug pattern resistance boxes
M2 DMM 20 V DC

From Figure II.7.1.1, there are four parts of the B-H curve:

1. **Part 1 is A_1 to A_2.** In this part we vary the current from $+I_{max}$ at A to a lower positive value on the curve A_1A_2.. The switch settings for this region are **A UP; C UP; and B is DOWN. Switch S_2 on the Integrator in the FORWARD Position.**

   Adjust the current to 0.8 A when all the plug keys in the box RB2 are plugged in. To change the current from $+I_{max}$ to $+I$ ($I<I_{max}$) pull out a plug from the 20 Ohms key. Note the maximum output voltage on the DMM connected to the output of the integrator. Note the final current I on the meter. Then push switch S_1 on the integrator to position S.

   Plug in the 20 Ohm key to restore the current to 0.80 A. Push the shorting switch S_1 on integrator to the position O. Then pull out resistance plug from the 50 Ohm key. Note the maximum reading on the DMM connected to integrator output.

   In this fashion we can change the current from I_{max} to a final positive value I down to zero and note the DMM readings.

2. **Part 2 is A_2 to A_3** : In this part we vary the current from $+I_{max}$ to $-I$ ($0<I<I_{max}$).

The switch positions are the following:
A UP; C DOWN, and B is pressed from UP to DOWN. Switch S_2 on the Integrator in the FORWARD Position.

Put shorting switch S_1 on the integrator to S. Put B down. Adjust the negative current to a desired value by pulling out appropriate plugs from the resistance box R_2. Then push B up to get a current of $+I_{max}$. Push switch S_1 to position O. Adjust the offset pot so that the reading on the DMM connected to the integrator is zero. Now push B down so that the current changes from $+I_{max}$ to $-I$. Note the maximum output voltage on DMM2. Repeat this operation changing $-I$ in steps of 0.05A till one reaches $-I_{max}$ by plugging in the keys in resistance box 2.

3. **Part 3 is A_3A_4.** Here the current is changed from $-I_{max}$ to $-I$ ($0<I<I_{max}$).

The switch settings are

A DOWN; C UP; B DOWN. Switch S_2 on the Integrator to the REVERSE Position. All the keys in Resistance box R_2 are plugged in to get the maximum current $-I_{max}$.

Check that the shorting switch is in position O and the offset pot is adjusted so that the DMM2 shows zero reading. As in step 1, pull out the plug from the 20 Ohm key and note the maximum reading on DMM2 and the current on DMM1. Repeat this process till the current I becomes zero.

4. **Part 4 is A_4A_1.** Here the current is changed from $-I_{max}$ to $+I$ ($0<I\leq I_{max}$).

The switch settings are

A DOWN; C DOWN; B pressed from UP to DOWN. Switch S_2 on the Integrator in the REVERSE Position.

To adjust the final current put switch S_1 in position S. Put B down and adjust the resistance in box RB2 till DMM1 shows the desired final current. Then B is pushed up, switch S_1 is put in position O and B is pushed down and maximum reading on DMM2 is noted.

When switch A is down and the current is changed from $-I_{max}$ the secondary coil voltage gives a negative pulse. The integrated value of this pulse will not pass through the diode in the peak detector. So we put switch S_2 on the integrator in the Reverse position to reverse the input voltage. The out put voltage read by DMM2 will always be positive. But when S_2 is in the reverse position we enter the voltage in the table with a minus sign.

We note the readings of the current and voltage in a table as shown below. The Table is divided into two parts. The top part is under the heading current decreasing from positive maximum value. This contains the readings for the part $A_1A_2A_3$ of the BH curve. The second part of the table is under the heading current increasing from $-I_{max}$. This part of the table traces the part $A_3A_4A_1$ of the B-H curve.

Table II.7.1.1
B-H Curve
Current decreasing from +0.8 A

Number of turns per m in primary coil	3900	
Number of turns in secondary coil n_s	100	
Area of the secondary coil a =	3.80E-05	m^2
Amplification of the integrator	4.84	
CR value of the integrator	0.01034	

Current decreasing from +0.81 A

		From I max of +0.80 Amp					
Switch	Posn	amp	ampturns/m	Volts	Webers	Webers	Tesla
A	UP	I	H	V_p	$\Delta\Phi$	Φ	B
B	Dn	0.8	3120	0	0	2.99E-05	0.787
C	UP	0.52	2028	0.01	2.14E-07	2.97E-05	0.781
S2	Forward	0.29	1131	0.09	1.92E-06	2.80E-05	0.736
Pull from	Plugs RB2	0.16	624	0.25	5.34E-06	2.46E-05	0.647
		0.09	351	0.3	6.41E-06	2.35E-05	0.618
		0.04	156	0.37	7.9E-06	2.20E-05	0.579
		0	0	0.43	9.19E-06	2.07E-05	0.545
A	UP	-0.08	-312	0.46	9.83E-06	2.01E-05	0.528
C	DN	-0.11	-429	0.55	1.18E-05	1.82E-05	0.478
S2	Forward	-0.15	-585	0.66	1.41E-05	1.58E-05	0.416
B	up dn	-0.22	-858	0.92	1.97E-05	1.03E-05	0.270
		-0.28	-1092	1.52	3.25E-05	-2.56E-06	-0.067
		-0.33	-1287	1.6	3.42E-05	-4.27E-06	-0.112
		-0.38	-1482	1.95	4.17E-05	-1.18E-05	-0.309
		-0.48	-1872	2.25	4.81E-05	-1.82E-05	-0.478
		-0.53	-2067	2.29	4.89E-05	-1.90E-05	-0.500
		-0.62	-2418	2.46	5.26E-05	-2.26E-05	-0.596
		-0.76	-2964	2.71	5.79E-05	-2.80E-05	-0.736
		-0.8	-3120	2.8	5.98E-05	-2.99E-05	-0.787

Table II.7.2.1.1 (contd)

Current increasing from -0.8 A

Switch	Posn	From I max of =-amp.80					
A	Dn	amp	ampturns/m	Volts	Webers	Webers	Tesla
B	Dn	I	H	V_p	$\Delta\Phi$	Φ	B
C	Up	-0.8	-1677	0	0	-3.15E-05	-0.83
S2	Rev	-0.43	-1677	-0.11	-2.40E-06	-2.91E-05	-0.77
Pull	Plugs	-0.29	-1131	-0.18	-3.80E-06	-2.77E-05	-0.73
from	RB2	-0.09	-351	-0.39	-8.30E-06	-2.32E-05	-0.61
		-0.04	-156	-0.43	-9.20E-06	-2.23E-05	-0.59
A	Dn	0	0	-0.44	-9.40E-06	-2.21E-05	-0.58
C	Dn	0.04	156	-0.47	-1.00E-05	-2.15E-05	-0.57
S_2	Rev	0.08	312	-0.54	-1.20E-05	-1.95E-05	-0.51
B	up $\downarrow$ dn	0.14	546	-0.63	-1.30E-05	-1.85E-05	-0.49
		0.2	780	-1.03	-2.20E-05	-9.50E-06	-0.25
		0.34	1326	-1.8	-3.80E-05	6.50E-06	0.17
		0.42	1638	-2.1	-4.50E-05	1.35E-05	0.36
		0.52	2028	-2.39	-5.10E-05	1.95E-05	0.51
		0.65	2535	-2.67	-5.70E-05	2.55E-05	0.67
		0.75	2925	-2.85	-6.10E-05	2.95E-05	0.78
		0.8	3120	-2.95	-6.30E-05	3.15E-05	0.83

The first column of the table gives the switch positions. The second column gives the maximum current $+I_{max}$ and the other set currents +I or –I on the branch $A_1A_2A_3$ of the B-H curve. The third column is the magnetic intensity field H corresponding to the current I. Since the primary coil has 3900 turns/m, H is obtained by multiplying the current in column 1 by 3900. Column 4 gives the peak output voltage as read on DMM2 when the current is changed from $+I_{max}$ to +I or –I by pressing switch B down.. The first row in this column will be zero as it will correspond to no change in current. From this the flux change $(\phi(I_{max}) - \phi(I)) = \Delta\phi$ is obtained by dividing the voltage in column 4 by 4.84 n_s / CR = (4.84x100)/(47μ fdx220 Ohms) . This flux change is in webers. The last reading in this column corresponds to a change $((\phi(I_{max}) - \phi(-I_{max})) = 2\phi(I_{max})$.

Column 5 gives the flux $\phi(I)$. The first row in this column is obtained by dividing the last value in column 4 by 2. The other rows in column 5 are obtained by subtracting from the first value in column 5 the corresponding $\Delta\phi$ value in column 4. Thus we obtain in column 5 the flux values at various currents. The sixth column gives the magnetic induction field B(I). This is obtained by dividing the flux value $\phi(I)$ in column 5 by a, the area of cross section of the secondary coil, which is 0.38×10^{-4} m^2. Thus we have obtained B as a function of H for the part $A_1A_2A_3$ of the B-H curve.

For the part $A_3A_4A_1$ the calculations are shown in the corresponding columns in the the Table above under the heading Current increasing from -0.8 A. Now we are increasing the current from $-I_{max}$. **Since we have put the switch S_2 on the integrator inposition REV for this part, the output voltage must be taken with a negative sign.** The flux $\phi(-I_{max})$ which is in the first row of bottom half of column 5 is (1/2) of the last value in the bottom half of column 4. The flux at any other value of current is obtained by subtracting the $\Delta\phi$ value in column 4 from this value.

The curve of B vs H is shown in Figure II.7.1.5 below.

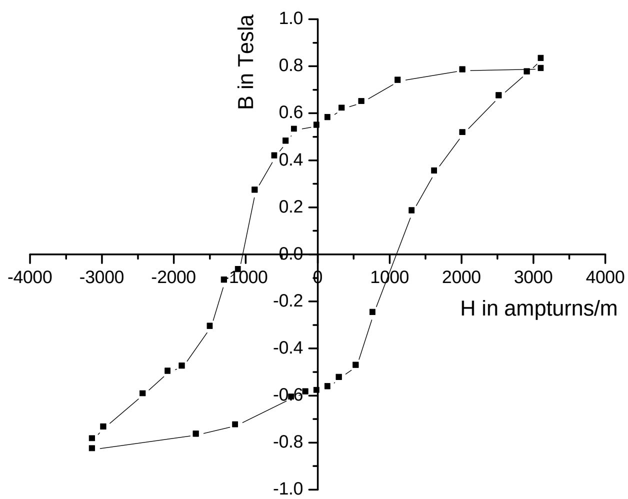


Figure II.7.1.5 B-H Curve

From the figure we get $H_C = (1140+1044)/2 = 1092$ Amp Turns/m, $B_r = (0.551+0.583)/2 = 0.567$ T. The area of the loop is 3390 J/m^3 . This is the energy loss per cycle due to magnetic hysteresis in the material. If the loop had been a square extending from $-H_c$ to $+H_c$ along the X axis and from $-B_r$ to $+B_r$ along the Y axis the area of the loop would be $4B_rH_c = 2477$ J/m^3. The actual area under the loop is 137% of this value. In fact the product $4B_rH_c$ roughly characterizes the hysteresis loss in a material.

For example, if a current of 0.80 A at 50 Hz flows through the Torus the heat generated per second would be 3390x50 = 169500 W/m^3. This will heat up the specimen. The higher the frequency the higher the heat generated by hysteresis loss. This is in addition to the eddy current loss in the material. To reduce hysteresis loss in a power transformer we use a soft ferromagnetic core with a low B_rH_c value and the eddy current loss is minimised by laminating the core.

II.7.2 CALIBRATION OF A SEARCH COIL MAGNETIC FIELD VARIATION ALONG THE AXIS OF A SOLENOID

1. INTRODUCTION

There are many methods for measuring a magnetic field. The simplest method is by using a search coil. A search coil is a coil of thin copper wire wound on a bobbin of small diameter. The coil consists of several hundred turns wound in different layers. The winding is such that the emf developed across each layer, when the magnetic flux changes through the coil, add together to give a substantial signal, which can be amplified and integrated. The search coil will measure the change in magnetic field averaged over the volume occupied by the coil.

When the magnetic field changes by ΔB, the search coil develops a voltage that is given by

$$v(t) = - n A \, dB/dt \qquad \text{(II.7.2.1)}$$

Here n is the total number of turns in the search coil, A its average cross sectional area and B is the magnetic induction field. This voltage is amplified by a factor α and connected to an integrator (see I.12 for a description of the integrator) where it charges a Capacitor C through a resistance R. The voltage V across the capacitor will vary with time as

$$dV(t)/dt = \alpha v(t)/CR = - (nA\alpha/CR) \, dB/dt \qquad \text{(II.7.2.2)}$$

The peak voltage to which the capacitor will be charged is given by

$$V_{max} = \int_0^{\infty} dV(t)/dt = - (nA\alpha/CR) \int_B^{B+\Delta B} dB \qquad \text{(II.7.2.3)}$$

$$= - (nA\alpha/CR) \Delta B$$

This voltage is further amplified by a factor μ and goes through a peak detector, which indicates the amplified value of V_{max} on a multimeter connected to the output terminals of the integrator. This output voltage V_{output} is proportional to ΔB. To measure B we either start with B = 0 and go to a final value of B or start with B and go to zero. Then the magnitude of ΔB is B and the search coil output is proportional to B.

Since the area of the search coil is not well defined, the search coil is calibrated to measure the output for known changes in B.

2. SEARCH COIL SET UP

For generating the magnetic induction field B, we use a solenoid about 30 cms in length and 5 cm in diameter. SWG 26 gauge enamel coated copper wire is wound closely on the former in two layers. In each layer there are 13 turns per cm. So the number of turns in both layers together is 2x13 = 26 turns/cm or 2600 turns/m. When a current I flows through the solenoid, a magnetic intensity field

$$H = 2600\ I \text{ amp turns/m} \qquad \text{(II.7.2.4)}$$

is generated at the center of the solenoid and is nearly uniform over a short length at the center. Since the solenoid is in air the magnetic induction field B corresponding to H is

$$B = \mu_0 H \qquad \text{(II.7.2.5)}$$

where $\mu_0 = 4\pi x 10^{-7}$ and B is in Tesla (1 Tesla = 10^4 Gauss).

The search coil is positioned at the center of the solenoid on its axis. The axis of the search coil is along the axis of the solenoid.

The panel of the solenoid contains a DPDT switch and two pairs of banana plugs as shown in Figure II.7.2.1 (a).

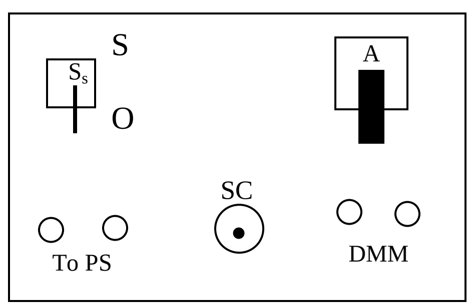


Figure II.7.2.1 (a) Front Panel of Solenoid-Search coil assembly. S_s is a shorting switch. A is a 5 A DPDT switch to reverse the current through the solenoid. Banana Terminals marked to PS are connected to a regulated power supply. A DMM in the DC 10 A range connected to the banana terminals marked DMM measures the current through the solenoid. SC is the output of the Search Coil coming to a RCA socket.

Diagram of connections is shown in Figure II.7.2.1 (b).

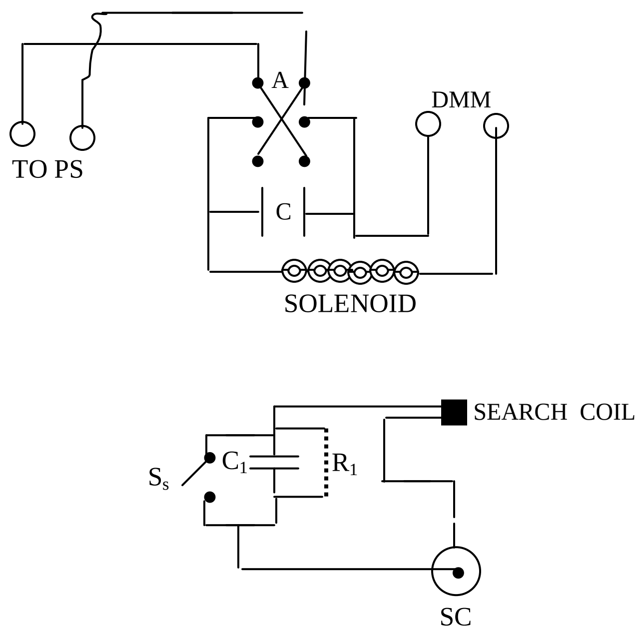


Figure II.7.2.1 (b) Connections to the front panel of solenoid and search coil. The switches and terminals have the same symbols as in Figure II.7.2.1 (a).
C is an electrolytic capacitor 1000μf 50 V.
C_1 is 1μf capacitor and R_1 is 500 Ω.

A regulated power supply is connected to the terminals marked PS. A DMM in the DC 10 Amp range is connected to the terminals marked DMM. A capacitor C of 1000 μf, connected across the middle terminals of the DPDT switch, makes the current reversal smooth. If this capacitor is not connected, the search coil output after integration and amplification will fluctuate widely. When the DPDT switch is in the UP position the current flows through the coil in the positive direction and the ammeter reads a current of +I amp. When the DPDT switch is pressed down the current through the coil is reversed and the ammeter reads –I amps. One lead of the search coil is connected through a RC network (C_1 1 μf and R_1 500 Ohms) to the central pin of the RCA socket marked SC on the front panel. The other lead is connected to the outer terminal of the RCA socket. A shorting switch S_s is connected across the capacitor C_1.

The search coil is mounted on a long rod of plastic and the search coil can be moved from the mid point of the solenoid to one end. An indicator on a meter scale indicates the position of the coil in the solenoid.

3. APPARATUS REQUIRED

A regulated DC power supply giving a maximum of 15 V and a maximum current of 2 A, search coil set up, a DMM to measure current in DC 10 A mode, integrator and a DMM to measure DC volts to two decimal places in the 20 V range.

4. PROCEDURE

a. Connect the regulated power supply to the Banana terminals marked power supply.
b. Connect A DMM in the DC 10 Amp range to the terminals marked DMM.
c. Connect the RCA socket marked SC to the input RCA socket on the integrator.
d. Position the search coil on the axis of the solenoid, near the mid-point of its length.
e. Set the band switch on the integrator to the position SC. Connect a DMM in the DC 2 Volts range to the output terminals of the integrator.
f. Put the shorting switch on the integrator (S_i) in the position S and adjust the offset pot so that the DMM connected to the integrator shows zero.
g. Put the DPDT switch on the solenoid up and the DPDT on the integrator in the forward position.
h. Switch on the power supply and adjust the voltage so that the current through the solenoid is 0.1 A.
i. Put the shorting switch on the integrator S_1 to the position O.
j. Check that the shorting switch S_s on the front panel of the solenoid assembly is in the open (O) position.
k. Press the DPDT switch on the front panel of the solenoid down and note the maximum reading at the output of the integrator.
l. Put S_1 up. Then put S_s in the shorting (S) position.
m. Restore the DPDT switch A on the solenoid assembly to the up position.
n. Then put S_s to position O and S_1 to position O.
o. If the DMM connected to the integrator shows a slightly different reading from zero, adjust the offset to make the reading zero.
p. Press the DPDT switch on the solenoid down and note the DMM reading.
q. Repeat operations 12 to 16 so that you get ten readings of the output of the integrator for the same current through the solenoid.
r. Change the current through the solenoid in steps of 0.1 A up to 1 A, and for each current repeat operations 12-16 ten times to get ten values of the output for each current.

s. Then put the DPDT switch on the integrator to reverse position. Now we will start with a current –I through the solenoid and reverse it to +I and note the integrator output following steps 12 to 16. Again the current is changed in steps of 0.1 amp up to a maximum of 1 A.

DO NOT PASS A CURRENT OF MORE THAN 1 AMP THROUGH THE SOLENOID.

A sample set of readings is shown in Table II.7.2.1 (a) and (b).

Table II.7.2.1 (a)
Calibration of the search coil
Forward: from I to -I

I amp→	0.2		0.3		0.4		0.5		0.6	
Sl. No.	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2
1	0.17	9.00E-04	0.32	4.84E-04	0.44	8.41E-04	0.59	9.00E-06	0.7	1.44E-04
2	0.21	1.00E-03	0.29	5.48E-04	0.47	8.42E-04	0.55	1.38E-03	0.71	1.48E-04
3	0.21	1.10E-03	0.29	6.12E-04	0.43	2.36E-03	0.59	1.39E-03	0.71	1.52E-04
4	0.2	1.10E-03	0.29	6.76E-04	0.49	2.80E-03	0.54	3.60E-03	0.73	4.76E-04
5	0.21	1.20E-03	0.31	8.20E-04	0.48	2.93E-03	0.58	3.64E-03	0.74	1.26E-03
6	0.2	1.20E-03	0.28	1.14E-03	0.5	3.89E-03	0.61	4.17E-03	0.73	1.58E-03
7	0.19	1.30E-03	0.33	2.17E-03	0.45	4.25E-03	0.6	4.34E-03	0.75	3.03E-03
8	0.25	3.80E-03	0.27	2.95E-03	0.45	4.61E-03	0.6	4.51E-03	0.67	4.79E-03
9	0.19	3.90E-03	0.29	3.02E-03	0.47	4.61E-03	0.6	4.68E-03	0.72	4.86E-03
10	0.17	4.80E-03	0.31	3.16E-03	0.51	6.29E-03	0.61	5.21E-03	0.66	7.56E-03
Average	0.2		0.298		0.469		0.587		0.712	
Variance	5E-04		4E-04		7E-04		6E-04		0.0008	
Std Dev	0.023		0.019		0.026		0.024		0.029	

I in amp→	0.7		0.8		0.9		1	
S. No.	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2
1	0.82	3.60E-05	0.92	1.23E-32	0.94	4.84E-04	1.18	8.10E-03
2	0.83	2.92E-04	0.93	1.00E-04	0.98	8.08E-04	1	1.62E-02
3	0.81	3.08E-04	0.91	2.00E-04	1.02	4.17E-03	1.17	2.26E-02
4	0.76	3.22E-03	0.88	1.80E-03	0.93	5.20E-03	1.1	2.27E-02
5	0.81	3.24E-03	0.92	1.80E-03	0.97	5.26E-03	1.03	2.63E-02
6	0.82	3.28E-03	0.92	1.80E-03	0.96	5.26E-03	1.14	2.88E-02
7	0.83	3.53E-03	0.95	2.70E-03	0.95	5.41E-03	1.11	2.92E-02
8	0.84	4.21E-03	0.92	2.70E-03	0.97	5.47E-03	1.08	2.93E-02
9	0.83	4.46E-03	0.96	4.30E-03	0.93	6.50E-03	1.05	3.09E-02
10	0.79	5.04E-03	0.89	5.20E-03	0.97	6.56E-03	1.04	3.34E-02
Average	0.814		0.92		0.962		1.09	
Variance	6E-04		6E-04		7E-04		0.004	
Std. Devn.	0.024		0.024		0.027		0.061	

In the above table the first row gives the current through the solenoid. For each current there are two columns. The first column marked V in volts gives the integrator output when the current is changed from +I to –I. The row marked average gives the average of the ten output voltages measured for a given current. The first row under error^2 in the second column gives $(V-\langle V\rangle)^2$. The second row adds the value in the previous row in column 2 to $(V-\langle V\rangle)^2$ for that row. Thus the tenth row will give the sum of squares of errors of all ten measurements in a column. The row marked standard deviation is obtained by dividing the sum of error^2 by (10-1) and taking its square root. Thus for each current the average output and standard deviation are calculated.

Table II.7.2.1 (b)
Calibration of the search coil
Reverse: from -I to +I

I amp→	0.2		0.3		0.4		0.5		0.6	
S. No.	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2
1	0.25	2.92E-03	0.33	8.10E-05	0.48	2.50E-05	0.58	3.24E-04	0.7	6.40E-05
2	0.23	4.07E-03	0.35	2.02E-04	0.48	5.00E-05	0.58	6.48E-04	0.72	2.08E-04
3	0.19	4.11E-03	0.35	3.23E-04	0.49	2.75E-04	0.62	1.13E-03	0.66	2.51E-03
4	0.17	4.78E-03	0.34	3.24E-04	0.47	3.00E-04	0.61	1.28E-03	0.74	3.54E-03
5	0.23	5.94E-03	0.33	4.05E-04	0.49	5.25E-04	0.61	1.42E-03	0.66	5.84E-03
6	0.17	6.62E-03	0.3	1.93E-03	0.48	5.50E-04	0.59	1.48E-03	0.7	5.90E-03
7	0.17	7.29E-03	0.36	2.37E-03	0.48	5.75E-04	0.61	1.63E-03	0.69	6.23E-03
8	0.17	7.97E-03	0.36	2.81E-03	0.46	8.00E-04	0.63	2.65E-03	0.73	6.71E-03
9	0.2	7.98E-03	0.31	3.65E-03	0.48	8.25E-04	0.55	4.96E-03	0.7	6.78E-03
10	0.18	8.24E-03	0.36	4.09E-03	0.44	2.05E-03	0.6	4.96E-03	0.78	1.20E-02
Average	0.196		0.339		0.475		0.598		0.708	
Variance	9E-04		5E-04		2E-04		6E-04		0.001	
Std Dev	0.030		0.021		0.015		0.023		0.036	

I amp →	0.7		0.8		0.9		1	
S.No.	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2	V_{out} V	Error^2
1	0.78	1.60E-05	0.83	8.41E-04	0.94	4.84E-04	1.18	8.10E-03
2	0.76	2.72E-04	0.88	1.28E-03	0.98	8.08E-04	1	1.62E-02
3	0.79	4.68E-04	0.88	1.72E-03	1.02	4.17E-03	1.17	2.26E-02
4	0.77	5.04E-04	0.86	1.72E-03	0.93	5.20E-03	1.1	2.27E-02
5	0.82	2.44E-03	0.84	2.09E-03	0.97	5.26E-03	1.03	2.63E-02
6	0.78	2.46E-03	0.83	2.93E-03	0.96	5.26E-03	1.14	2.88E-02
7	0.75	3.13E-03	0.86	2.93E-03	0.95	5.41E-03	1.11	2.92E-02
8	0.81	4.29E-03	0.82	4.45E-03	0.97	5.47E-03	1.08	2.93E-02
9	0.76	4.54E-03	0.9	6.13E-03	0.93	6.50E-03	1.05	3.09E-02
10	0.74	5.84E-03	0.89	7.09E-03	0.97	6.56E-03	1.04	3.34E-02
Average	0.776		0.859		0.962		1.09	
Variance	6E-04		8E-04		7E-04		0.004	
Std Dev	0.025		0.028		0.027		0.061	

From the current I one can calculate B from the equations II.7.2.4 and II.7.2.5. For each B the average output voltage and the standard deviation are collected in Table II.7.2.2.

Table II.7.2.2
Data culled from tables II.7.2.1 (a) and (b)

		Forward		Reverse		Average	
I amp	B T	V_{av}	Std devn	V_{av}	Std devn	V_{av}	Std Devn
0.2	0.000653	0.2	0.023	0.196	0.03	0.198	0.0265
0.3	0.00098	0.298	0.019	0.339	0.021	0.3185	0.02
0.4	0.001307	0.469	0.026	0.475	0.015	0.472	0.0205
0.5	0.001634	0.587	0.024	0.598	0.023	0.5925	0.0235
0.6	0.00196	0.712	0.029	0.708	0.036	0.71	0.0325
0.7	0.002287	0.814	0.024	0.776	0.025	0.795	0.0245
0.8	0.002614	0.92	0.024	0.859	0.028	0.8895	0.026
0.9	0.002941	0.962	0.027	0.962	0.027	0.962	0.027
1	0.003267	1.09	0.061	1.09	0.061	1.09	0.061

A plot of V_{av} against B is shown in Figure II.7.2.2 (a) for both forward and reverse measurements.

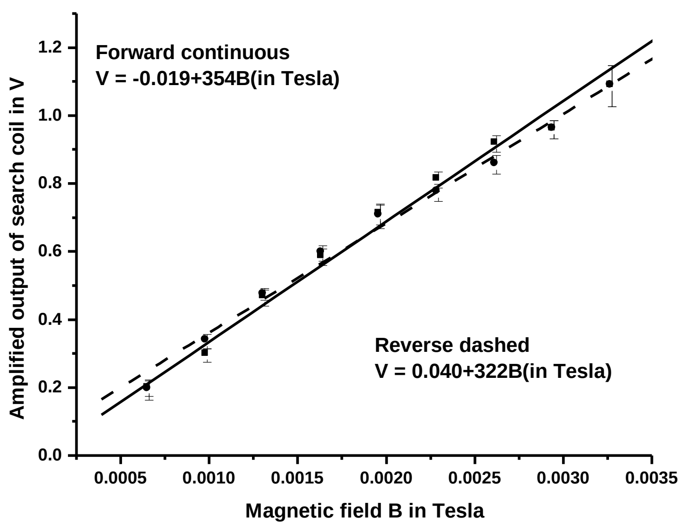


Figure II.7.2.2 (a) Plot of average integrator output voltage of the search coil against the magnetic field B in Tesla for forward (continuous) and reverse (dashed) situations. Linear fits are shown

Figure II.7.2.2 (b) shows the average of the forward and reverse outputs plotted against B.

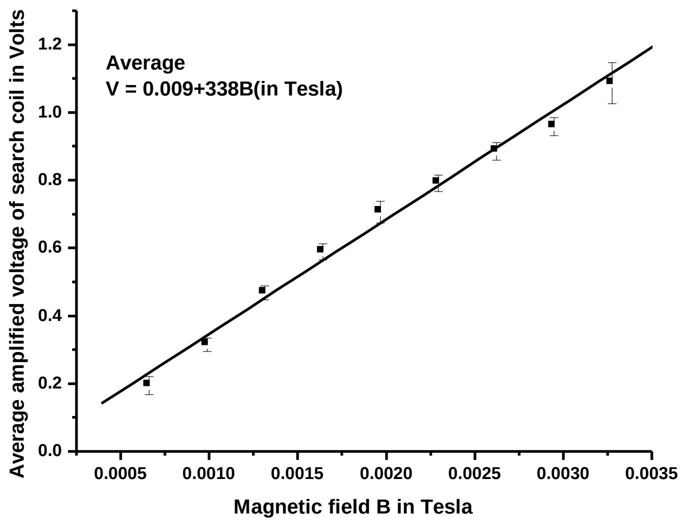


Figure II.7.2.2 (b) Plot of average of the forward and Reverse outputs plotted as a function of B.

A reasonably good straight line is obtained with the average output voltage given by

$$V_{out} = 0.009 + 338\ B$$

with B in Tesla. The calibration will be accurate to ± 5%. Since 1 Gauss is 10^{-4} T the slope of the graph gives a change of 33.8 mV for a change in B of 1 Gauss.

5. FIELD VARIATION ALONG THE AXIS OF A SOLENOID

With the calibrated search coil we measure how the field varies along the axis of the solenoid. We start with the search coil at the center of the solenoid. Set the current through the solenoid to be about 0.5 A. Then we carry out the following steps.

1. Operations 12 to 16 described in the previous section are repeated five times for forward and five times for reverse settings to get ten readings for the output voltage for a given location of the search coil.
2. The search coil is moved forward along the axis of the coil in steps of 2 cm and operations 12 to 16 are repeated each time. Thus we get ten readings for the output for each position of the search coil. This is done till the search coil reaches the forward end of the solenoid.

A sample set of data is shown in Table II.7.2.3 on the next page. The first row gives the position of the search coil on the axis of the solenoid measured from the center of the solenoid. The rest of the columns and rows have the same significance as in Table II.7.2.1.

Table 7.2.3
Field along the axis of the solenoid
Solenoid center is at 170 mm reading on scale
Current 0.45 A

	end							
Scale Rdg	50 mm		70 mm		90 mm		110mm	
	V output	Error^2	V output	Error^2	V output	Error^2	V output	error^2
Forward	0.36	0.0001	0.45	0.000361	0.49	0.001764	0.47	0.0009
	0.29	0.0037	0.41	0.0038	0.45	0.0018	0.47	0.0018
	0.33	0.0041	0.43	0.0054	0.42	0.0026	0.51	0.0019
	0.29	0.0077	0.45	0.0057	0.42	0.0033	0.5	0.0019
	0.34	0.0078	0.47	0.0057	0.39	0.0067	0.49	0.0020
Reverse	0.37	0.0082	0.49	0.0062	0.44	0.0068	0.46	0.0036
	0.35	0.0082	0.51	0.0078	0.41	0.0082	0.51	0.0037
	0.4	0.0107	0.5	0.0088	0.49	0.0100	0.53	0.0046
	0.39	0.0123	0.49	0.0092	0.49	0.0117	0.54	0.0062
	0.38	0.0132	0.49	0.0097	0.48	0.0128	0.52	0.0066
Av	0.350		0.469		0.448		0.500	
Variance	0.001467		0.00108		0.00142		0.00073	
Std dev	0.038297		0.03281		0.03765		0.02708	

					Center	
Scale Rdg	130mm		150mm		170 mm	
	V output	Error^2	V output	Error^2	V output	Error^2
Forward	0.49	0.000225	0.47	4.9E-05	0.54	0.0041
	0.43	0.0023	0.42	0.0019	0.46	0.0044
	0.42	0.0053	0.47	0.0019	0.42	0.0075
	0.5	0.0059	0.42	0.0038	0.46	0.0077
	0.48	0.0059	0.42	0.0056	0.52	0.0097
Reverse	0.48	0.0060	0.5	0.0070	0.5	0.0103
	0.48	0.0060	0.46	0.0070	0.48	0.0103
	0.51	0.0072	0.49	0.0078	0.47	0.0103
	0.47	0.0072	0.49	0.0085	0.44	0.0116
	0.49	0.0075	0.49	0.0092	0.47	0.0116
Av	0.475		0.463		0.476	
Variance	0.000828		0.001023		0.001293	
Std dev	0.028771		0.03199		0.035963	

Using the calibration of the search coil

$$V_{average} = 0.009 + 338B \text{ (in Tesla)}$$

the magnetic field B is calculated in Tesla at various distances X from the center of the solenoid. In Table II.7.2.4 the values of X, $V_{average}$ and B are shown in the first three columns. From the Field B (0) at the center and B(X) at position X the function F(X) = B(x)/B (0) is calculated and given in column 4. The error in F(X) is calculated from the error in B(X). This error arises (a) due to standard deviation in the measured output voltages and (b) the error in slope in the calibration equation. The fractional error due to fluctuations in readings is Stand. Dev/$V_{average}$. The fractional error due to error in slope in calibration is 16/338. The total fractional error in B(X) is the sum of these two fractional errors. The error ΔF(X) is taken as

$$\Delta F(X) = F(X) \text{ x Fractional error in } B(X)$$

This is given in column 5. For a solenoid of length 2L and radius a, the function F(X) can be calculated from theory and is given by

$$F(X) = \{[(L-X)/\ \{1+ (L-X)/a)^2\}^{0.5}] + [(L+X)/\ \{1+ (L+X)/a)^2\}^{0.5}]\}/2L/\ [1+ (L/a)^2)^{0.5}]$$

This function is calculated for different values of X and shown in the last column. This function is the same for all currents i.e. all values of B (0).

Table II.7.2.4
Field along the Axis of the solenoid
Current 0.45 A

X in cm	Av Volts	Std dev	B(X) T	F(X) = B(X)/B(0)	ΔF(X)	F(X) Th
12	0.35	0.039	0.00101	0.73	0.12	0.823
10	0.469	0.033	0.00136	0.99	0.12	0.936
8	0.448	0.038	0.0013	0.94	0.12	0.973
6	0.05	0.027	0.00145	1.05	0.12	0.989
4	0.475	0.029	0.00138	1.00	0.12	0.996
2	0.463	0.032	0.00134	0.97	0.12	0.999
0	0.476	0.036	0.00138	1.00	0.12	1.000

Figure II.7.2.3 shows a plot of F(X) as a function of X. The error in measurement is about ± 12%. The theoretical function calculated with 2L = 28 cm and a = 2.5 cm is plotted as a continuous curve. The theoretical values are for a search coil with infinitely small dimensions. The actual search coil has a diameter of 5 mm and a length of 1 cm and measures the average value of B in this volume. The experimental measurements are in agreement with theoretical function within the error of measurement.

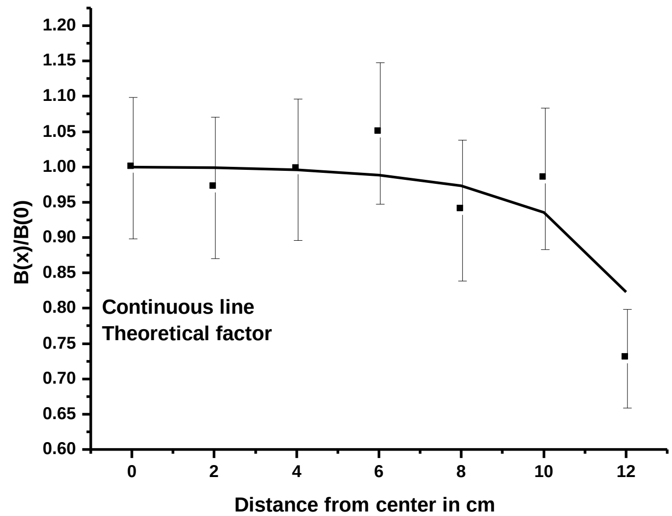


Figure 7.2.3: Ratio of the field B(X) at a distance X from the center to the field B(0) at the center of the solenoid as a function of X. Continuous curve is from theory

The purpose of the experiment is to show

1. how the search coil can be used to measure the magnetic field,
2. how the field varies along the axis of a solenoid

and

3. even with random errors as large as 10 to 20% one can get meaningful results if one takes a large number of readings and takes the average.

II.8 RELAXATION EXPERIMENT

II.8.1 MEASUREMENT OF THERMAL RELAXATION TIME CONSTANT OF A SERIAL LIGHT BULB

1. INTRODUCTION

Relaxation phenomena occur very commonly in physics. Take a spherical ball of radius *a* moving with an initial velocity v (0) in a medium with viscosity η. The equation of motion for the ball is

$$m\,(dv/dt) + 6\pi\eta a v = 0 \qquad \text{(II.8.1.1)}$$

Its velocity will decrease exponentially with time as

$$v\,(t) = v(0)\,\exp\,(-t/\tau) \qquad \text{(II.8.1.2)}$$

and it will come to rest eventually. τ, called the relaxation time, is given by

$$\tau = m/6\pi\eta a \qquad \text{(II.8.1.3).}$$

Here is another example. A sample is connected to a bath at temperature T_0. It is heated to a temperature T. Then heat is cut off. The sample will cool and its temperature T (t) as a function of time t satisfies the equation

$$ms\,dT/dt + E\,(T-T_0) = 0 \qquad \text{(II.8.1.4)}$$

Here m is the mass of the sample, s its specific heat and E the constant in Newton's law of cooling. The second term in the above equation represents the heat lost in one second by the sample to the bath. The solution to this equation is

$$T\,(t) = (T-T_0)\,\exp\,(-t/\tau) + T_0 \qquad \text{(II.8.1.5)}$$

The temperature of the sample will approach the temperature of the bath exponentially with a characteristic relaxation time τ given by

$$\tau = ms/E \qquad \text{(II.8.1.6)}$$

A third example is the following. An electric field E is applied to a polar dielectric medium like acetone. This creates an electric polarization P in the medium. When the electric field is switched off at time t = 0, the polarization decays to zero exponentially with time. There is a characteristic relaxation time associated with this process. The mechanism of relaxation here is more complicated than in the other two cases considered above.

This is also true of magnetization created in a paramagnetic material on the application of a magnetic field. When the magnetic field is switched off the magnetization in the specimen decreases exponentially to zero with a characteristic relaxation time.

Relaxation is a phenomenon characteristic of dissipative mechanical, thermal, electric and magnetic systems. If X is the material property, which relaxes with a characteristic relaxation time τ, it satisfies the equation

$$dX/dt + X/\tau = 0 \qquad \text{(II.8.1.7)}$$

in the absence of a 'Force'.

Let us now consider forcing the system at a frequency f. For the ball moving in a viscous medium this is achieved by applying a mechanical force F exp ($\iota\omega t$) where

$$\omega = 2\pi f \qquad \text{(II.8.1.8)}$$

In the case of the thermal system, forcing is done by applying a periodic heat $Q = Q_0 \exp(\iota\omega t)$ to the system. In the dielectric example forcing is done by applying a periodic electric field E exp ($\iota\omega t$). In the magnetic example forcing is done by applying a magnetic field B exp ($\iota\omega t$). A relaxation system forced by a simple harmonic force will satisfy the equation

$$dX/dt + X/\tau = F \exp(\iota\omega t) \qquad \text{(II.8.1.9)}$$

The solution to this equation is

$$X(t) = [F\tau/(1+\iota\omega\tau)] \exp(\iota\omega t) \qquad \text{(II.8.1.10)}$$

The response of the system is also simple harmonic with the same frequency f. But the response is not in phase with the force. If we write

$$F\tau/(1+\iota\omega\tau) = A\exp(-i\phi) \qquad \text{(II.8.1.11)}$$

Then $$A = F\tau/(1+\omega^2\tau^2)^{1/2} \qquad \text{(II.8.1.12)}$$

and $$\tan(\phi) = \omega\tau \qquad \text{(II.8.1.13)}$$

For a fixed amplitude of 'force', F, the amplitude A of the response decreases with frequency as shown by equation (II.8.1.12). The amplitude, as the frequency tends to zero, is A (0) = Fτ. If $\omega\tau >> 1$, the amplitude A (ω) at frequency ω will decrease as

$$A(\omega) = A(0)/\omega\tau \qquad \text{(II.8.1.14)}$$

2. THEORY OF THE EXPERIMENT

Let us consider a serial light bulb. We pass a current

$$I = I_0+I_1 \sin \omega t \qquad \text{(II.8.1.15)}$$

through the bulb, where I_1 is small compared to I_0. If we choose I_0 and I_1 suitably, and the frequency f of the oscillating current is small (about 10 Hz), we can see the light blinking. Let m be the mass of the filament and s its specific heat. The bulb will lose heat through the thermal conductance of the support connecting the filament to the cap of the bulb and through radiation. We model the heat loss by Newton's law of cooling. We measure the temperature difference θ of the filament over that of the cap, which is at the ambient temperature T_0. The heat power input to the filament is

$$Q = I^2 R = (I_0^2 + (1/2)I_1^2)R + 2I_0I_1R \sin(\omega t) -(1/2)I_1^2 R \cos(2\omega t) \qquad \text{(II.8.1.16)}$$

$$Q = Q_0 + Q_1 \sin \omega t - Q_2 \cos 2\omega t \qquad \text{(II.8.1.17)}$$

The temperature θ of the filament satisfies the equation

$$msd\theta/dt + E\theta = Q_0 + Q_1 \sin (\omega t)- Q_2 \cos (2\omega t) \qquad \text{(II.8.1.18)}$$

E is the constant in Newton's law of cooling.

Writing $E/ms = 1/\tau$, $Q_j/ms = \theta_j$ the above equation can be written as

$$d\theta/dt + \theta/\tau = A_0 + A_1 \sin\omega t - A_2 \sin 2\omega t \qquad \text{(II.8.1.19)}$$

Integrating this equation from t = 0, when $\theta = 0$, we get

$$\theta(t) = \theta_0(1- \exp (-t/\tau) + (\theta_1 /[1+\omega^2\tau^2]^{1/2})\sin(\omega t+\phi_1)$$

$$- (\theta_2/ [1+4\omega^2 \tau^2]^{1/2} \sin(2\omega t + \phi_2) \qquad \text{(II.8.1.20)}$$

where

$$\tan (\phi_1) = \omega\tau \text{ and } \tan (\phi_2) = 2\omega\tau \qquad \text{(II.8.1.21)}$$

If $I_1/I_0 = 0.1$, then $\theta_1/\theta_0 \approx 0.2$ and $\theta_2/\theta_0 \approx 5 \times 10^{-3}$. So the second harmonic signal is less than 1% of the first harmonic. We shall neglect the second harmonic term.

The intensity of the radiation emitted from the filament will vary as θ^α where α is nearly four. So the intensity of the emitted light will vary as

$$I_L = I_{L0} + I_{L1} \sin(\omega t) \qquad \text{(II.8.1.22)}$$

where I_{L1} will be proportional to $\alpha\theta_0^{\,(\alpha-1)}\,\theta_1[1/(1+\omega^2\tau^2)^{1/2}]\,\sin(\omega t + \phi_1)$

We pick this intensity fluctuation with a photodiode circuit in the linear mode. The photodiode BPW 34 will give an output voltage proportional to the intensity of the light. The rms AC output voltage V_{out} of the photo-detector is measured with a multimeter.

The frequency, f, of the AC signal is changed, keeping the AC voltage across the bulb constant. The resulting set of measurements are fitted to a formula

$$V_{AC}(f) = V(0)/[1+(2\pi f\tau)^2]^{0.5} \qquad \text{(II.8.1.23)}$$

The value of V(0) and τ are found as described below.

3 THERMAL RELAXATION SET UP:

The photodiode is BPW 34. The photodiode circuit is shown in Figure II.8.1.1.

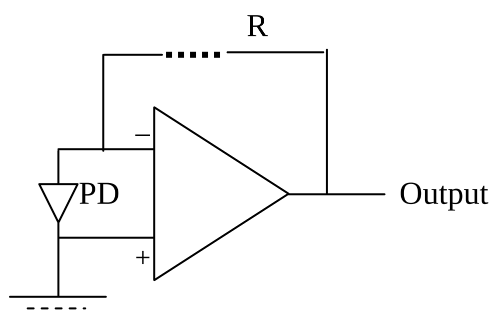


Figure II.8.1.1: Photodiode circuit

The blinking light source is a 3 V serial light bulb. The connection to the light bulb is as shown in Figure II.8.1.2. A resistance of 6 Ohms (1 W) and a resistance of 2 Ohms (1W) are connected in series with the bulb and a 5 V regulated DC supply as shown in Figure II.8.1.2. When the switch SW1 is thrown to one side, the DC voltage across the 6 Ohm resistance can be measured by a DMM2 in the DC 2 V range connected between the terminals T and G. This voltage divided by 6 gives the DC current through the bulb. With the switch SW2 is in the LC position, the bulb glows a dull red. One can connect an external plug pattern resistance box (0.1 to 10 Ohm) across the two Ohms resistor and can vary the current through the bulb by adjusting the resistance in the box. When the switch SW2 is in the HC position it shorts the resistance of 2 ohms and increases the current through the bulb. The bulb glows brightest when SW2 is in HC position. The distance of the photodiode from the bulb is adjusted so that the output DC voltage measured by DMM1 at the terminals marked PD O/P is about 4.5 to 5 V in the high current position of SW2. A signal generator is connected through two 50 Ohm resistances and two 1000 μf capacitors to the bulb.

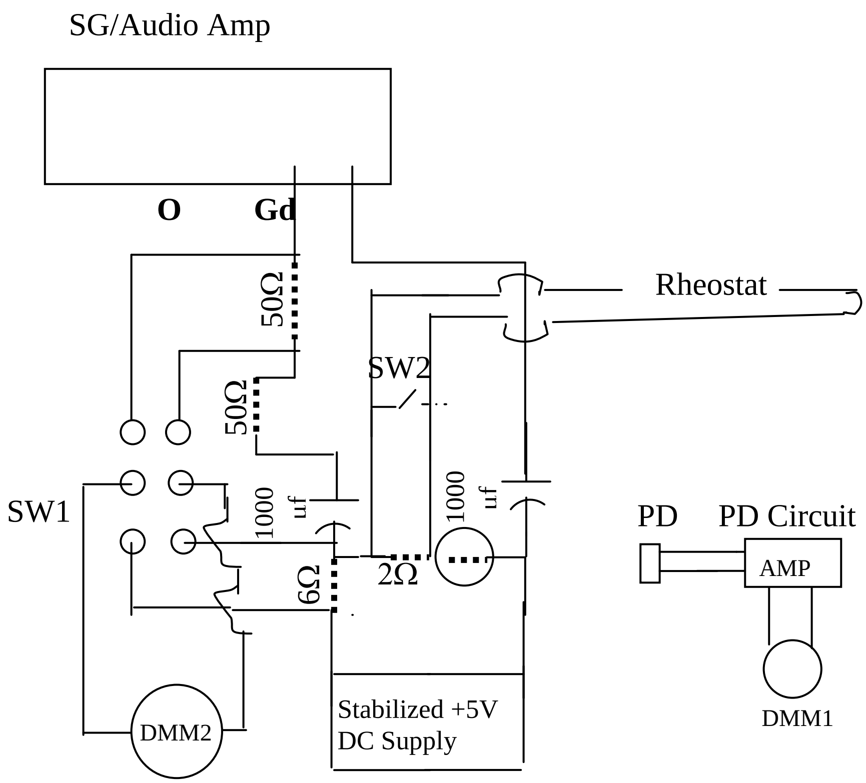


Figure II.8.1.2 Connections for Relaxation Experiment

The signal generator is switched on and its frequency is adjusted to about 20 Hz. This frequency can be measured on DMM2 when it is put in Hz position and the switch SW1 is thrown in the AC position. The rms AC voltage across 50 Ohms can be measured by changing the range switch on the DMM to AC 2 V position. The output of the signal generator is adjusted so that the RMS AC voltage across the 50 Ohm resistor as measured by DMM2 is 1 Volt. The AC current is about 10 to 20% of the DC current. AC frequency is changed keeping the AC voltage across the 50 Ohm resistor constant by adjusting the signal amplitude knob on the signal generator. The AC current causes the temperature of the filament in the bulb to oscillate about a mean temperature determined by the DC current. This produces a fluctuating AC output of the photo-diode at the frequency of the AC current. The RMS value of the fluctuating voltage is measured by connecting DMM1 set to measure AC 200 millivolts to the terminals marked PD O/P. The frequency of the AC voltage is varied in steps and the AC output voltage is measured.

For serial light bulbs the relaxation time is of the order of 0.1 s. So even at 20 Hz frequency, $(\omega\tau)^2$ is more than a hundred times larger than 1. So equation II.8.1.23 becomes to a good approximation

$$V_{AC}(f) = V(0)/ (2\pi f\tau) \qquad \text{(II.8.1.24)}$$

The Front panel diagram of the relaxation set up is shown below.

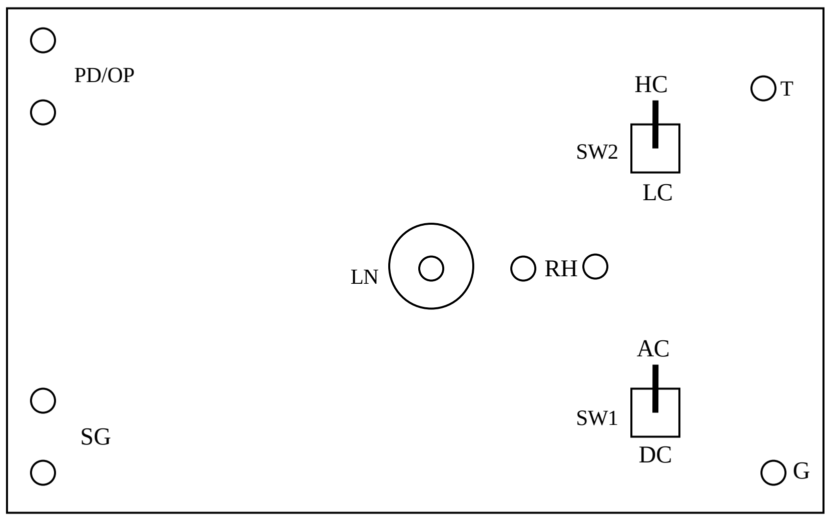


Fig. II.8.1.3 Front panel diagram of thermal relaxation set up

4. APPARATUS REQUIRED:

Thermal relaxation set up, a signal generator, a DMM which will measure DC Volts, AC Volts in the 200 mV range and frequency correct to one decimal place in the range 20 to 100 Hz.

5. PROCEDURE:

(a) AC PART:

Switch on the thermal relaxation set up., Put the DPDT switch SW2 at the right top to position HC. Put the DPDT switch SW1 at the right bottom to position DC. Connect a DMM between the banana terminals marked T and G on the right. Set it in DC 2 volts range to measure the voltage to three decimal places.. The meter will show a reading around 1.8 V. This is the DC voltage across 6 Ohms in series with the bulb. If the same meter (or another DMM1) is connected between terminals marked P/D OP on the top left of the panel, the meter will show an amplified Photo diode output DC voltage. The distance between the photodiode and the bulb can be changed by releasing the locking nut LN and turning the screw at the center of the panel. The reading on the meter will increase as the photodiode is brought closer to the bulb and it will decrease as the photodiode is moved farther from the bulb. Adjust the screw so that the DC output of the photodiode is between 4 and 5 volts in magnitude. Lock the nut.

Connect a signal generator between the two banana plugs at the bottom left marked SG. The bottom banana terminal should be connected to the earth of the signal generator. Set a frequency of about 20 Hz on the signal generator and the output voltage around 2 V AC. Connect the DMM2 between the terminals T and G and put the DPDT switch SW1 at the bottom right corner to AC. The meter should be selected to read AC volts. The meter will read the voltage across 50 Ohms in series with the bulb. Adjust the output amplitude of the signal generator so that the meter reads 1 V AC. RMS value of the AC current through the bulb is 1/50 = 0.020 A. Turn the knob on the DMM2 to the frequency position and measure the frequency of the AC voltage. Adjust this frequency to 20 Hz by turning the frequency adjusting knob on the signal generator. Again check that the AC voltage read by the DMM is 1 V. Then read the AC voltage at the terminals marked P/D OP either using DMM2 (or DMM1in the AC mode). Note this reading. Again connect the DMM2 to the terminals T and G, set it to measure frequency and adjust the AC frequency from the signal generator to be 22 Hz. Check that the AC voltage read by DMM2 is 1 V. Measure the AC output voltage of the photodiode now. Repeat these measurements as the frequency increases from 20 to 100 Hz and as the frequency decreases from 100 to 20 Hz. **ALWAYS KEEP THE AC VOLTAGE ACROSS T-G TERMINALS TO 1 V AT EACH FREQUENCY.**

The average signal output in mV is taken at each frequency. Table II.8.1.1 shows a sample set of readings. This is $V_{AC\ av}$ in column 4 of the table. $1/V_{AC\ av}$ is given in column 5.

Table II.8.1.1 RELAXATION MEASUREMENT

pd dc o/p	5.2	V			
dc Voltage across 6 Ohms			1.862	V	
AC voltage across 50 Ohms			1	V	
freq	V_{AC} mV	V_{AC} mV	V_{AC} mV average	$1/V_{ACav}$	V_{AC} cal
	f increasing	f decreasing			mV
20	117	119	118	0.00848	113
22	108	106	107	0.00935	103
24	100	102	101	0.0099	94
26	92	92	92	0.0109	87
28	83	85	84	0.0119	81
30	81	81	81	0.01234	75
35	65	67	66	0.01515	65
40	58	58	58	0.01724	56
45	52	52	52	0.01923	50
55	47	47	47	0.02128	41
60	38	38	38	0.02632	38
70	33	33	33	0.0303	32
80	28	28	28	0.03571	28
90	26	26	26	0.03846	25
100	23	23	23	0.04348	23

$1/V_{ACav}$ is plotted against f. Such a plot is shown in Figure II.8.1.3. The slope of this graph $2\pi\tau/V_{rms}(0)$ is 4.39×10^{-1} s/V. The AC voltages measured on the DMM are RMS voltages. To find τ we should find $V_{rms}(0)$. This is done by doing a DC experiment described below.

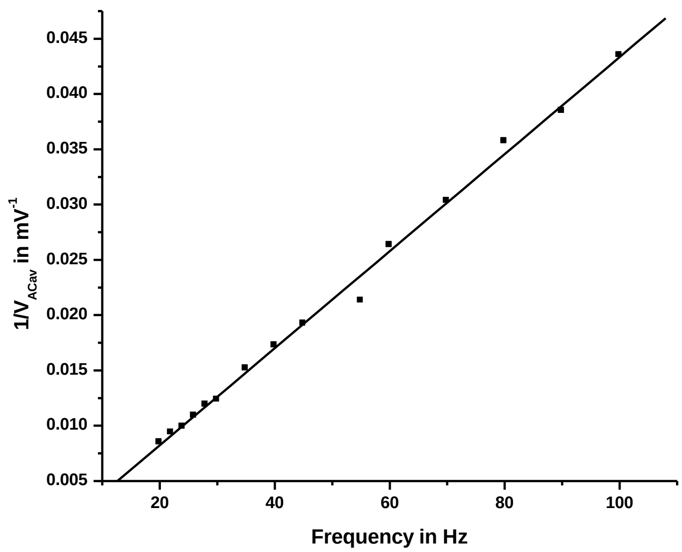


Figure II.8.1.3 Plot of $1/V_{ACav}$ against frequency

b) DC PART

Remove the signal generator connections. Put the DPDT switch SW2 on the right top to LC. Connect a plug pattern resistance box (0.1 to 10 Ohms) to the terminals marked RH. Put the bottom DPDT switch SW1 to position DC. Connect the DMM in the DC Volts mode to the terminals T and G. Pull out 0.1 Ohm from the plug pattern box. Measure the voltage V on the DMM2. The current through the bulb is now V/6 A. Connect the same DMM2 (or use another DMM if available) to the terminals P/D OP and measure the output DC Voltage $V_{DC,PD}$ of the photodiode. Repeat the measurement as the resistance in the box is increased in small steps of 0.1 or 0.2 ohms till the P/D output voltage comes down to 2.5 V.

A sample set of readings are given in the following table II.8.1.2. First column gives the DC voltage measured at terminals T and G. This voltage divided by 6 is the current I through the bulb given in the second column. The third column gives the square of I. The fourth column is the DC voltage measured at the terminals PD OP. A plot of the photo-diode voltage against I^2 is given in Figure II.8.1.4.

Table II.8.1.2

DC Photodiode output as a function of DC Current through the bulb

DC Volts across 6 Ohms	Current I in amp	I^2	DC O/P of PD in Volts
1.843	0.307	0.09435	3.459
1.837	0.306	0.09374	3.321
1.817	0.303	0.09171	2.858
1.771	0.295	0.08712	1.993
1.761	0.294	0.08614	1.824
1.742	0.290	0.08429	1.559
1.728	0.288	0.08294	1.331
1.72	0.287	0.08218	1.232
1.704	0.284	0.08047	1.045
1.696	0.283	0.07990	0.953

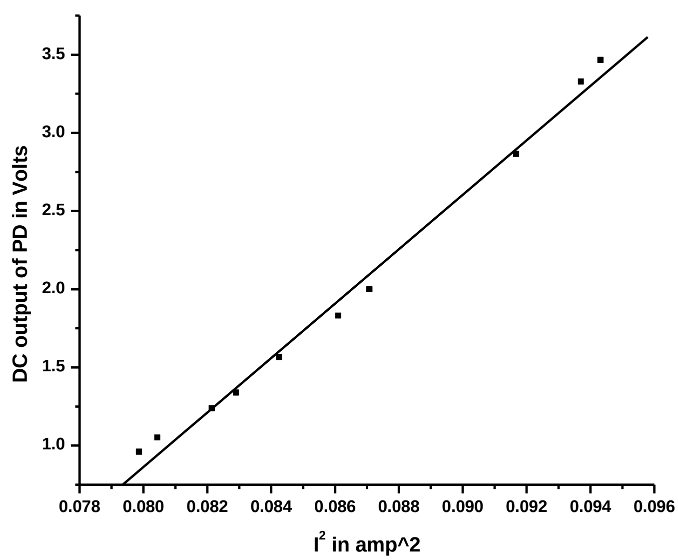


Figure II.8.1.4 Plot of DC Output voltage of photodiode against the DC current through the bulb.

The equation to this line is

$$V_{DCOP} = A + BI^2 \qquad \text{(II.8.1.25)}$$

where A, the intercept, is -13.06 and B, the slope, is 174.1.

This may be written as

$$V_{DC\ O/P} = B(I^2 + A/B) = 174.1(I^2 - 0.075)$$

Written this way the equation shows that a minimum current is required through the bulb for the photo diode to respond.

In the AC experiment the rms AC current through the bulb is 1/50 = 0.02 A. So peak amplitude of AC current is 1.414*0.02 = 0.028 amp. This is superposed on the DC current of 1.862/6 = 0.310 Amp Superposition of AC with DC currents leads to a maximum current of 0.310+0.028 = 0.338 Amp through the bulb and to a minimum current of 0.310-0.028 = 0.282 amp through the bulb. As the frequency tends to zero, these currents will produce an output of the photo diode of 6.83 and 0.78 V respectively when substituted in equation (II.8.1.25). In the limit of low frequencies (f tending to zero) this excursion of the photo diode output voltage, (6.83-0.78) V, is equal to $2V(0)_{peak}$. So $V(0)_{peak}$ = 3.02 V. In the AC part of the experiment we measured RMS AC voltages on the multimeter. So we must find $V(0)_{rms}$, which is equal to $V(0)_{peak}$*0.707. This means that the rms value, $V_{rms}(0)$ in the slope of AC experiment, is 3.02x0.707 = 2.14 V. τ can be calculated from the slope of the line in Figure II.8.1.3 ($2\pi\tau/V(0)_{rms}$ = $4.39\text{x}10^{-1}$) s/V and $V(0)_{rms}$ = 2.14 V. τ comes out to be 0.15 s.

Using the values of V_{rms} (0) and τ and the Debye formula

$$V(f) = V_{rms}(0)/[1+4\pi^2 f^2 \tau^2]^{0.5}$$

we calculate the values of V at the frequencies of measurement. These values are shown in the last column of Table II.8.1.1 under $V_{AC\ cal}$ in mV. Figure II.8.1.5 shows the experimental values of V_{AC} as a function of frequency and the values calculated from Debye formula are plotted as a continuous curve. The agreement is good.

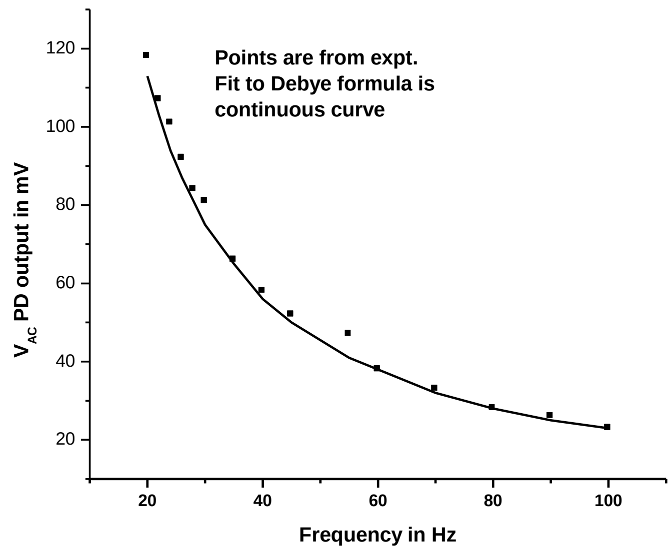


Figure II.8.1.5 Comparison of experiment with Debye Formula

NOTE:

Different makes of the serial light bulbs have different values of the relaxation time and the photodiode output will vary with the make of the bulb for the same current through the bulb and the distance of photo-diode from the bulb. So the above readings must be taken as a sample only.

Questions:

1. Give examples of relaxation in physics.

2. A capacitance C is connected in series with a resistance R. A DC voltage V is applied at time t = 0. How will the voltage across the capacitor change with time? On what factors does the relaxation time depend?

3. There are two thermal systems. The first system has a large thermal capacity and is connected to a thermal reservoir at temperature T through a weak link. The second system has a low thermal capacity and is connected to the reservoir with a strong thermal link. Which system will relax faster and why?

II.9 EXPERIMENTS WITH THE LOCK IN AMPLIFIER

II.9.1 PRINCIPLE OF PHASE SENSITIVE DETECTION

1. INTRODUCTION

Amplifiers are used to amplify a weak AC signal. However the amplifier also amplifies noise. Amplifiers have a bandwidth of several kilohertz. Usually noise is present over a range of frequencies while the signal is at a single frequency. If the signal is very weak, amplification alone will not enable us to pick out the signal against noise.

There are different sources of noise in an amplifier. Flicker noise is present in all electronic instruments and its power spectrum varies inversely with the frequency. This noise becomes a problem only if one works at very low frequencies. Then we have noise due to electromagnetic pick up from running motors, tube lights and so on. Such a noise has a peak in the power spectrum only at the frequency of the motor or the frequency of the mains. Such noise can be reduced by electromagnetic shielding. The third source of noise is thermal noise. This is of thermodynamic origin and cannot be avoided. Its power spectrum extends over all frequencies. So it is called white noise. If we have resistance R through which a current I is flowing the voltage across the resistance will vary randomly about the average value $V_0 = IR$. The mean square fluctuation of the voltage is defined by $< (V-V_0)^2>$ where the average is taken over a long period of time. If we measure the noise over a bandwidth W, the mean square voltage will be

$$< (V-V_0)^2> = 4\, k_B\, T\, R\, W \tag{II.9.1.1}$$

To reduce this noise we either reduce the temperature T or reduce the bandwidth W of the amplifier. For detecting very weak signals at room temperature we have to effectively reduce the bandwidth to a few Hertz though the amplifier has a large bandwidth in kilohertz range. This is achieved by Phase Sensitive Detection.

2. SIGNAL AND REFERENCE

Let us have an AC sinusoidal cause. For example, the cause may be the current through the primary coil of a mutual inductance. It may be a light signal whose intensity is modulated periodically. This cause will have an effect that will have the same frequency as the cause but may differ in phase from the cause. The effect in the case of mutual inductance is the induced emf in the secondary coil. This will be 90^0 out of phase with the current in the primary coil, but it will have the same frequency as the current. The effect will be weak of the order of a few micro-volts or nanovolts. If we merely amplify the weak signal a million times we will also amplify the noise. In an ordinary amplifier the noise is collected over a wide frequency range and so may even swamp the weak signal due to the cause.

To overcome the noise vis-à-vis the weak effect signal, we take a reference signal, which is derived from the cause signal and is in phase with it. For example in the mutual inductance, a resistance is connected in series with the primary coil so that the current through the primary also passes through the resistance R. The voltage across the resistance will be the reference signal. By choosing R suitably one may generate a reference signal of a few tens of a millivolt or more.

Figures II.9.1.1 shows the weak effect signal and Figure II.9.1.2 shows the reference signal.

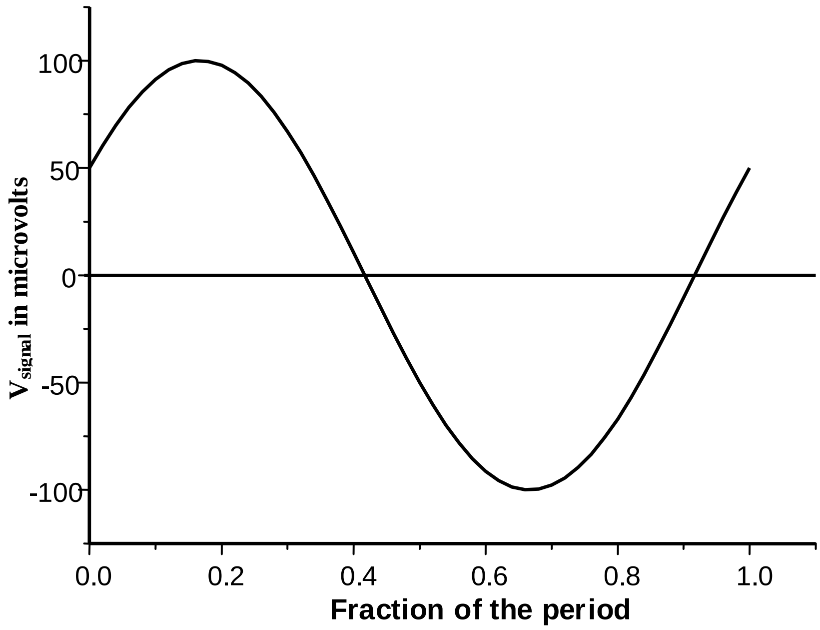


Figure II.9.1.1 Weak signal as a function of time (t/T) where T is the Period.

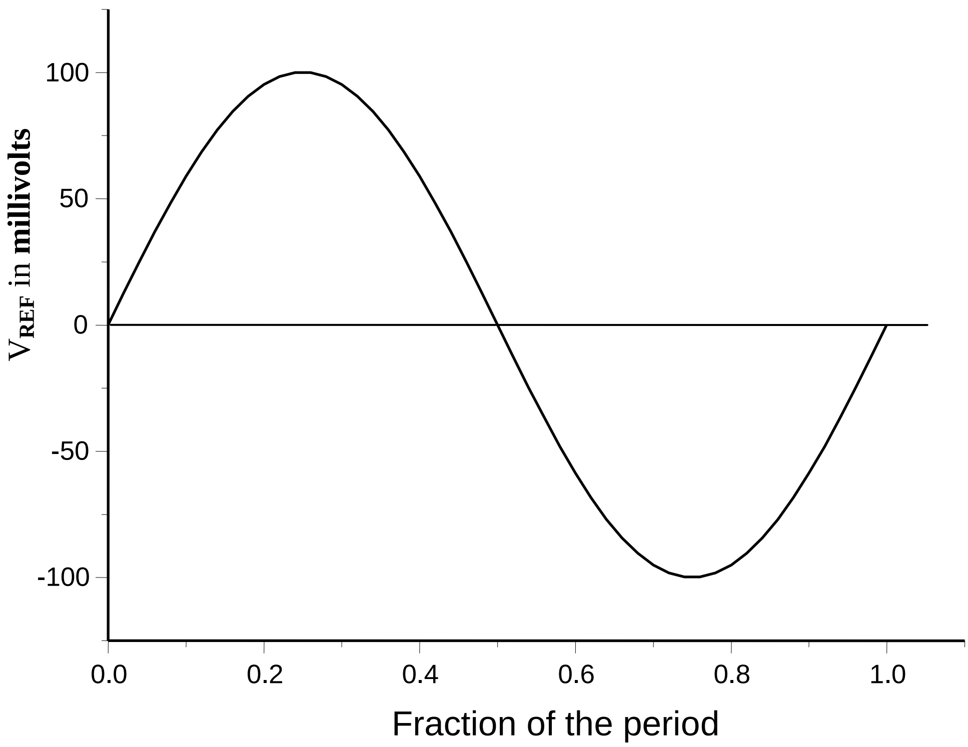


Figure II.9.1.2 Reference voltage as a function of time (t/T) where T is the Period.

From the above figures it is seen that the weak signal is advanced in phase relative to reference signal by an angle ϕ. In the above example $\phi = \pi/6$.

We may write

$$v_{signal} = v_0 \sin(\omega t+\phi) \qquad \text{(II.9.1.1)}$$

and

$$V_{ref} = V_0 \sin(\omega t) \qquad \text{(II.9.1.2)}$$

Let us say these are the magnitudes of the signal and reference voltage after some amplification in our circuit.

These signals are then fed to the chip AD 630. This is a phase sensitive detector. In this chip there are two identical amplifiers. One is a direct amplifier. This means that the output is in phase with the input. The other is an inverse amplifier i.e. the output is 180^0 out of phase with the input. There is a switch operated by a comparator. The comparator senses the reference signal. When the reference signal is positive (i.e. for time t $0<t<T/2$) the comparator connects the weak signal v_{signal} to the direct amplifier so that the output of the amplifier is

$$V_{out} = \mu\, v_{signal} \qquad 0<t<T/2 \qquad \text{(II.9.1.3)}$$

When the reference signal is negative (i.e. for time t between T/2 and T) the weak signal is connected to the inverse amplifier so that the output voltage is

$$V_{out} = -\mu\, v_{signal} \qquad T/2<t<T \qquad \text{(II.9.1.4)}$$

Figure (II.9.1.3) shows the amplified signal V_{out} assuming an amplification of 1000.

We see from Figure II.9.1.3 that the output signal is positive for most of the period but there is a small negative portion. So the output signal will have an average DC part superposed on an AC part at the frequencies ω, 2ω …. If we have a filter that will bypass the AC components to earth, we will get a DC output V_{DC}. We may calculate the DC output from the equation

$$V_{DC} = (1/T)\left\{ \int_0^{T/2} \mu v_0 \sin(\omega t+\phi)dt - \int_{T/2}^{T} \mu v_0 \sin(\omega t+\phi)dt \right\} \qquad \text{(II.9.1.5)}$$

$$= \{4\mu v_0/2\pi\} \cos(\phi) \qquad \text{(II.9.1.6)}$$

remembering $\omega T = 2\pi$.

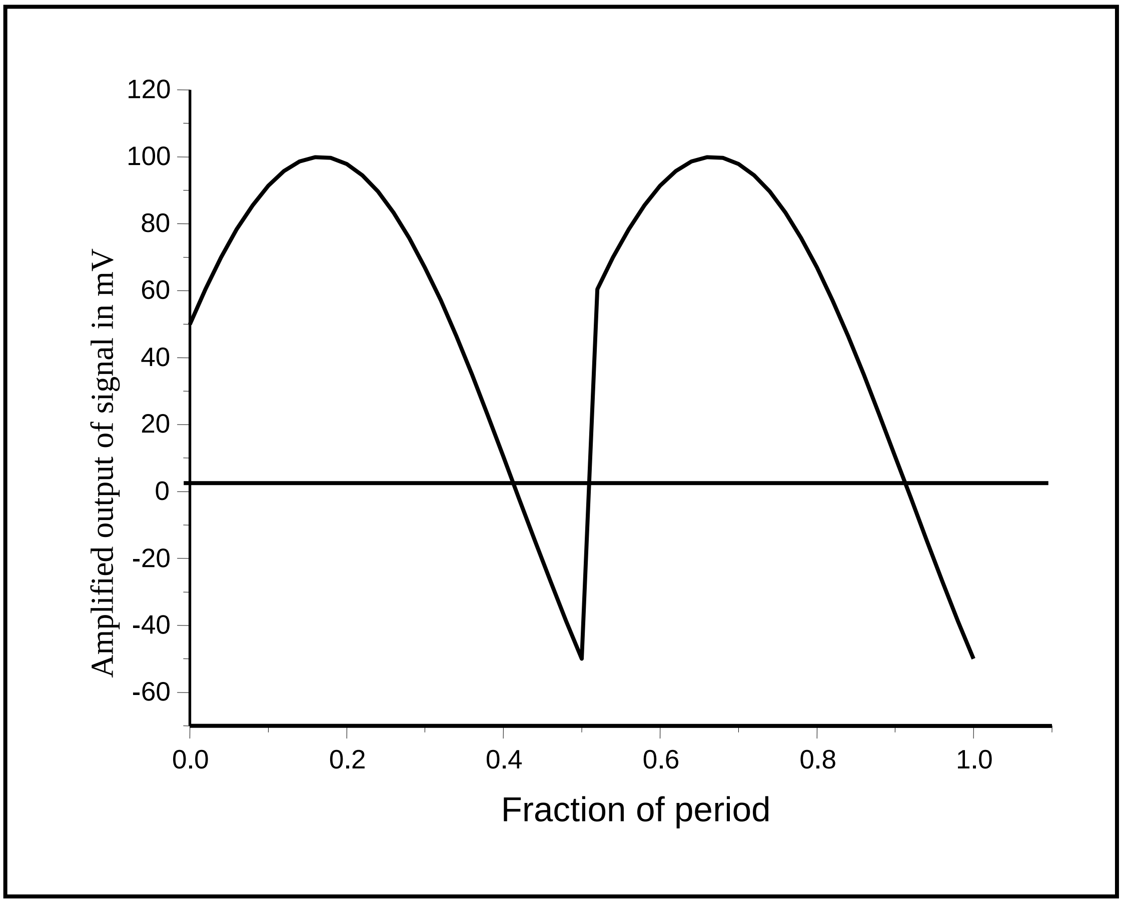


Figure II.9.1.3 Amplified output V_{out} when the reference signal is fed to the comparator.

If we introduce a circuit to phase shift the reference signal and then send the phase shifted reference signal to the comparator in ADL 630, the output DC voltage will vary as the phase shift is increased from zero. It will reach a maximum when the reference signal is phase shifted by ϕ and the phase-shifted reference signal is in phase with the weak signal $v_{signal.}$ Then the output voltage will look like the one shown in Figure II.9.1.4. The average DC voltage, V_{DC}, will then reach a maximum value $2\mu v_0/\pi$.

That is why we call this phase sensitive detection and the amplifier a lock-in amplifier because we make the weak signal lock in phase with the phase-shifted reference to give maximum DC output voltage. If we measure on an oscilloscope how much we have to shift the phase of the reference signal to get maximum DC output, that gives the phase difference between effect and cause.

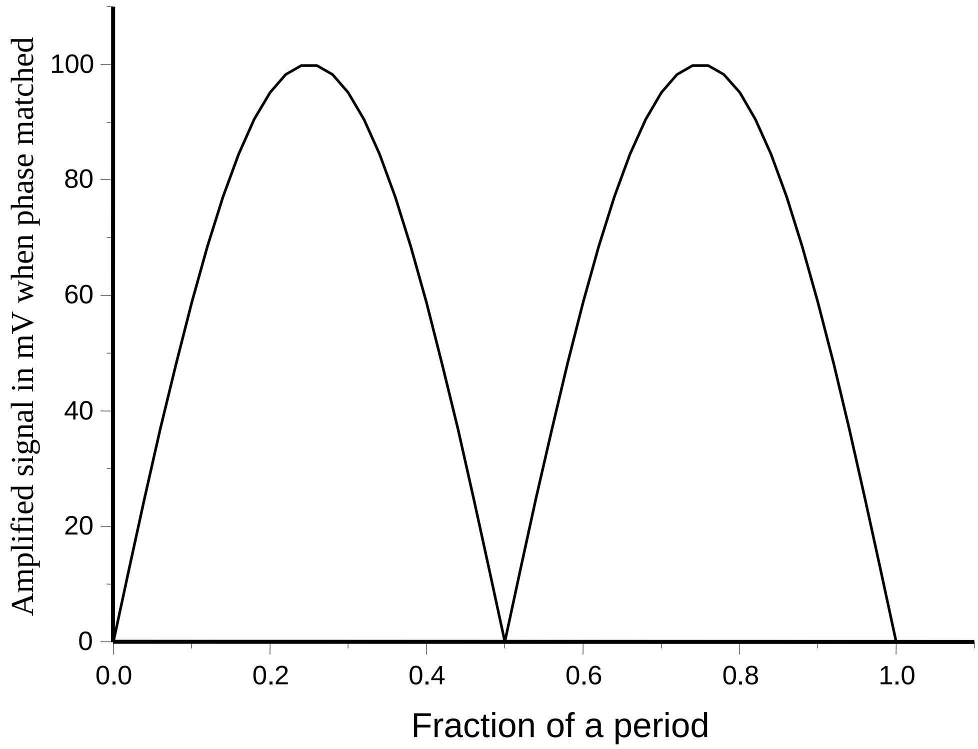


Figure II.9.1.4: Amplified output signal when the reference signal is phase shifted by ϕ and fed to AD 630 chip

Suppose we have noise at a frequency ω' different from the frequency ω of the reference signal. We have a large integration time (several times the period of the reference signal). Then we can show that the contribution of this noise to V_{DC} drops rapidly as ω' differs from ω. If τ, the integration time is large compared to the period of the reference signal, then only noise frequencies differing from ω by n/τ (where n is a small number) will make a contribution to V_{DC}. This means that the effective bandwidth of the lock-in amplifier is n/τ. If the integration time is 1 second, this implies that the effective bandwidth W is a few Hz. So one can understand how thermal noise and other noises are suppressed by the lock-in amplifier.

II.9.2 CALIBRATION OF THE LOCK-IN AMPLIFIER

1. INTRODUCTION:

In Section I a lock-in amplifier is described. Before using the lock-in amplifier for experiments, one must calibrate the lock-in amplifier. Calibration means measuring the maximum output DC voltage of the lock-in amplifier for a small known AC signal voltage as a function of the frequency of the signal.

2. APPARATUS REQUIRED:

Signal generator, lock-in amplifier, two channel oscilloscope and a DMM to measure DC volts to three decimal places in the 2 V range.

3. PROCEDURE

1) Connect a signal generator (described in Section I) to the RCA socket marked SIG GEN at the bottom of the front panel of the LIA. The current drawn from the Signal generator will be very low. There is no need for a power amplifier.

2) Throw the switches SW1 and SW2 to the position marked CAL. Then the internal calibration circuit is connected to the LIA chip.

3) Set the frequency of the signal generator at 500 HZ. Adjust the amplitude v_{app} of the signal generator at about 4 V.

4) Inside the Lock in amplifier, this voltage will be applied to three resistances in series, namely 220 k, 10 Ohms and 5 k. The voltage across 10 Ohms will be $v_{signal.}$

$$v_{signal} = v_{app} \text{ x } 10/\,(225\text{x}10^3) = 44.4 v_{app}\ \mu\text{V} \qquad \text{(II.9.2.1)}$$

The reference voltage is the voltage across the 5 k resistor, one end of which is grounded.

5) Put the switch SW3 up. Turn the phase shifter potentiometer knob to the right extreme (resistance is zero and phase shift is zero).

6) Connect the RCA sockets marked REF and REF‘ to the two channels of an oscilloscope. There will be two sinusoidal traces at the frequency of the signal generator. These are amplified reference signals, one before phase shifting and the other after phase shifting. Since the phase shift is zero when the phase adjusting pot is at the extreme right, the two traces will be superposed exactly indicating zero phase shift. Note the DC volts on a DMM in the 2 V range connected to the RCA socket marked OUTPUT. This DC voltage will be positive. Turn the phase shifter potentiometer knob anti-clockwise and observe what

happens to the signal coming from REF' relative to the signal coming from REF. You will see that the signal from REF' moves to the right relative to the signal from REF. This indicates a phase shift between the two signals. Note that as you turn the knob, the DC voltage on the multimeter comes down. Turn the knob and adjust to the position when the DC voltage on the multimeter is zero. You will see that the maxima of the REF' signal now occur at the position of zeroes of the REF Signal. This indicates a phase shift of 90^0. You can check this by feeding the two signals to the X and Y plates of the oscilloscope and noting that the Lissajous pattern is a circle on the oscilloscope. (There is distortion of the reference signal after phase shifting. So the circle will be distorted.) If the knob is turned further, the phase shift increases beyond 90^0, the panel meter reading becomes negative. When the knob of the phase shift potentiometer is to the left end the phase shift is nearly (but less than) 180^0 and the panel meter indicates the negative largest value.

In the calibration circuit, the signal and reference voltages are derived from the potential differences across two resistances in series. They are in phase. So the panel meter voltage is largest when the phase shift is zero.

7) Connect the RCA socket marked PIN 13 to the oscilloscope and see how the output of the lock-in chip changes as the phase shifter knob is turned. As the knob is turned the output at PIN 13 develops a larger and larger negative part.

8) Turn the phase adjustment potentiometer knob to the right extreme and wait till the DMM connected to the output socket of the lock in shows a steady DC value. At the right extreme the phase difference between the signal and the reference is zero. Note the reading on the DMM. This is called V_{DC}.

9) Repeat operations 8 for various values of the output of the signal generator from 1.5 V to 4V in steps of 0.5V, keeping the frequency fixed.

10) Plot V_{DC} vs. v_{signal} calculated from v_{app} using equation (II.9.2.1). Find the slope μ. This is the amplification factor at the frequency f (500 Hz).

11) Change the frequency from 500 Hz to 1000, 1500, 2000, 2500, and 3000 Hz. For frequencies at and above 1000 Hz, put the switch S_3 down. At each frequency repeat step (9).

Sample readings are shown in Table II.9.2.1.

Table II.9.2.1
Calibration of the Lock-in-amplifier

f Hz →	500			1000			1500	
Volts	μV	Volts	Volts	μV	Volts	Volts	μV	Volts
VAC	Vsig	VDC	VAC	Vsig	VDC	VAC	Vsig	VDC
1.03	45.8	0.266	1.02	45.3	0.261	1.02	45.3	0.260
1.51	67.1	0.383	1.52	67.6	0.387	1.51	67.1	0.372
2.00	88.9	0.500	1.99	88.4	0.495	2.00	88.9	0.487
2.53	112.4	0.624	2.51	111.6	0.618	2.52	112.0	0.619
3.04	135.1	0.745	3.02	134.2	0.736	3.03	134.7	0.724
f Hz →	2000			2500			3000	
Volts	μV	Volts	Volts	μV	Volts	Volts	μV	Volts
VAC	Vsig	VDC	VAC	Vsig	VDC	VAC	Vsig	VDC
1.02	45.3	0.26	1.03	45.8	0.252	1.03	45.8	0.246
1.51	67.1	0.372	1.53	68.0	0.353	1.53	68.0	0.354
2.01	89.3	0.487	2.01	89.3	0.473	2.01	89.3	0.466
2.52	112.0	0.619	2.53	112.4	0.612	2.53	112.4	0.581
3.05	135.6	0.724	3.03	134.7	0.729	3.03	134.7	0.741

In converting V_{Ac} into V_{signal} we use the conversion factor $10/225x0^3$ = 44.4 μV for one Volt from the signal generator.

Figure II.9.2.1 shows the calibration curve for 500 Hz. From this figure we see that the Lock-in amplifier output in DC Volts is proportional to the signal. The linear fit has a slope of $(5.35 \pm 0.022)x10^3$. Similar fits are done at all the other frequencies and the slopes are collected in Table II.9.2.2.

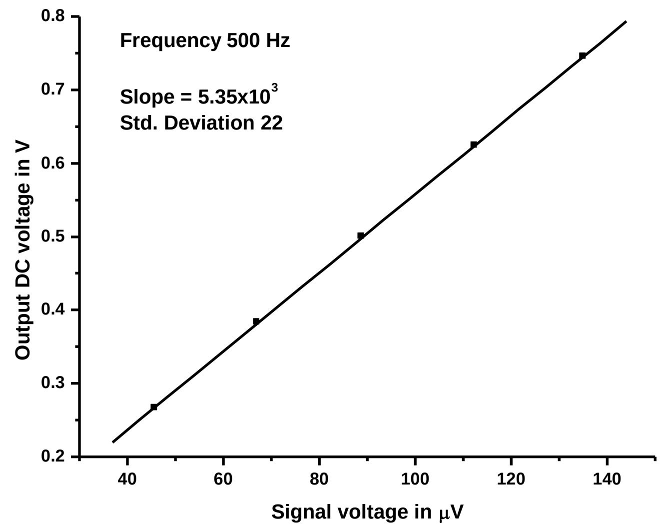


Figure II.9.2.1 V_{DC} in V against v_{signal} in μV
For 500 Hz

Table II.9.2.2
Amplification factor μ at different frequencies

	x10^3	x10^3
f kHz	μ	Devn
0.5	5.35	0.022
1.0	5.32	0.048
1.5	5.26	0.087
2.0	5.21	0.108
2.5	5.46	0.140
3.0	5.48	0.240

We see from the above table that the amplification factor can be taken to be a constant with a value $(5.35 \pm 0.10) \times 10^3$ independent of frequency.

THE AMPLIFICATION FACTOR MAY VARY FROM LOCK-IN AMPLIFIER TO LOCK-IN AMPLIFIER. THE ABOVE READINGS ARE SAMPLE READINGS FOR ONE LOCK IN AMPLIFIER.

II.9.3 MUTUAL INDUCTANCE WITH LOCK- IN-AMPLIFIER

1. INTRODUCTION

When two coils are placed side by side and an AC current is passed through one coil (called the primary), an AC voltage at the same frequency is induced in the other coil (called the secondary). If the primary current varies as

$$I = I_0 \sin(2\pi f\, t) \qquad (II.9.3.1)$$

where f is the frequency in Hertz, the emf induced in the secondary is

$$V = -M\, dI/dt = -2\pi M f I_0 \cos(2\pi f\, t) \qquad (II.9.3.2)$$
$$= -2\pi M f I_0 \sin(2\pi f\, t + \pi/2)$$

So
(1) The phase difference between the primary current and the induced emf is $\pi/2$
(2) The secondary emf is proportional to the amplitude I_0 of theprimary current
(3) The induced emf is proportional to the frequency f.

In this experiment all these factors will be verified using the lock in amplifier.

2. THE MUTUAL INDUCTANCE COIL

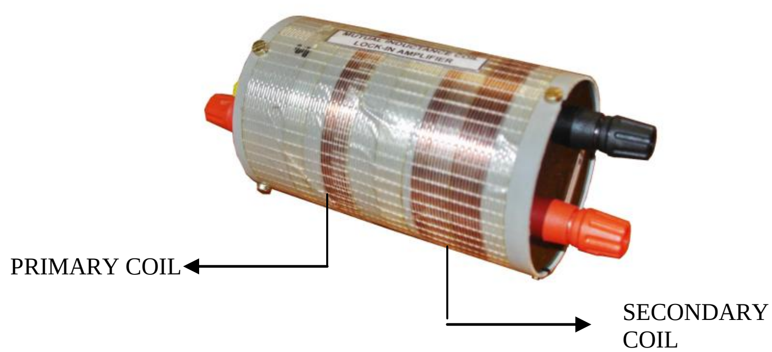


Figure II.9.3.1 Mutual inductance coils

On an insulating former a coil of about 20 turns is wound using insulated copper wire, which can carry a current of about a few milli-ampere. There are three banana terminals red, yellow and black at primary end of the coil. A 4.7 k resistor and the primary coil are connected between red and yellow banana terminals. Between the yellow and black a hundred ohm resistor is connected. When a signal generator is connected between the red and black terminals and a rms AC voltage of V_{RMS} volts is applied, a rms current of

$$I_{RMS} = V_{RMS}/R_{Total} = V_{RMS}/4.8.x10^3 \text{ amps} \qquad \text{(II.9.3.3)}$$

flows through the primary coil. The resistance of the primary coil, which is of the order of 1 Ohm, is neglected in the denominator compared to 4800 Ohms. The voltage across 100 ohms will be used as the reference signal. The signal generator ground must be connected to the black terminal on the primary side of the coil box and the other terminal of the signal generator must be connected to the red banana terminal on the primary side of the coil box.

On the same former a second coil of about 100 turns is wound at a distance of a few centimeters from the primary coil. The terminals of the secondary coil are brought to two banana terminals on the other end of the insulating former. The emf generated by mutual inductance of the coils will appear at these terminals and will be measured by the Lock-in amplifier.

3. APPARATUS REQUIRED

Signal generator, lock-in amplifier, two-channel oscilloscope, mutual inductance coil, one DMM to measure AC frequency exactly, and one DMM to measure DC output of lock in amplifier to two decimal places in DC 20 V range.

4. PROCEDURE

Since we are measuring an external signal, the switches SW1 and SW2 on the LIA front panel should be set to the positions marked EXT.

The red terminal on the signal generator is connected to the red terminal and the black terminal on the signal generator to the black terminal on the primary coil side of the insulating former. Connect the two terminals (Yellow and Black) on the primary coil side of the insulating former to the RCA socket marked **EXT REF** on the front panel of the LIA.

Connect the red terminal on the secondary side of the insulating former to the red banana terminal marked **EXT SIG** on the LIA. Connect the black terminal on the secondary side of the insulating former to the black banana terminal marked EXT SIG on the front panel of the LIA.

Connect a DMM in the DC 20 V range to the RCA socket marked OUTPUT on the front panel of the LIA. Connect the two RCA sockets marked REF and REF’ on the front panel of the LIA to the two channels of an oscilloscope.

Switch on the signal generator. Set the amplitude on its panel meter to about 4 Volt and the frequency to about 400 Hz. Switch on the Lock-in Amplifier and the oscilloscope. Put the toggle switch SW3 up. You will see two sinusoidal traces at the frequency of 400 Hz on the oscilloscope. When the phase-shift adjust pot on the LIA is to the extreme right the two traces will be in phase. Note that the DMM reading is small.

Now turn the phase adjust pot to the left. The trace of REF' will shift relative to the trace of REF and the DMM reading will increase. The sign of the DMM reading will be negative. The DMM takes some time to reach a steady value. So turn the phase adjusting pot little by little and wait for the DMM to reach a steady reading. The magnitude of the reading on the DMM will increase and reach a maximum at one position of the phase adjusting pot. A further turning of the phase adjusting pot will reduce the magnitude of the reading. Keep the phase adjusting pot at the position when you get the maximum reading on the DMM. On the oscilloscope screen we see that the phase difference between the two traces is 90^0. **To show this more dramatically press the XY button on the oscilloscope. Now one sees the Lissajous figure due to the signals Ref and Ref'**. When the DMM shows the maximum value this figure on the oscilloscope screen is a distorted circle. The reference signal is in phase with the current. A maximum output of the LIA indicates that the phase-shifted reference signal is in phase with the mutual inductance emf. The Lissajous figure shows that the phase-shifted reference is 90^0 out of phase with the reference signal. **SO THE MUTUAL INDUCTANCE EMF IS 90^0 OUT OF PHASE WITH THE PRIMARY CURRENT.**
(NOTE: Due to the distortion of the phase shifted REF' signal the Lissajous Figure may appear more like a square with rounded corners.)

Note the DMM reading. Keeping the frequency of the signal generator the same, change the amplitude from 4 to 1 V in steps of 0.5 V. There is no need to adjust the phase shift pot for each amplitude at a given frequency. Once the phase shift is adjusted to get a circle as the Lissajous figure on the oscilloscope screen, changing the amplitude only changes the radius of the circle but does not change its shape.

Having taken the readings of the DMM for different signal generator amplitudes at 400 Hz, repeat the experiment at different frequencies in steps of 200 Hz. When the frequency is 1 kHz or more, switch SW3 will have to be pressed to the down position.When the frequency is changed the phase-shift pot needs to be adjusted to get the maximum magnitude for the DMM reading. At this maximum magnitude the Lissajous figure will be a distorted circle.

A sample set of readings is given below in Table II.9.3.1. In this table, we give the values of the signal generator output voltage V_{AC} and the DC output voltage V_{DC} in volts of the Lock-in amplifier.

Table II.9.3.1

f Hz→	400		600		800		1000	
	V	V	V	V	V	V	V	V
	VAC	VDC	VAC	VDC	VAC	VDC	VAC	VDC
	2.02	0.40	2	0.60	2	0.81	2.00	1.00
	2.5	0.50	2.51	0.75	2.51	1.00	2.50	1.27
	3.01	0.61	3	0.91	3.01	1.23	3.01	1.52
	3.51	0.72	3.51	1.0	3.53	1.44	3.51	1.80
	4.01	0.84	4	1.23	4.03	1.70	4.00	2.10
f Hz→	1200		1400		1600			
	V	V	V	V	V	V		
	VAC	VDC	VAC	VDC	VAC	VDC		
	2	1.21	1.99	1.42	2	1.63		
	2.5	1.52	2.5	1.76	2.5	2.02		
	3.02	1.83	3	2.14	3	2.48		
	3.5	2.18	3.5	2.50	3.5	2.96		
	4.04	2.55	4.05	2.93	4.01	3.40		

Plot for each frequency V_{DC} against V_{AC}. These plots are shown in Figures II.9.3.2 and II.9.3.3.

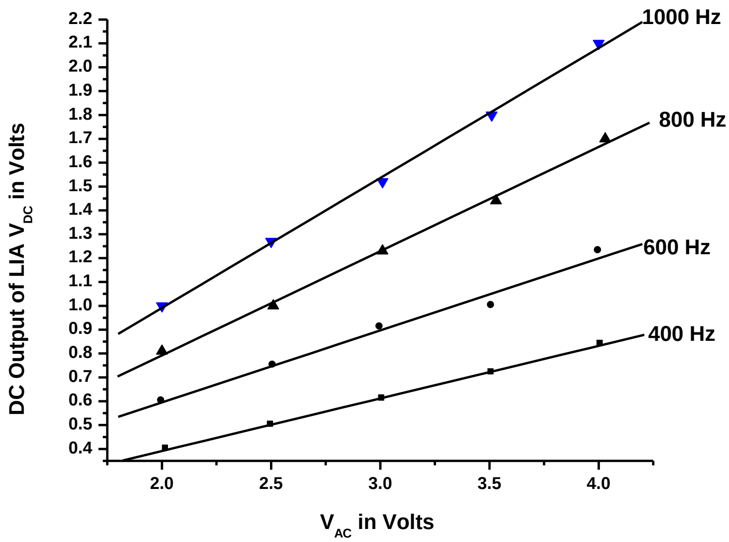


Figure II.9.3.2: DC Output of LIA vs AC voltage applied to primary circuit

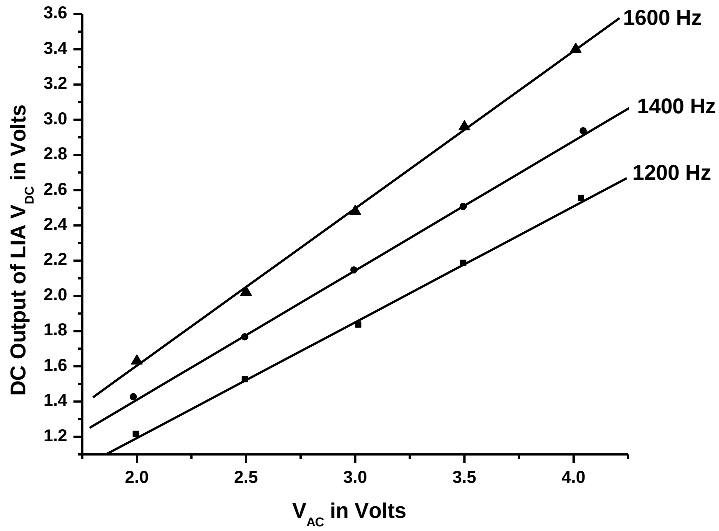


Figure II.9.3.3: DC Output of LIA vs AC voltage applied to primary circuit

The slopes of these curves at different frequencies are given in Table II.9.3.2.

Table II.9.3.2

f Hz	Slope
400	0.221
600	0.302
800	0.437
1000	0.545
1200	0.658
1400	0.735
1600	0.892

The slopes of V_{DC} vs V_{AC} curves in Figures II.9.3.2 and II.9.3.3 are plotted against frequency in Figure II.9.3.4.

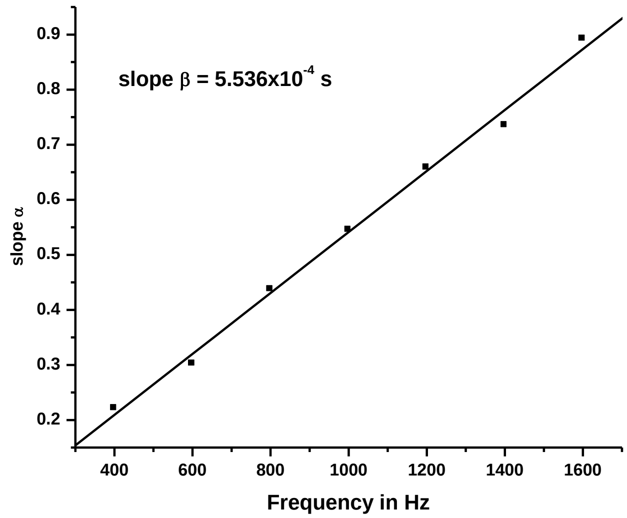


Figure II.9.3.4 Plot of slopes of V_{DC} vs V_{AC} curves in Figures II.9.3.2 and 3 vs frequency

This curve is linear with a slope 5.536×10^{-4} s. The figures II.9.3.2, 3 show that the mutual inductance emf is proportional to the current through the primary coil, and figure II.9.3.3 to the frequency of the AC current.

The slope β of the graph in Figure II.9.3.4 must be equal to

$$\beta = 2\pi M\mu/R \qquad \text{(II.9.3.4)}$$

where M is the mutual inductance of the coil, μ is the amplification factor of the LIA and R is the resistance in the primary circuit (4.8 k). μ for the LIA is 5.35×10^3 from II.8.2. Putting these values in (II.9.3.4) we get

$$M = 79.0\ \mu\text{Henries}$$

You learn from this experiment that:

1. The mutual inductance EMF is 90^0 out of phase with the current in the primary coil
2. The EMF is proportional to the current in the primary coil
3. The EMF is proportional to the frequency.

[NOTE: The mutual inductance will vary from coil to coil. Since it depends inversely on the third power of the distance between the primary and secondary coils, the variation will be appreciable. The above readings are sample readings only.]

Questions:

1. If the number of turns in the secondary coil is increased, will the mutual inductance increase or decrease? Give a reason.

2. Keeping the number of turns constant in the primary coil, I increase the length of the primary coil. Will the mutual inductance increase, decrease or remain a constant? Give a reason for your answer.

3. If the distance between the primary and secondary coils is increased will the mutual inductance increase or decrease? Give a reason.

4. If the frequency of the signal is decreased what will happen to the induced EMF in the secondary for the same current in the primary?

5. To get a DC signal voltage of 1 Volt what should be the current through the primary coil in the mutual inductance set up at a frequency of 500 Hz?

II.9.4 MEASUREMENT OF LOW RESISTANCE

1. INTRODUCTION:

Measurement of low resistance (resistance less than an Ohm) with the DC technique would require a high current. Also since the voltage developed across the resistance will be small, it will be affected by noise when it is amplified.

The AC technique using a lock-in-amplifier provides a solution to the problem. This is illustrated by the following experiment.

2. LOW RESISTANCE BOX

Figure II.9.4.1 Low Resistance Box

In this box a resistance of 4.7 k (1/4 W) is connected with the low resistance and a 100-Ohm (1/4 W) resistor. The free end of the 4.7 k resistor is connected to the red banana terminal, the free end of the low resistance to the middle terminal (Green). A 100 Ohm resistor is connected to the middle terminal (Green) and the end terminal (in the above box this terminal is yellow). Potential leads across the low resistance come to the RCA socket marked signal on the left top of the box. The voltage across the 100-Ohm resistor serves as the reference signal. This reference signal can be taken between the middle and end banana terminals on the box or from the RCA socket marked reference at the right top of the box.

The low resistance is made of insulated thin copper wire (SW 42). One free end of the wire is soldered to the 4.7 k resistor and the other free end to the 100-Ohm resistor. Two potential leads are soldered on the copper wire between the current leads. The free ends of the potential leads are soldered to the terminals of the RCA socket marked Signal.

The 4.7 k resistor limits the current through the wire to a few hundred microamperes when the signal generator output is connected to the terminals of the box.

3. APPARATUS REQUIRED

Signal generator, lock-in amplifier, two-channel oscilloscope, low resistance box, one DMM to measure AC frequency and one DMM to measure output of lock in amplifier to two decimal places in the DC 20 V range.

4. PROCEDURE:

The output terminals of the signal generator are connected to the two end banana terminals on the box. The reference signal, taken from the RCA socket marked reference is connected to the RCA socket marked EXT REF on the front panel of the Lock-in amplifier. The RCA socket marked SIGNAL on the box is connected to the banana terminals marked EXT SIG on the Lock-in amplifier front panel. Connect the socket marked OUTPUT on the front panel of the LIA to a DMM in the DC 20 V range. Put the switches SW1 and SW2 on the LIA to the positions marked EXT and the switch SW3 up.

Switch on the signal generator. Adjust its frequency to be about 200 Hz and its amplitude about 4 V (V_{AC}). Switch on the lock-in amplifier. A DC voltage will appear on the DMM which will reach a positive maximum value as the phase adjust pot is turned to the right extreme position. Note the value of this maximum DC voltage V_{DC}. Change the signal generator output V_{AC} in steps of 0.5 V down to 2 V and note the voltage V_{DC}. Repeat the experiments at frequencies, 400, 600, 800 Hz.

At each frequency plot a graph between V_{DC} and V_{AC}. Fit the points to a straight line and get the slope dV_{DC}/dV_{AC}.

$$dV_{AC} = R\, dI_{AC} \qquad \text{(II.9.4.1)}$$

R is the total resistance in the primary circuit (4.8 k) and dI_{AC} is the change in the current through the low resistance when the signal generator voltage is changed by dV_{AC}. The output DC signal V_{DC} is proportional to the voltage V_r across the low resistance r. When the current through the low resistance is changed by $dI_{AC,}$ the voltage V_r changes by dV_r where

$$dV_r = r\, dI_{AC} \qquad \text{(II.9.4.2)}$$

and this causes a change in V_{DC} by dV_{DC} given by

$$dV_{DC} = \mu dV_r = \mu r\, dI_{AC} = (\mu\, r/R)\, dV_{AC} \qquad \text{(II.9.4.3)}$$

So $$dV_{DC}/ dV_{AC} = (\mu\ r/R) \tag{II.9.4.4}$$

To ensure that the contribution to V_r from any residual inductance of the low resistance is small, we work at low frequencies and we check that the slope dV_{DC}/dV_{AC} is independent of the frequency.

A set of sample readings is shown in Table II.9.4.1.

Table II.9.4.1

Low resistance measurement

Frequency	200 Hz		400 Hz		600 Hz		800 Hz	
	V_{AC} V	V_{DC} V	V_{AC} V	V_{DC} V	V_{AC} V	V_{DC} V	V_{AC} V	V_{DC} V
	1.99	0.34	1.99	0.34	2.01	0.40	2.02	0.32
	2.49	0.42	2.49	0.42	2.50	0.48	2.5	0.40
	2.99	0.5	2.99	0.50	2.99	0.55	3.02	0.48
	3.49	0.58	3.49	0.58	3.49	0.64	3.51	0.56
	4.00	0.67	3.99	0.66	3.98	0.72	4.00	0.65
	4.50	0.75	4.49	0.75	4.50	0.80	4.53	0.74

A plot of V_{DC} against V_{AC} for 400 Hz is shown in Figure II.9.4.3. From the linear fit the slope is found to be (0.1638±0.0013). The slopes for all four frequencies are collected in Table II.9.4.2.

Table II.9.4.2

Freq Hz	Slope
200	0.164
400	0.163
600	0.164
800	0.167

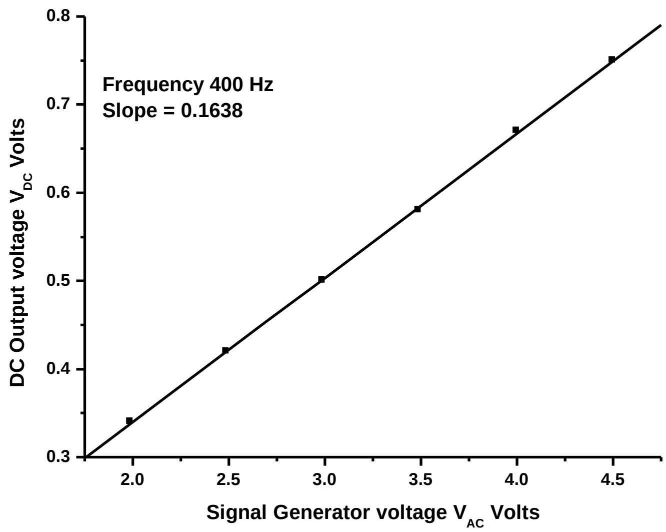


Figure II.9.4.2 Plot of V_{DC} vs. V_{AC} at 400 Hz
The slope of the graph is marked.

We see that the slope is nearly constant around 0.165 ± 0.002. From this average slope we calculate the low resistance r from the formula II.9.4.4. In that formula $\mu = 5.35x10^3$ for the lock in amplifier used, R = 4.8 k. Substituting these values in the equation we find

$$r = 0.147 \pm 0.002 \text{ Ohms.}$$

The value of the small resistance will vary from box to box.

Questions:

1. If you do not use voltage leads but measure the voltage at the points where the two ends of the low resistance are soldered to 4.7 k and 100 Ohms, will you get the correct value of the low resistance? Justify your answer,

2. What is the heat dissipated in the low resistance when V_{AC} is 2 V?

3. If the residual inductance of the low resistance is 1 micro-Henry, what will be the contribution of the inductance to the voltage V_r at a frequency of 10 kHz?

II.10.0 EXPERIMENTS IN NON-LINEAR DYNAMICS

II.10.1 DYNAMICS OF NON-LINEAR SYSTEMS

FEIGENBAUM CIRCUIT

1. INTRODUCTION:

The Feigenbaum circuit is described in the following paper:

"Some Experiments in Chaotic Dynamics" by Keith Briggs, American Journal of Physics, **55**, 1084-1089, (1987).

The circuit has been adopted from this paper with some modifications.

Consider the population of a species whose growth with time from year to year is represented by the equation

$$X_{n+1} = rX_n - sX_n^2 \qquad \text{(II.10.1.1)}$$

Here X_n is the population in the year n, r is its rate of growth and s is the rate of decay due to the sharing of the scarce resources among the population. Given X_1, the population in year 1, r and s, this equation will predict how the population will vary from year to year.

We may use a normalized population by substituting

$$Y_n = (s/r)\, X_n \qquad \text{(II.10.1.2)}$$

Equation (II.10.1.1) becomes

$$Y_{n+1} = r\, Y_n(1-Y_n) \qquad \text{(II.10.1.3)}$$

The condition on Y_n is that it should be positive. So Y_n must always be a positive number between 0 and 1.

Let us see how this equation behaves for $Y_1 = 0.2$ for different values of r. First we choose r = 1.22 and calculate on a computer the values of Y_n for n up to 72. It is convenient to plot the result in a polar diagram taking 1 year equal to an angle of 30^0. We show such a plot in Figure II.10.1.1. We see that as n increases the population moves along a tighter and tighter spiral approaching the limiting value $Y_\infty = 1 - 1/r$, which is 0.18180330551464600000. This limiting value is called the attractor.

Since the equation is deterministic, one would expect a single limit point or attractor for all values of r. Let us do the calculation for r = 3.0. The calculation shows that for odd years we get one limiting value and for even years we get a different limiting value.

We plot in Figures II.10.1.2 (a) the evolution of population for the odd years starting from n=3 to n = 71. This is a spiral like the one shown in Figure II.10.1.1.

In Figure II.10.1.2 (b) we plot ($1-Y_n$) for even years from 4 to 72. (Note: If we plot Y_n it will spiral out to reach a constant radius as n tends to infinity. By plotting $1-Y_n$ against n we get an inward spiral which tends to a point as n tends to infinity. This illustrates clearly the idea of an attractor.)

From these figures we see that the one attractor for r =1.22 has **bifurcated** to two attractors at r = 3.

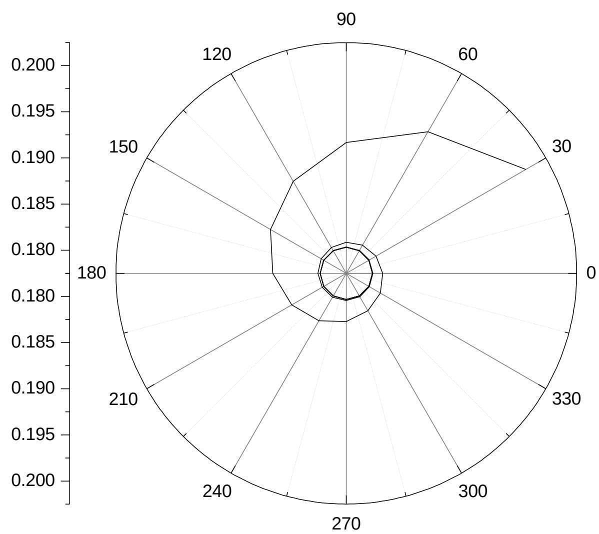


Figure II.10.1.1 Variation of reduced population from 1 to 72 years.
An increase of 30^0 corresponds to an increase of 1 year.

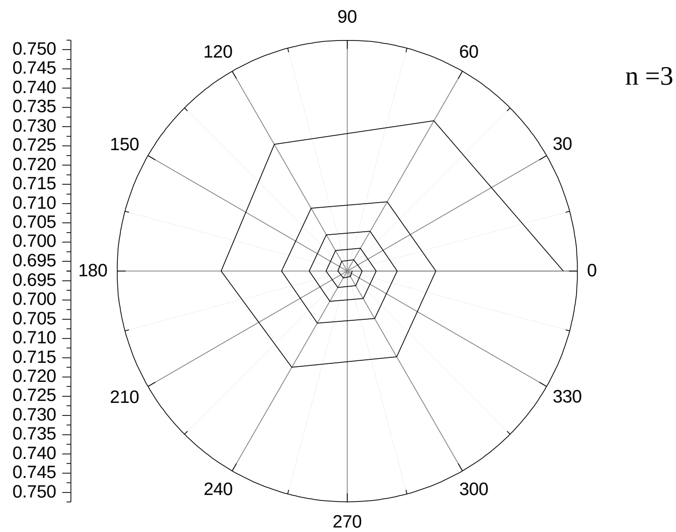


Figure II.10.1.2 (a) Variation of Y_n in the odd years from 3 to 71

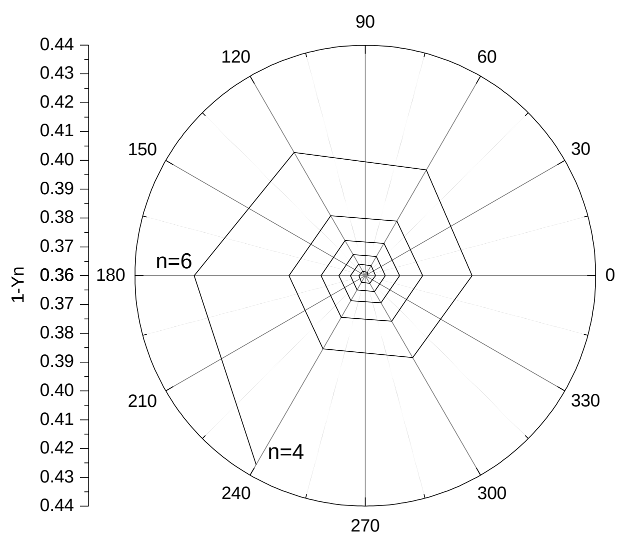


Figure II.10.1.2 (b) Variation of $1\text{-}Y_n$ in the even years from 4 to 72

We see that each attractor occurs once in two years. We call this **period doubling.**

As r is increased further the two attractors will bifurcate successively into 4, 8, 16,… attractors. Figure II.10.1.3 shows the plot of Y_n as a function of n for r = 4 for 72 years. We see that Y_n takes values between 0 and 1 randomly. If we had calculated for n tending to infinity we would have had values for Y_n distributed between 0 and 1 and filling the unit circle. We say we have reached chaos.

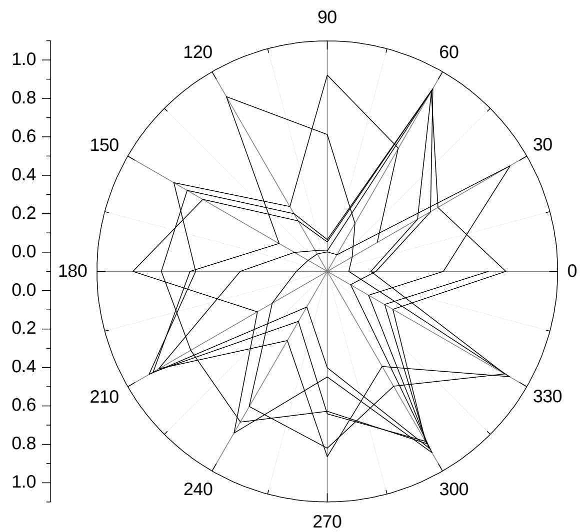


Figure II.10.1.3: Plot of Y_n as a function of n from 1 to 72 years for r = 4.

In all the above figures, the starting value Y_1 is 0.2. We see that the deterministic behaviour for r = 1.22 goes to chaotic behaviour for r = 4 through a series of bifurcations of the attractors at critical values of r. This is the one of the routes taken by a deterministic non-linear system when it goes from ordered to chaotic behaviour.

The following table gives the value of $Y_{n\to\infty}$ for different values of r as calculated on a computer up to the first three bifurcations. When r is 3.84 we see that there are only three values of Y_∞.

TABLE II.10.1.1.
Computer calculations for $Y_{n\to\infty}$
for various values of r

r	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y8
2	0.5							
2.2	0.5455							
2.4	0.5833							
2.6	0.6154							
2.8	0.6429							
3	0.6738	0.6593						
3.1	0.7646	0.558						
3.2	0.7995	0.513						
3.3	0.8236	0.4794						
3.4	0.8422	0.4519						
3.44	0.8485	0.4422	0.8485	0.4422				
3.46	0.8613	0.4132	0.839	0.4675				
3.48	0.8694	0.3951	0.8317	0.4872				
3.5	0.875	0.3828	0.8269	0.5009				
3.52	0.8795	0.3731	0.8233	0.512				
3.54	0.8833	0.3648	0.8203	0.5218				
3.56	0.8899	0.3488	0.8086	0.5508	0.8808	0.3738	0.8333	0.4945
3.84	0.9594	0.488	0.1494					

The bifurcation diagram up to the first three bifurcations is shown in Figure II.10.1.4.

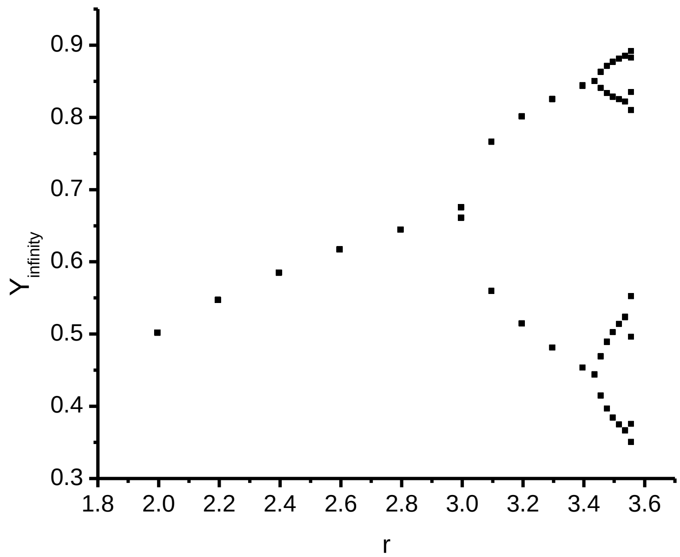


Figure II.10.1.4: Bifurcation diagram for the equation $Y_{n+1} = rY_n(1-Y_n)$

To illustrate these points the Feigenbaum circuit is used...

2. FEIGENBAUM CIRCUIT

A block diagram of this circuit is shown in Figure II.10.1.5. From a small positive voltage X_1 between 0 and 10 V, and a voltage regulator giving +10 V, a signal of magnitude $(10-X_1)$ volts is generated using an inverting amplifier and a summing amplifier. The voltage X_1 and the voltage $(10-X_1)$ are fed to a multiplier M_1. The output of the multiplier is $Z_1 = X_1(10-X_1)/10$. A variable voltage from 0 to 5 V is generated manually using a potentiometer. This voltage is the value of r. This voltage r and Z_1 are fed to a second multiplier M_2. The output of this multiplier is $Y_2 = rZ_1/10 = rY_1(1-Y_1)$ where $Y_1 = X_1/10$. This is the equation (II.10.1.3). The output of multiplier 2 is fed to a sample and hold chip S_1. The value is held in the chip for a short time. It is then passed on to another sample and hold chip S_2 and S_1 receives a new value from M_2. S_2 will hold the value for a short time, pass it to the direct amplifier and receive a new value from S_1. The direct amplifier will amplify the value of Y_2 by a factor of 10 before feeding it into the input X. The sample rate and hold time is determined by a 555 timer.

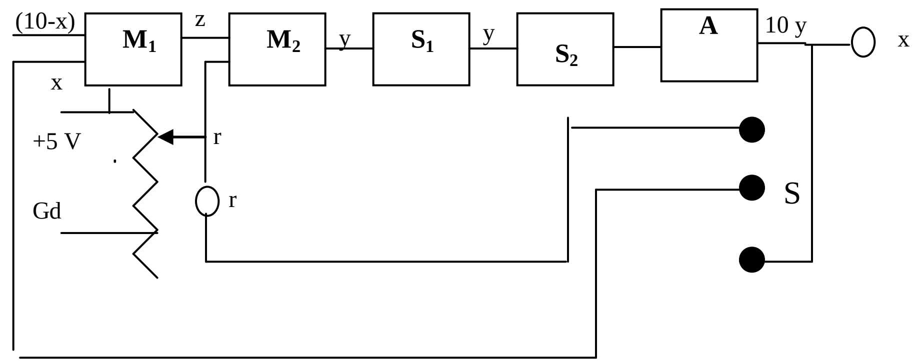


Figure II.10.1.5: Block diagram of the Feigenbaum Circuit

This process of iteration is carried out at 250 Hz. The value of X which we measure at the terminal to the right of the amplifier gives $10Y_n$ as the number of iterations becomes very large. This is the value of the attractor for a given value of r.

We connect the terminal marked X to an oscilloscope. For each value of r, the oscilloscope will give a horizontal line with the value of $X_n = 10\ Y_n$ as n tends to infinity. For a given value of r, the oscilloscope will show one horizontal line if there is one value of the attractor. As r is increased this line will become double at a critical value of r showing that there are two attractors and a bifurcation has taken place. As r is further increased each of these lines will become double. When r reaches a value 4, the whole screen of the oscilloscope will be filled with dots showing that the limiting value of X_n is chaotic.

3. EXPERIMENTAL PROCEDURE:

The front panel of the Feigenbaum Circuit Box is shown in Figure II.10.1.6. At the lower left corner is the mains switch. At the top is a Toggle switch S. When it is up, the starting value of X is the value r set by the POT. This value of r can be measured by connecting a DMM to the RCA socket marked r. Iteration will start when the switch is put down and proceed 250 times in one second. The limiting value of X after much iteration can be measured by connecting the RCA socket marked X to one channel of an oscilloscope. The Inputs 1 and 2 to the multiplier 1 can be measured at the banana terminals marked Input 1 and Input 2 under the tag Mult 1. The output of the multiplier 1 can be measured at the banana terminal marked Output 1. Banana terminals are provided to measure the inputs and output of multiplier 2. The banana terminal marked Com provides the common ground terminal.

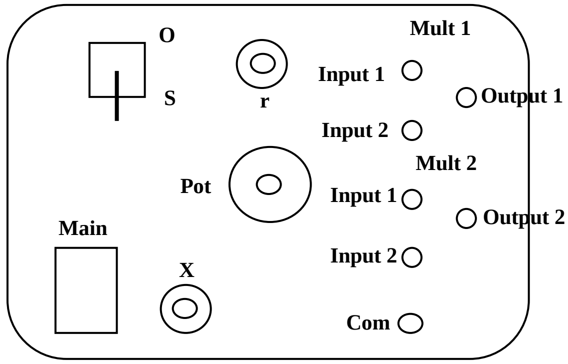


Figure II.10.1.6 Front Panel of Feigenbaum Circuit Box

The following procedure is with a GW Instek storage oscilloscope. With other storage oscilloscope one should study controls of the oscilloscope and use them appropriately.

Connect the RCA socket marked X to channel 1 of the two channel storage GW INSTEK oscilloscope. Connect the RCA socket marked r to a DMM in the DC 20 V range. Set the oscilloscope to roll and the scale on channel 1 to 2 V.

Adjust the pot so that the value of r is 2 V as read on the DMM. Put SPDT switch S UP so that the starting value of x is r. The output at socket X will be r^2 (10-r)/10 at the end of the first iteration. On the oscilloscope screen you will see a horizontal line. Put the switch S down to start the iteration. The horizontal line will shift up. We press the button marked Cursor on the oscilloscope front panel. Two yellow lines will appear on the screen. If the lines are vertical press button X→Y on the right side of the screen. The lines will become horizontal. Press the select button Y_1.on the side of the screen. Then turn knob marked variable and adjust the Y_1 cursor in coincidence with the trace of X on the screen. Screen indicates Y_1 value. Note this value of X Increase r in steps of 0.2 and note the value of X on the oscilloscope for each value of r. Near r =3 the trace on the oscilloscope becomes double indicating that bifurcation has started. Note the value r_1 of r at which first bifurcation occurs. Now change r in steps of 0.1 and measure the two values of X by moving the cursor Y_1 to the two traces of X successively. Near r = 3.43 each of the lines on the screen will bifurcate. Note the value r_2 of r at which this happens. Change r in steps of 0.02 and for each value of r note the four values of X. When r is around 4.53 one of the traces will show bifurcation. Note the value r_3 of r at which this third bifurcation occurs. One will not be able to measure the values of X as r is changed because bifurcations will occur rapidly and the differences between the values of X will be masked by noise. Change r to 3.84. Then you will see only three traces on the screen. Note the three values of X. As r approaches 4, you see dots on the screen occupying all values between X = 0 to X = 10 V. This indicates chaos.

The bifurcation diagram constructed from the measured values of X as a function of r up to the third bifurcation is shown in Figure II.10.1.7.

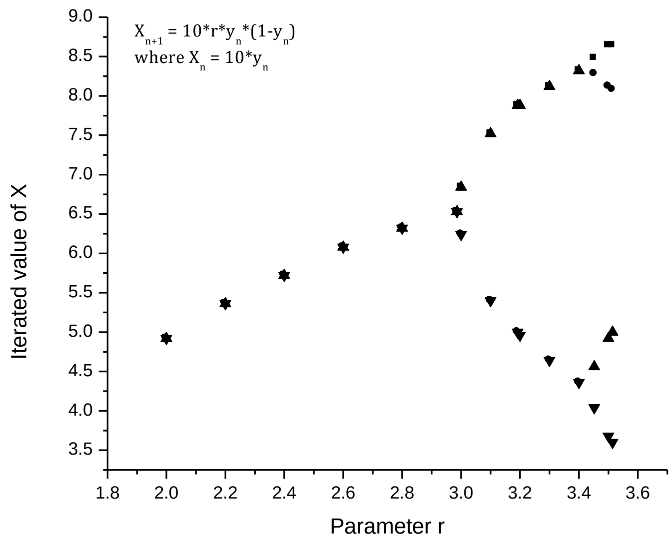


Figure II.10.1.7: Bifurcation diagram measured with the Feigenbaum circuit.

The values r_1, r_2 and r_3 at which the first, second and third bifurcations occur are 2.988, 3.434 and 3.526. The ratio $(r_2 - r_1)/(r_3 - r_2)$ comes out to be 4.80 in agreement with the theoretical value of 4.703.

II.10.2 CHUA's CIRCUIT FOR NON-LINEAR DYNAMICS

1. INTRODUCTION:

The Chua's Circuit is shown in Figure II.10.2.1. This circuit is taken with minor modifications from **"Robust Op-amp realization of Chua's circuit: Michael Peter Kennedy, http://www.physics.smu.edu/scalise/chaoscircuit.pdf".**

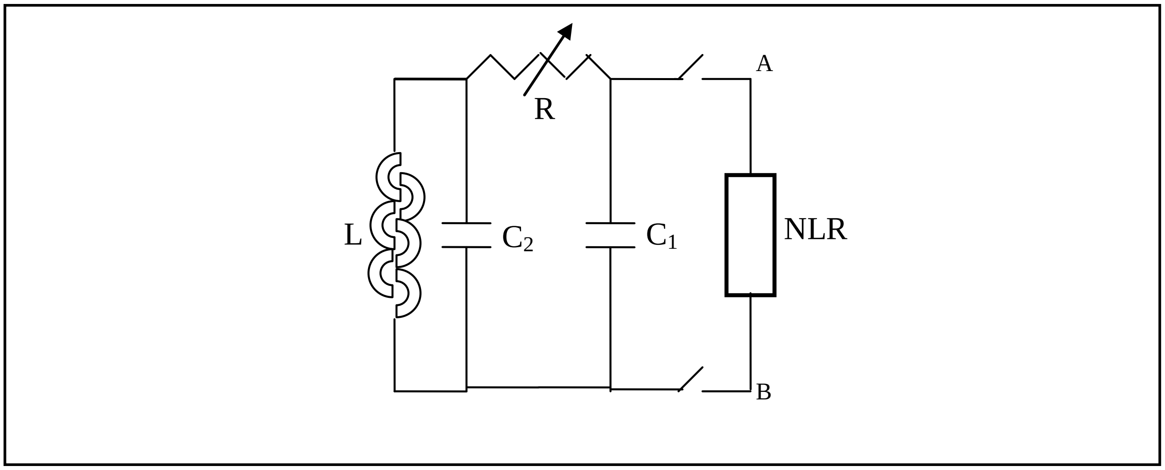


FIGURE II.10.2.1: Chua's Circuit

The circuit consists of four linear circuit elements L, C_1, C_2, and R and a non-linear negative resistor NLR. The non-linear resistor has the following current – voltage characteristic:

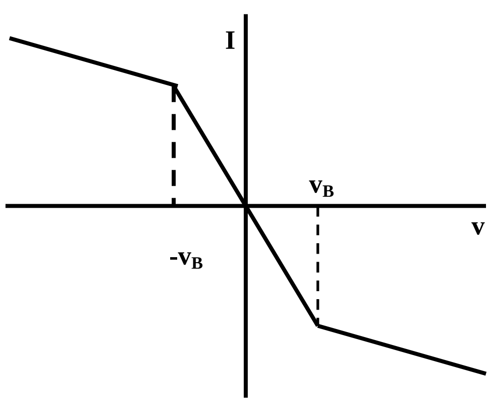


Figure II.10.2.2: I-v characteristic of the non-linear resistor NLR

Note that the resistor has a negative resistance characteristic in the voltage range indicated.

For $v < -v_B$ the current through NLR as a function of the voltage is given by the relation

$$i_{NLR} = -m_0(v+v_B) + m_1 v_B \qquad (\text{II}.10.2.1)$$

For $-v_B<v<v_B$ $\qquad i_{NLR} = -m_1 v \qquad$ (II.10.2.2)

For $v>v_B$ $\qquad i_{NLR} = -m_0 (v-v_B)-m_1 v_B \qquad$ (II.10.2.3)

m_0 and m_1 are positive quantities with $m_1 > m_0$ and they have the dimensions of electrical conductance (Siemens).

There are three potential drops, namely, v_{C2} across the capacitance C_2, v_R (= v_{C2}-v_{C1}) across the resistance and voltage drop across the non-linear resistance v_{NLR} (= v_{C1}). The current through the inductance is i_L and the current through the non-linear negative resistance is i_{NLNR}. These quantities are related by the following three differential equations;

$$-L\,di_L/dt = v_{C2} \qquad (\text{II}.10.2.4)$$

$$C_2\, dv_{C2}/dt = i_L - (v_{C2}-v_{C1})/R \qquad (\text{II}.10.2.5)$$

$$C_1\, dv_{C1}/dt = (v_{C2}-v_{C1})/R - i_{NLR} \qquad (\text{II}.10.2.6)$$

When R is more than a certain value the system is quiescent (i.e. it is not oscillating) and v_{C1} and v_{C2} are zero. When R is reduced oscillations start and the system shows a variety of non-linear behaviour. The behaviour can be seen by connecting v_{C2} to the Y plates of an oscilloscope and v_{C1} to the X plates of an oscilloscope.

Non-linear dynamical behaviour leading from order to chaos is seen in many non-linear mechanical systems and electrical systems. Chua’s circuit provides a simple experimental demonstration of the different types of behaviour.

2. CONSTRUCTION OF A NEGATIVE RESISTANCE ELEMENT

Ordinary resistors are always positive and are dissipative. Negative resistance devices can be constructed using diodes, transistors etc. A simple way of realizing negative resistance with an operational amplifier is shown in Figure II.10.2.3.

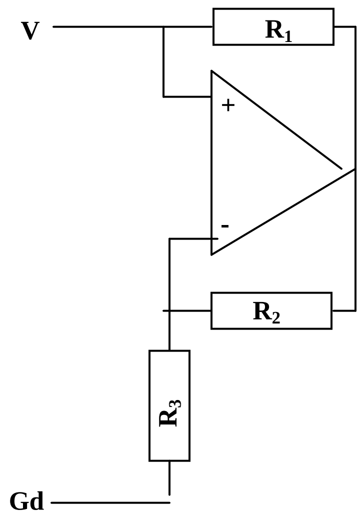


Figure II.10.2.3: Realizing a negative resistance with an Op-amp

For an ideal Op-amp, the current I, when a voltage is applied to the terminals V is

$$I = - (1/R_3)\, V$$

when $R_2 = R_1$ and when V lies between $-V_B$ and $+V_B$. The I-V characteristic is shown in Figure II.10.2.4. $+V_B$ and $-V_B$ are called the breakpoints. V_B is related to the saturation voltage of the Op-amp.

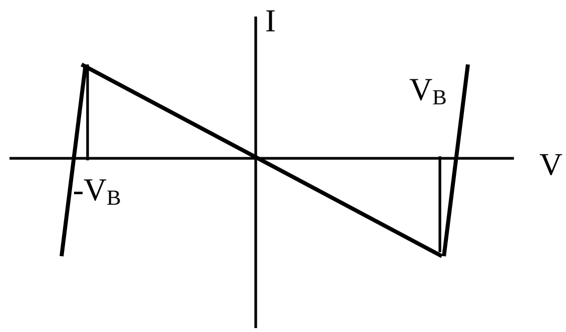


Figure II.10.2.4: I-V characteristic for a negative resistance device using an Op-Amp

Using two Op-Amps in parallel with different V_B values and different R_3 values we can have the non-linear characteristic shown in Figure II.10.2.2 over a range of voltages.

3. FRONT PANEL OF CHUA'S CIRCUIT:

The front panel of the Chua's circuit is shown in Figure II.10.2.5.

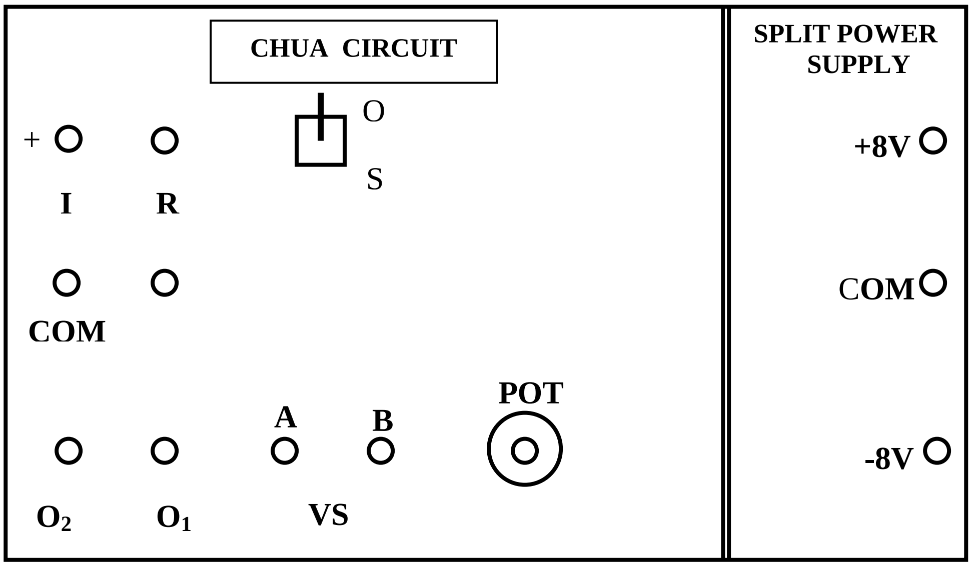


Figure II.10.2.5: Front Panel of Chua's Circuit Box

The right side of the panel is a split power supply giving +8 and -8 V and is not part of Chua's circuit. Chua's circuit is to the left of the double line. The various terminals and controls on the front panel are explained in I.14.

4. APPARATUS REQUIRED

The Chua's circuit box, a regulated power supply, one DMM to measure current in mA, one DMM to measure DC voltage up to 20 V and a two channel oscilloscope.

5. PROCEDURE

a) To measure the I-V characteristic of the non-linear resistor device made of two op-amps:

On the top of the front panel there is a DPDT switch which can be put in position O or S. In Position O it isolates the non-linear resistor part of the circuit in Figure II.10.2.1 from the rest of the circuit. Put switch S in position O. Connect the regulated power supply to the terminals marked VS (A and B) through a DMM to measure up to DC 3000 μA. When positive of the DC supply is connected to A and negative to B, the voltage applied to the non-linear resistive device is +V where V is the voltage of the DC source as read on a second DMM connected to A and B.. If the connections at A and B are reversed the voltage applied is –V volts.

Table II.10.2.1 shows a sample set of readings of I vs. V when the voltage is varied from -6.5 V to +6.5 V.

Table II.10.2.1
I-V characteristic of the non-linear device

VDc	I microamp	VDc	I microamp	VDc	I microamp	VDc	I µamps
-6.5	-1120.5	-2.7	1430	0	0	2.9	-1555.5
-6.3	-671	-2.5	1346.5	0.1	-93	3.1	-1638
-6.1	-258	-2.3	1261	0.2	-166	3.3	-1720.5
-5.9	185.5	-2.1	1175.5	0.3	-244.5	3.5	-1808
-5.7	614.5	-1.9	1091	0.4	-325.5	3.7	-1891.5
-5.5	1026.5	-1.7	1012	0.5	-407	3.9	-1972.5
-5.3	1444	-1.5	927.5	0.6	-479	4.1	-2057.5
-5.1	1829	-1.3	843	0.7	-561.5	4.3	-2143
-4.9	2194	-1.1	759.5	0.8	-636	4.5	-2224.5
-4.7	2265.5	-1	716.5	0.9	-717.5	4.7	-2308.5
-4.5	2182.5	-0.9	672	1	-759	4.9	-2389.5
-4.3	2097	-0.8	616.5	1.1	-801.5	5.1	-2475.5
-4.1	2014	-0.7	543	1.3	-885	5.3	-2556.5
-3.9	1933	-0.6	462	1.5	-967.5	5.5	-2123
-3.7	1845.5	-0.5	386	1.7	-1054.5	5.7	-1682.5
-3.5	1766.5	-0.4	305.5	1.9	-1139	5.9	-1230
-3.3	1679.5	-0.3	228	2.1	-1222.5	6.1	-782.5
-3.1	1597.5	-0.2	149	2.3	-1303.5	6.3	-343
-2.9	1510.5	-0.1	73	2.5	-1388	6.5	113.5

(If a micro-ammeter is not available, one can measure the DC voltage at the terminals marked I. This gives the voltage across a 10 Ohm resistor. The current is voltage divided by 10. If required the voltage across the terminals can be connected to a DC differential amplifier which is put in the amplification range 10.)

Figure II.10.2. 6 shows the I-V characteristic of the non-linear device.

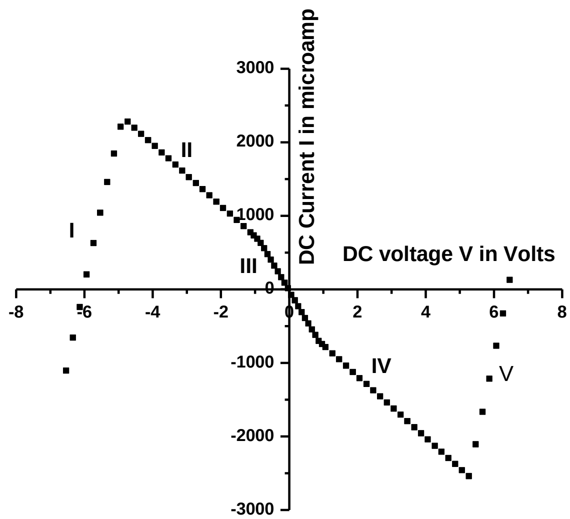


Figure II.10.2.6: I-V characteristic of the non-linear device

There are five regions in the curve marked I to V on the figure. In regions I (V<-5.3 V) and V (V>5.3 V) the device has a positive resistance with a slope of about $+2.2\times10^{-3}$ Siemens. Between -5 and +5 Volts the device exhibits a negative resistance behaviour. This region is divided into three parts II, III and IV. In regions II and IV the slope of the negative resistance curve is -4.18×10^{-4} Siemens (expected value according to circuit parameters is -4.55×10^{-4} Siemens) and in region III the slope is -7.71×10^{-4} Siemens (expected value from circuit parameters is -7.67×10^{-4} Siemens). The region III lies between -1 to +1 Volt.

b) To see the non-linear dynamics of the system as the resistance R is varied:

Remove the power supply connection to VS (and the DDA connection to I). Put the DPDT switch in position S. Now the non-linear resistive device is connected to the rest of the circuit in Figure II.10.2.1. O_2 measures the voltage across C_2 in Figure II.10.2.1 relative to ground (terminal marked COM in Figure II.10.2.6). O_1 measures the voltage across C_1 relative to the ground. Connect O_2 to Channel 1 of a dual channel oscilloscope and O_1 to channel 2 of the oscilloscope. The ground of the oscilloscope is connected to the terminal marked COM on the front panel. The potentiometer knob marked POT on the front panel can be turned right to decrease the resistance from about 1.4 k to 0. The resistance can be measured at the terminals marked R when the power to the Chua's circuit is turned off.

(The oscilloscope used in this work is the 25 MHz storage oscilloscope from GWINSTEK with the probes set at x1)

Put Channels 1 and 2 in the AC coupling mode. Set channels 1 and 2 in the scale 500 mV. Put the oscilloscope in the XY mode. The oscilloscope screen shows a dot at the center when the pot is at the extreme left, (i.e. Resistance of the pot is a maximum). This indicates that the circuit is not oscillating.

Turn the pot to the right slowly. At one point of the pot, a loop appears on the oscilloscope screen. This is shown in Figure II.10.2.7 (a) below. Turn off the power to the Chua circuit and measure the resistance of the pot by connecting a DMM in the 2000 Ohms range to the terminals marked R on the front panel. The resistance is 1.01 kΩ. The loop grows as the resistance is reduced slowly from this value. The appearance of the loop indicates that voltages across C_1 and C_2 are oscillating with a single frequency but with a difference in phase. If we go from the XY to the Main tab on the oscilloscope, the two voltages are seen as a function of time. Two sinusoidal traces appear. The frequency of the sine waves is about 3 kHz.

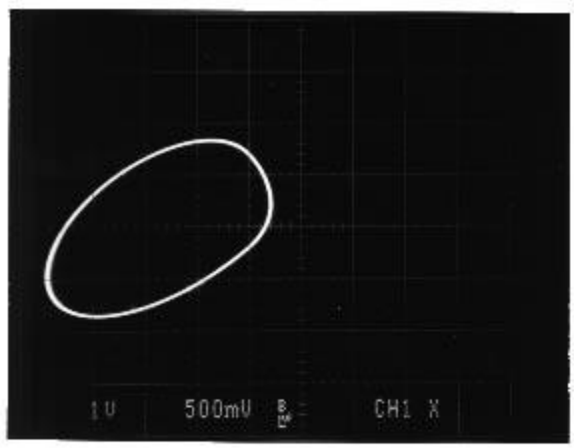


Figure II.10.2.7(a) Appearance on the oscilloscope screen in the XY mode when the resistance of pot is 1.1 k.

As the resistance is decreased the loop becomes double at a resistance of approximately 0.975 k . This is shown in Figure II.10.2.7 (b). This indicates that the system is oscillating at two periods close to each other. The period of oscillation has bifurcated. The loop is repeated every second cycle indicating period doubling. If we go to the depiction of two voltages as a function of time, we see a distortion of the pure sine wave.

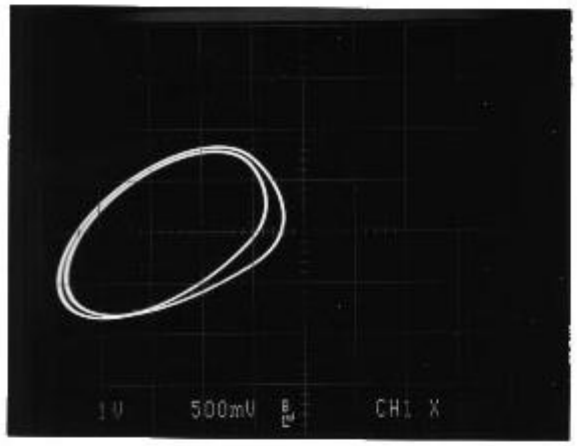


Figure II.10.2.7(b) Appearance of a double loop in the XY mode on the oscilloscope screen when resistance is 0.975 k.

As the resistance is further decreased the separation of the two loops grows and at a resistance of about 0.963 k four loops appear. This is shown in Figure II.10.2.7(c). This indicates further doubling of the two periods. Bifurcation has occurred again. One of the routes to chaos from order in a deterministic system is generally through such a bifurcation process. We now see that the points representing (v_{c1},v_{c2}) wanders over a larger region of the phase space.

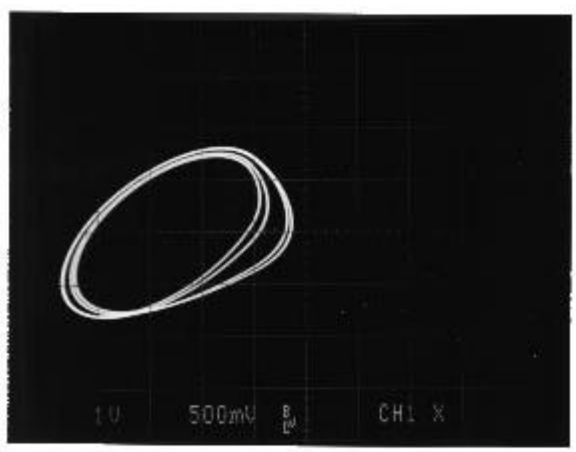


Figure II.10.2.7(c) Appearance of four loops at a resistance of 0.963 k.

As the resistance is reduced further the protuberance extends farther and farther as shown in the figure II.10.2.7(d). This is called a Rossler type attractor.

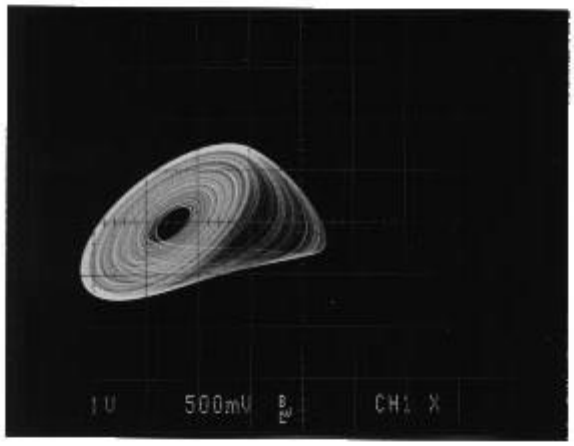


Figure II.10.2.7(d) Rossler type attractor .

With a further decrease in resistance the figure on the oscilloscope changes to the one shown in Figure 7(e).

Figure II.10.7.7(e) Double Scroll Attractor

This is called a double scroll attractor. For part of the time the voltages are fluctuating about one DC point (v_{C1},v_{C2}) and then suddenly switches to fluctuating about another DC point (v_{c1}’,v_{c2}’).

Finally as the resistance is reduced further we get a large figure as shown in II.10.2.7(f):

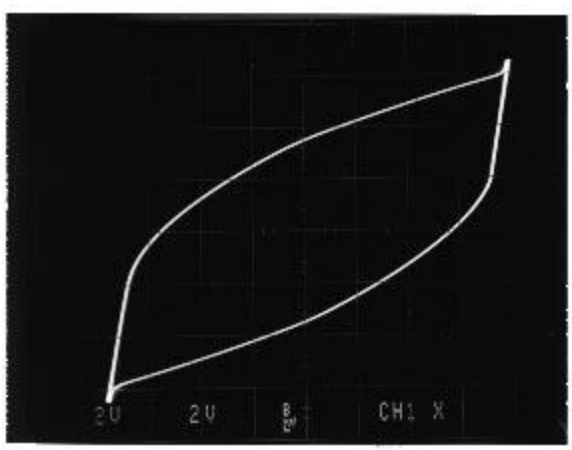


Figure II.10.2.7(f) Large limit cycle

This corresponds to the large limit cycle corresponding to outer segments of the v-I characteristic of the non-linear resistor.

The figures II.10.2. 7(a to f) appearing on the oscilloscope screen can be photographed with a mobile phone with camera attachment.

II.11.0 RESISTOGRAPH

EXPERIMENTS IN PHASE TRANSITIONS

T G Ramesh and V Shubha
Materials Science Division

National Aerospace Laboratories

Bangalore 560017

Council of Scientific & Industrial research

II.11.1 Tracking of the Ferromagnetic –Paramagnetic transition in Nickel through Electrical Resistivity

1. ABSTRACT

A Manual / PC based Resistograph (Electro - Thermal Analyser) has been designed and fabricated for studying the resistivity of metals, alloys and semiconductors as a function of temperature.

2. INTRODUCTION

This system can be used for measuring the resistance of metals, alloys and semiconductors as a function of temperature over the range 25 to 600°C. Figure II.11.1.1 gives the block diagram of the system.

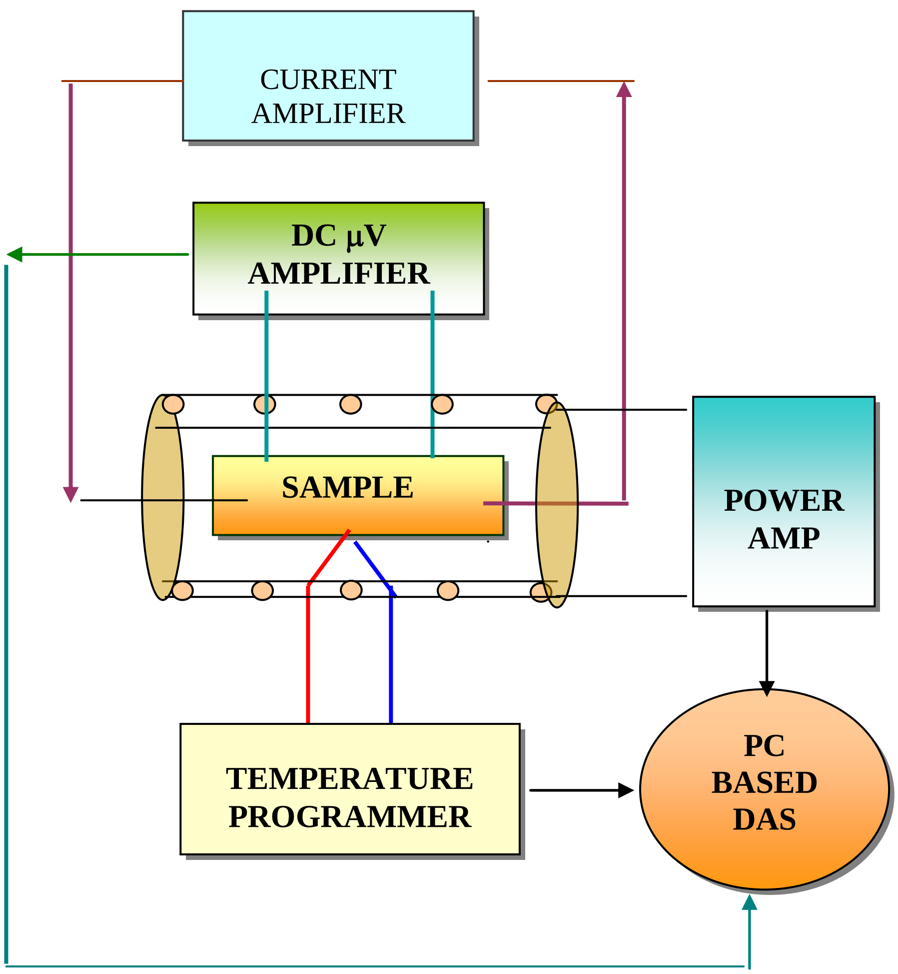


Figure.II.11.1.1 Block diagram of Resistograph

The main sub systems are

1. **Resistograph** unit consisting of

- Constant DC Current Source with high voltage compliance
- Second order Temperature Lineariser for the linearisation of thermo emf versus temperature for Chromel-Alumel thermocouple
- High precision DC μV Amplifier with variable gain

2. **Compact Furnace with sample holder**

3. **Power Amplifier** with 100 watts DC power output used for heating the furnace

4. **PC Data Acquisition Card (16 bit A/D and D/A).**

METHOD

1. RESISTOGRAPH

This system is based on the well-known four-probe method for resistivity measurement along with a thermocouple probe in good thermal contact with the sample for direct temperature measurement.

A DC Constant current source has been designed with a voltage compliance of 25 V. The magnitude of the constant current is selectable from the front panel of the Resistograph unit (1,10 & 50 mA).

A linearising circuit has been designed for the Chromel-Alumel thermocouple. This propriety circuit employs a second-degree polynomial curve fitting technique for correcting the non-linear thermo-emf versus temperature characteristic of Chromel-Alumel thermocouple. The thermo-emf of the Chromel-Alumel thermocouple, which is in good thermal contact with the sample under study, is amplified with a gain of 200 in a high quality front end Instrumentation Amplifier configured around INA 101 (Burr-Brown make). Since a single junction thermocouple is employed, Cold Junction Compensation (CJC) is effected through the usage of an IC temperature transducer like AD 590. The linearised voltage directly corresponds to the temperature (1mV corresponds to 1°C) and read through a 3½ digit DPM with a resolution of 1 mV.

A high stability DC μV Amplifier built around INA 101 is used to measure accurately the voltage developed across the sample. The input stage of this amplifier incorporates a passive differential input low pass filter for improved signal to noise ratio. The gain of the amplifier can be set from the front panel to values 10, 100 or 1000. The amplified voltage from the DC μV Amplifier is read through a 4 ½ digit DPM located in the front panel of the Resistograph.

2. COMPACT FURNACE

A compact furnace with a nominal resistance value of 6 Ω has been designed for reaching temperatures up to 600°C. Kanthal wire is wound round an alumina tube and forms the heating element of the furnace. A screw terminal board is provided at the top of the furnace for easy connection of the leads coming from the sample holder.

Different sample holders can be designed to carry out a variety of measurements in this set up. Although primarily this system was developed for **Resistivity measurements at high temperatures, the same set up can also be used for**

1. Differential Thermal Analysis

2. Study of thermo emf versus temperature of standard and non standard thermocouples

3. Study of energy gap of a thermistor.

A four probe arrangement for measuring the electrical resistivity of low resistance samples like Nickel, alloys like Nitinol etc., with a Chromel-Alumel thermocouple probe for measuring the temperature of the sample has been designed to study temperature induced continuous or first order phase transitions.

3. DC POWER AMPLIFIER

A variable DC Power supply (30V, 4A) has been designed to power the compact furnace. A **ten-turn potentiometer** in the front panel of the Power Amplifier is used for smooth variation of the power input to the furnace. Unlike conventional power supplies, this unit is essentially a power operational amplifier and can be easily interfaced with a PC for automated power control. Terminals have been provided in the front panel for remote operation.

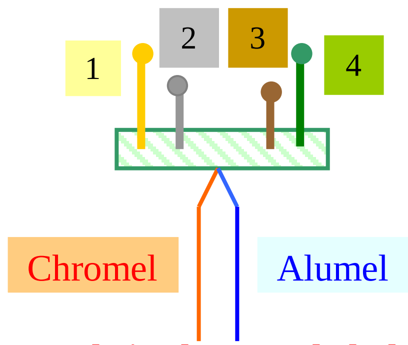
4. PC DAC (16 BIT A/D AND D/A)

This add-on card is a high resolution 16bit A/D converter with a front end 8 channel (differential) or 16 channel (single ended) multiplexer operating at a maximum speed of 40 kHz. The gain of the preamplifier is optimized prior to A/D conversion to enhance the accuracy of measurement. The card also features two 12 bit D/A converters which are useful for close loop control of parameters such as temperature. This card with the menu driven software developed in LabView environment provides the linkage between the PC and the hardware described earlier.

CONNECTION DIAGRAM FOR Ni SAMPLE HOLDER

Experiment 1:

Tracking the Ferromagnetic – Paramagnetic transition in Nickel through Electrical Resistivity.



1	Current High
2	Voltage High
3	Voltage Low
4	Current Low

Colour code in the sample holder:

Current High	:	**Yellow Teflon sleeve**
Current Low	:	**Green Teflon sleeve**
Voltage High	:	**White Teflon sleeve**
Voltage Low	:	**Brown Teflon sleeve**
Chromel	:	**Red Teflon sleeve**
Alumel	:	**Blue Teflon sleeve**

Connection details:

1. Inset the given sample holder in the furnace and connect the leads to the screw terminal boards provided at the top of the furnace.

2. Connect **Chromel and Alumel** to the **TC INPUT** terminals at the back panel of the Resistigraph unit.

Chromel is the $+^{ve}$ element: Connect it to the **RED** terminal.
Alumel is the $-^{ve}$ element: Connect it to the **BLACK** terminal.

3. Connect **Yellow sleeve wire** to the **White terminal** of the **CURRENT O/P** provided at the back panel. Connect **Green sleeve wire** to the **Green terminal** at the back.

4. Connect **White sleeve wire** to the **AMPLIFIER INPUT** terminal (**Blue terminal**) and **Brown sleeve wire** to the **Black terminal.**

5. Set the **CURRENT** value to **10 mA** and the **GAIN** to **10^2**

PC MODE OF OPERATION

1. The thermocouple connection to the Resistograph remains the same as described earlier.
2. The linearised temperature out put **T (O/P)** available at the rear panel is connected to **Channel 0** of the PCI Data Acquisition card.
3. The measured parameter of interest to the user like a voltage signal corresponding to the property of interest like resistivity is connected to **Channel 1**
4. The **D/A out put in channel selector box** should be connected to the **INPUT** terminals at the rear panel of the **DC Power Amplifier** and in the front panel the toggle switch should be kept in **Auto** mode.
5. After switching on the PC click on **nalres.exe** file.
6. The data collected during each experimental run is stored in a file called **datares.xls** which has to be renamed after each experiment.
7. The program can be terminated anytime by using the **stop button**.
8. The software incorporates real time graphic display of resistance versus temperature data with auto scaling along both X and Y axes.

Ferromagnetic-Paramagnetic Transition in Nickel

The experimental data of resistivity of Nickel over the temperature range from 300 to 425°C collected using the Resistograph is given in Figure 2.

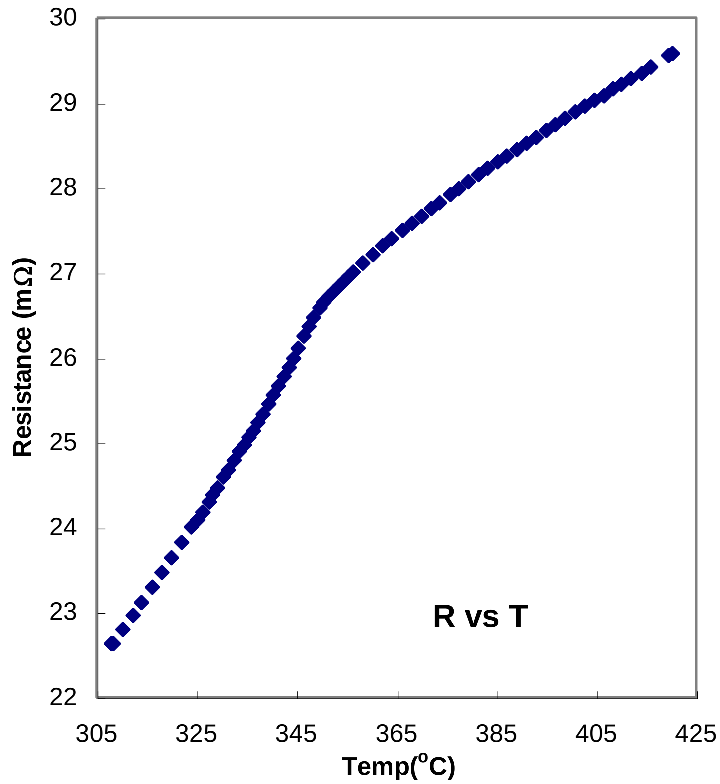


Figure II.11.1.2: Electrical resistivity of Nickel as a function of temperature across the Ferromagnetic-Paramagnetic Phase Transition

The main feature of this plot is the anomalous variation of resistance near 360°C. There is a marked change in the slope around the Curie temperature which is around 357°C for Nickel. This can be seen more clearly in the plot of 1/R dR/dt versus temperature (Figure 3). This curve bears close resemblance to the specific heat anomaly near second order transition (λ-type)

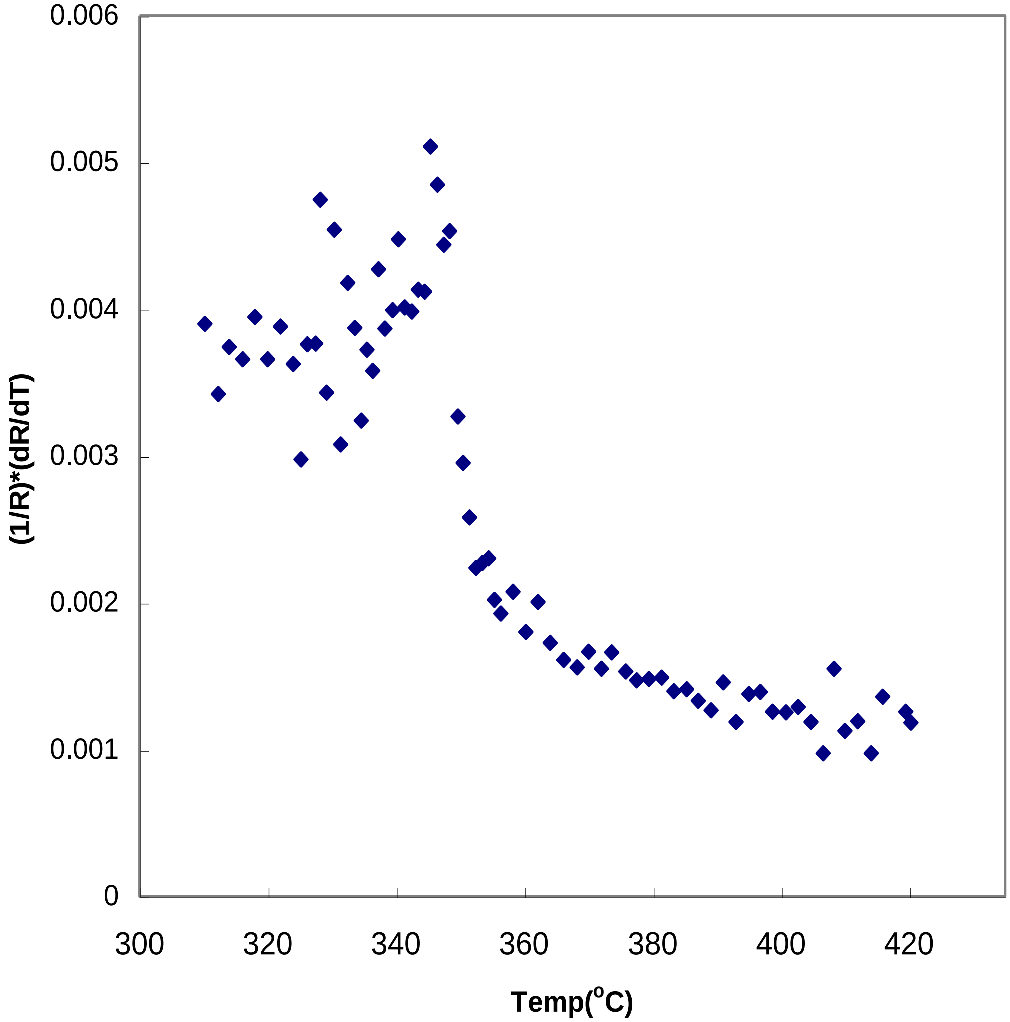


Figure II.11.1.3: Plot of 1/R dR/dt versus temperature evaluated using the data of Figure II.11.1.2.

Energy Band structure and Magnetic transition in Nickel

The high electrical resistivity of transition metals like Nickel as compared to Cu, Ag and Au was first explained by N F Mott. It may be noted that the conduction electrons in metals like Ni, Co, Fe (which belong to the transition metal series) have wave functions derived mainly from s states just as in Cu, Ag and Au and that the effective number of conduction electrons is not much less than in the noble metals. However the mean free path in transition metals is much smaller due to scattering of electrons from s states to vacant states in the d band. The unoccupied d states are responsible for the ferromagnetism in transition metals and there is a direct connection between the magnetic properties and their electrical conductivity.

The band structure of ferromagnetic nickel can be represented rather schematically as in Figure 4 .The spin-up electrons (spin parallel to directin of magnetization) consist of a 4s band accommodating about 0.3 electrons / atom and a partially filled 3d band containing 0.6 holes / atom .This 3d band has a very high density of states at the Fermi energy E_F and this is a typical feature of a transition metal. The spin-down electrons also have an s band containing about 0.3 electrons / atom. However the Fermi level lies above the spin-down 3d band and there are no d holes in this band. The overlap of 3d and 4s bands in Nickel is responsible both for the ferromagnetism and its high electrical resistivity.

The most important feature of the band structure of a transition metal like Ni, from the point of view of electron transport, is that the spin-up electrons have a high density of states at the Fermi energy. In contrast, the spin-down electrons have a low density of states at the Fermi energy. Essentially the current in Ni is carried by high mobility 4s electrons , but they are scattered predominantly into the vacant states of 3d band characterized by a high density of states. This s-d scattering is at the heart of the high resistivity of transition metals in comparison to noble metals.

The variation of the resistivity of Ni as a function of temperature in the range 300 to 425°C is shown in Figure 2. The important feature of this curve is that the resistivity of Ni varies nonlinearly with temperature in the ferromagnetic phase with the temperature coefficient of resistivity increasing with temperature reaching a maximum near the Curie temperature which is around 357°C.The ferromagnetic – paramagnetic phase transition manifests as a marked decrease in the temperature coefficient of resistivity near T_C followed by a linear increase of resistivity with temperature. A qualitative xplanation of the anomalous behavior of resistivity with temperature across the magnetic phase transition can be given based on the band structure diagram in Figure 4. If there is no spin flip during scattering, then for temperatures lower than the Curie temperature we would expect only **half** of

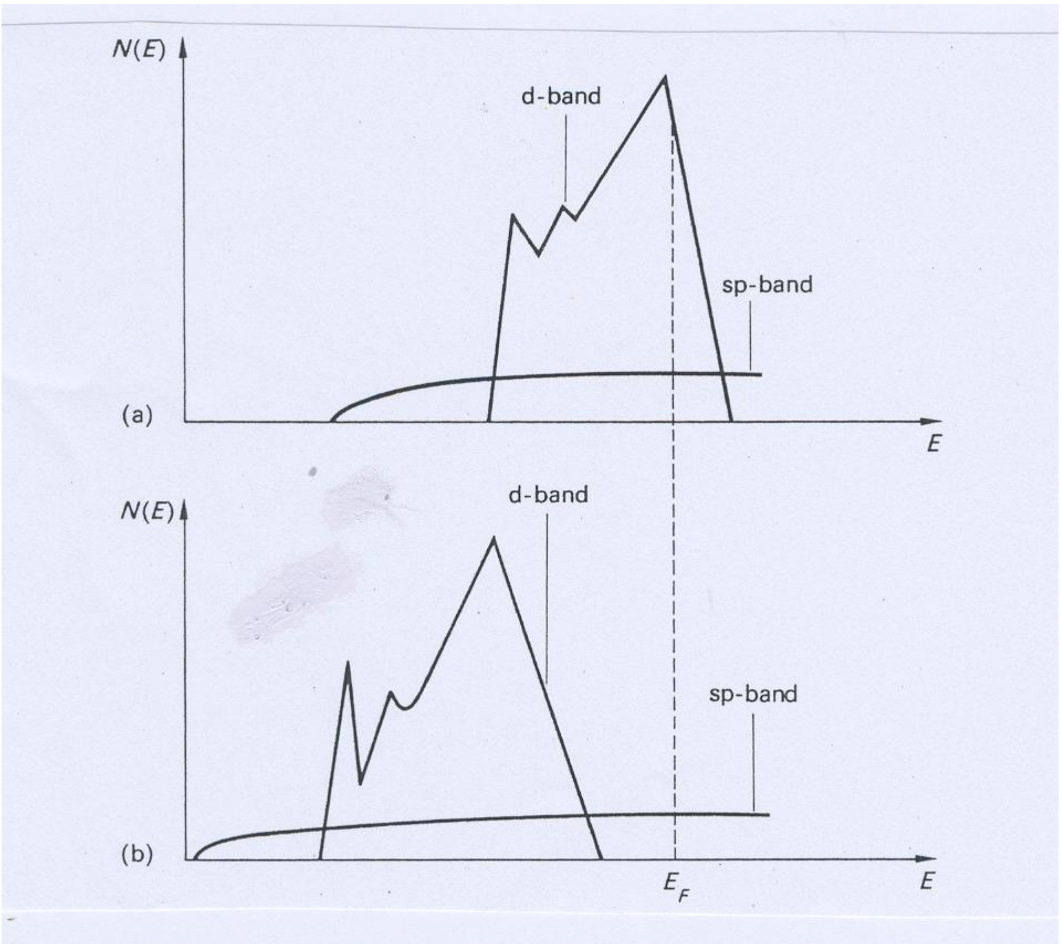


Figure II.11.1.4: Band Structure of Ferromagnetic Nickel (Schematic)
a) Spin - up electrons   b) Spin-down electrons

the conduction electrons to make transitions to the empty states of the 3d band. On the other hand, above T_C , when the spin-split 3d bands coalesce, holes with both spin directions will be present and **all** the conduction electrons can make transitions to the empty states in the 3d band. Thus the destruction of ferromagnetism leads to an increase in the electrical resistivity.

II.11.2 MARTENSITE TO AUSTENITE PHASE TRANSITION IN SHAPE MEMORY ALLOY NITINOL

1. INTRODUCTION:

The Shape Memory Alloy NITINOL (Nickel Titanium Naval Ordnance Laboratory) was discovered in 1959 by William J. Buehler of the U.S. Naval Ordnance Laboratory. This alloy exhibits two remarkable effects viz., ‘shape memory’ wherein the alloy can ‘memorize’ a predetermined shape and return to this shape under certain temperature conditions and ‘pseudo-elasticity’ where large strains of the order of 8-10 % can be recovered in applications. These unique features have made NITINOL a remarkable engineering material with applications in such diverse fields as cardiovascular surgery, orthodontics, solid-state heat engines, aerospace and toy industry.

The serendipitous ie., accidental discovery of ‘memory metal’ took place at the U.S.Naval Ordnance Laboratory when Buehler noticed a remarkable acoustic damping change of a Ni-Ti ingot with temperature change near room temperature. This unusual event unfolded when an assistant of Buehler was transporting several melted Ni-Ti bars from an arc furnace to a table. One of the bars which had cooled to near room temperature accidentally fell on the concrete floor and made a ‘dull thud’ sound while a bar at a higher temperature made a characteristic ‘metallic sound’ on being dropped to the floor. This quick test of Buehler to determine the damping capacity of the Ni-Ti alloy has now come to be known as ‘Pseudo-Elasticity’. Intuitively Buehler reasoned that the startling acoustic damping must be related to a major atomic structural change, related only to minor temperature variation. Further metallurgical studies like micro-hardness and microstructure led Buehler and his group to a significant conclusion that in this alloy major atomic movements occur in a rather low temperature regime near room temperature.

The revelation of the unique ‘Shape Memory Effect’ in NITINOL came a little later. In the early 1960s, Buehler prepared a long thin Nitinol strip for use in demonstrations of the material’s unique damping properties. The strip was bent into short folds longitudinally, forming a sort of metallic accordion. The strip could be repeatedly compressed and stretched (as an accordion) at room temperature. In a review meeting this strip was passed around the conference meeting and everyone flexed the strip repeatedly. One of the technical Directors, who was a pipe smoker accidentally heated the compressed strip which to everyone’s amazement transformed at once to the original longitudinal strip. The mechanical memory discovery, while not made in Buehler’s metallurgical laboratory, was the missing piece of puzzle of the earlier mentioned acoustic damping and other unique changes during temperature variation. This serendipitous discovery became the ultimate payoff for NITINOL. It is now well recognized that the low temperature Martensite to high temperature Austenite phase change occurring near room temperature is responsible for the remarkable ‘Shape Memory’ and ‘Pseudo-Elastic’ behavior of NITINOL.

2. MARTENSITE TO AUSTENITE TRANSFORMATION

The martensite-austenite transformation involves co-operative movement of atoms unlike nucleation and growth mechanism commonly encountered in first-order phase transitions.

Figure II.11.2.1. depicts the structural relationship between the austenitic and martensitic phases that leads to the 'Shape Memory' effect. It may be noted that a co-operative movement of atoms through a shear transformation can bring about the cubic (high temperature austenite phase) to monoclinic (low temperature martensite phase) structural transition. This type of diffusionless transformation is also known as "military transformation" and is characterized by a low enthalpy change. Further the martensitic phase is 'twinned' so that the overall shape is retained. The volume change accompanying the martensite-austenite first-order phase transition is less than 1% and emphasizes the subtle nature of this phase transformation. The process of 'detwinning' on application of stress in the martensitic phase accounts for the large recoverable strain in this system.

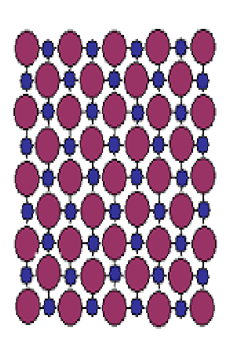
Another unique feature of this transition is that the phase change occurs over a temperature range so that during the heating cycle one can define the austenite start (A_S) and austenite finish (A_F) temperatures. Martensitic start(M_S) and Martensitic finish (M_F) temperatures characterize the reverse transition during the cooling cycle. Figure II.11.2.2 shows these characteristic temperatures in the thermally induced phase transition in Nitinol.

What are Shape Memory Alloys?

•Shape Memory Alloys (SMAs) are metallic alloys that undergo a solid-to-solid phase transformation which can exhibit large recoverable strains.
Example: Nitinol

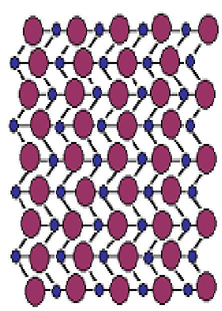
Austenite

- High temperature phase
- Cubic Crystal Structure

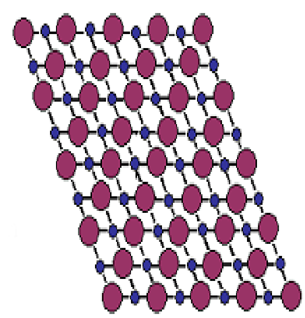


Martensite

- Low temperature phase
- Monoclinic Crystal Structure



Twinned Martensite



Detwinned Martensite

Fig.II.11.2.1 Structural changes involved in Martensite-Austenite phase transition.Note that a co-operative movement of 0.8 A^0 brings about this phase change. The process of detwinning leads to large recoverable strains.

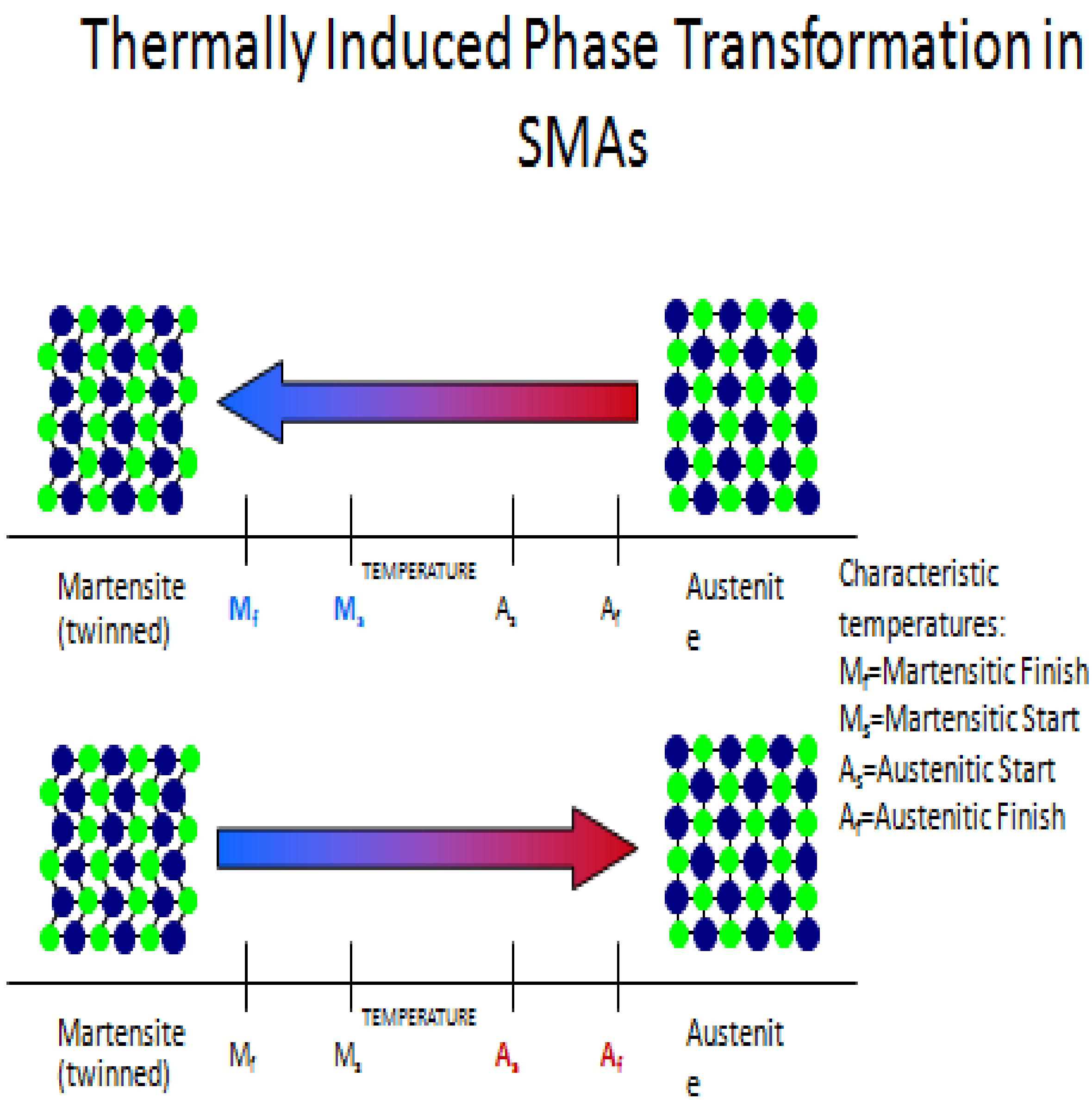


Fig.II.11.2.2 Schematic showing the characteristic temperatures involved in Martensite-Austenite phase transition in NITINOL.

3. RESISTIVITY STUDIES ACROSS THE MARTENSITE-AUSTENITE TRANSITION IN NITINOL

Electrical resistivity measurements provide a convenient probe to track the ‘first order’ martensitic phase transformation in a shape memory alloy. The experimental arrangement essentially consists of a Nitinol wire with the conventional four probes spot welded as indicated in Fig II.11.2.3. A Chromel-Alumel thermocouple is also spot-welded at the centre for measuring the temperature of the sample.

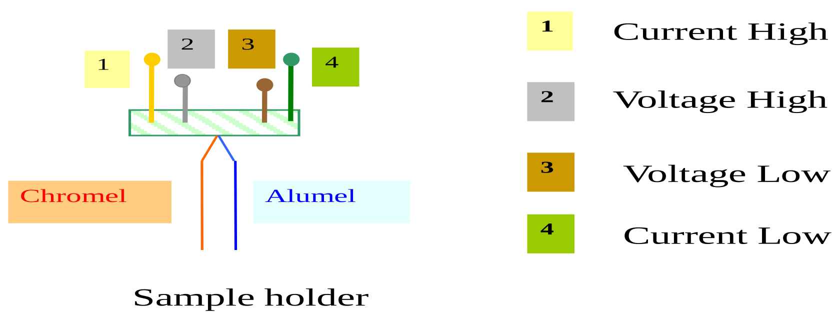
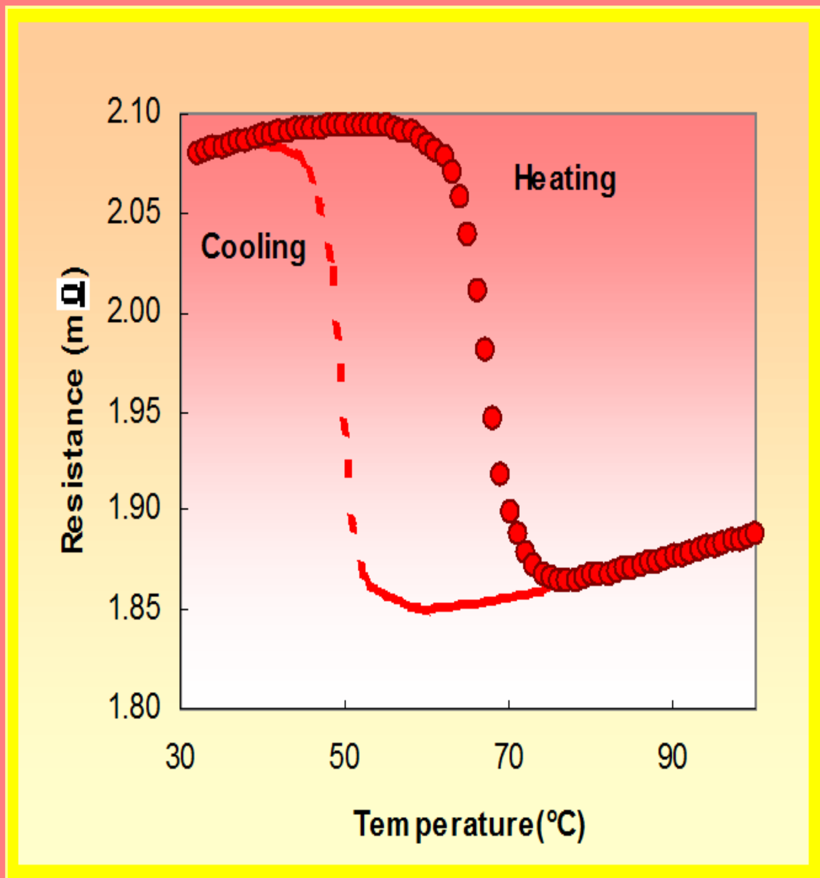


Fig II.11.2.3. Sample holder arrangement for four probe resistivity measurement as a function of temperature.

This sample holder is sandwiched between two nichrome heaters which are connected in parallel. A 10 Volt, 1.5 Amp Power supply is adequate to heat the sample to around 150^{o}C. A constant current of 10 milliampere is passed through the leads 1 and 4 from D.C Constant current source. The voltage developed across the leads 2 and 3, which is proportional to the resistance of the sample, is amplified in a high quality D.C.Amplfier (typically with a gain of 100) and its output is read in a Digital Panel Meter. The thermo-emf from the Chromel-Alumel thermocouple, after cold-junction compensation, is again amplified in a D.C Amplifier and its output read in a Digital Panel Meter. NBS Table for Type-K thermocouple is used to convert the thermo-emf to the appropriate temperature value. A typical plot of the resistivity vs temperature data is given in Fig II.11.2.4.



Martensitic (monoclinic) to Austenite (Cubic) phase transformation

Fig 4.Resistivity behaviour as a function of temperature in a Shape Memory Alloy $Ni_{49}Ti_{45}Cu_6$

The salient features of this plot are (1) the drop in the resistivity accompanying the phase transition (2) near- ideal reversibility in the resistivity value after a thermal cycle (3) hysteresis of around 20^{o} C which is typical of a first-order transition. It may also be noted that both the forward and reverse transformation occur over a small temperature range as indicated in Fig II.11.2.2. The characteristic temperatures A_S, A_F,M_S and M_F can be estimated using the data in this plot.

II.11.2 DIFFERENTIAL THERMAL ANALYSER

T G Ramesh and V Shubha
Materials Science Division

National Aerospace Laboratories
Bangalore 560017
Council of Scientific & Industrial research
CSIR-NAL

II.11.2 STRUCTURAL AND MELTING TRANSITION IN KNO_3 USING DIFFERENTIAL THERMAL ANALYSER

1. **General description**

Differential Thermal Analysis (DTA) is a technique to record a thermal event in the sample by comparing it with a reference material, which is devoid of any physical or chemical changes in the temperature range of interest. In the conventional experimental arrangement, a single block with symmetrical cavities for the sample and the reference is heated uniformly in the constant temperature zone of a tubular furnace. The block acts as a heat sink, ensuring that the sample and reference are heated at the same rate so that there is no temperature difference between them in the absence of a thermal event. Further the block also exchanges heat with the sample so that the sample temperature re-equilibrates with the furnace temperature after a thermal event in the sample. However, the temperature of the sample must not re-equilibrate too rapidly because the DTA signal (the differential temperature) will then be very small. It is thus recommended to have a sample container of low thermal conductivity so as to get an adequate DTA signal. It follows that the temperature change accompanying a transition is directly proportional to the enthalpy change of the transition. An absolute quantitative measurement of the heat of transition requires a mathematical treatment of the heat flow between the sample and the block. A practical method of evaluating the change of enthalpy is to calibrate the DTA instrument with a material whose change in enthalpy is known from other studies and comparing the peak areas.

In the conventional DTA technique the thermocouples are housed in the sample and reference cavities in a block made out of Aluminum for the temperature range 25-500°C and in a stainless steel block for the temperature range up to 800°C. This sample block is located in the constant temperature zone of a tubular furnace. For the temperature range up to 800°C, the conventional type K (Chromel-Alumel) thermocouple is employed.

The Differential Thermal Analyser (DTA) system consists of four sub systems viz.,

1. PRECISION TEMPERATURE PROGRAMMER

Consisting of a) Temperature Lineariser Card
b) ΔT Amplifier card
c) 4 ½ Digit DPM card

2. POWER AMPLIFIER

3. FURNACE (specially designed)

4. PC Add - on card (16 bit A/D and 12 bit D/A).

1. PRECISION TEMPERATURE PROGRAMMER

The Precision Temperature Programmer is the heart of the DTA system. Figure II.11.2.1 gives the block diagram of the setup.

a) Temperature Lineariser Card

The thermocouple input from the Chromel-Alumel thermocouple in thermal contact with the reference material (Alumina) is routed through a differential input filter followed by a high quality instrumentation amplifier configured for a gain of 200. Since temperature measurements are carried out using a single junction thermocouple, reference point compensation is required and an IC temperature transducer provides the necessary cold junction compensation (CJC). The amplified thermocouple voltage with CJC is linearised using a propriety circuit that employs a fourth order polynomial curve fitting for the thermoemf versus temperature data. The fitting accuracy is ± 0.5°C for the temperature range 25-800°C.

The linearised temperature is read through a **4½ digit DPM** with a resolution of 0.1°C. The analog output of the linearised temperature (1mV corresponding to 1°C) is available at the rear panel of the unit as **TEMP O/P.**

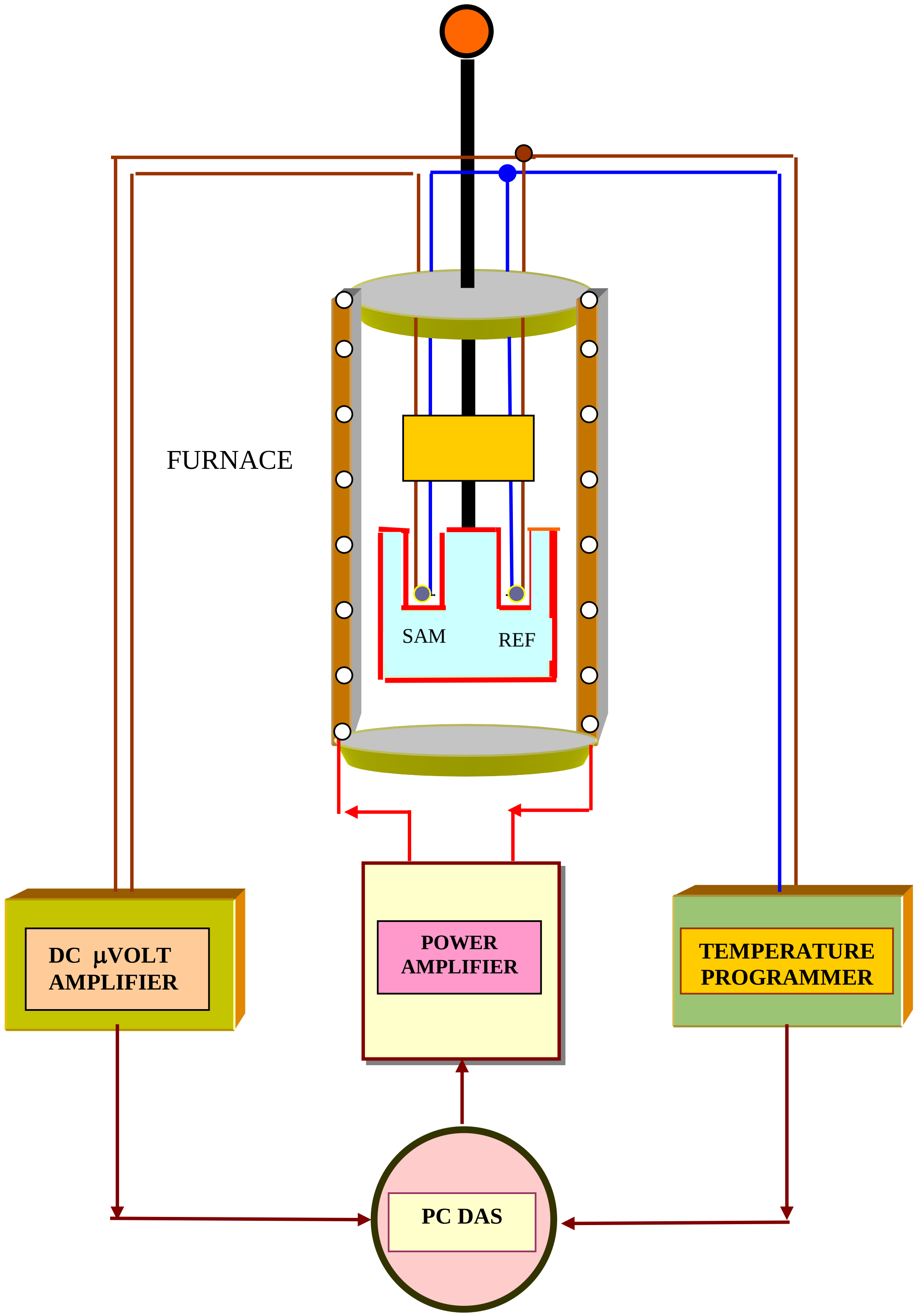


Figure II.11.2.1. Block Diagram of DTA Apparatus

b) ΔT Amplifier card

The temperature differential between the reference and the sample, ***ΔT***, is the central quantity of interest. This signal is fed to the terminals marked ***ΔT INPUT*** at the rear panel of the unit. The signal is initially filtered using a special differential filter followed by a high stability, high gain differential Instrumentation amplifier based on INA 101. The gain of the amplifier can be set from the front panel of the instrument to 200,500 or 1000. The DC offset is less than 1μV referred to the input stage even when a high gain of 1000 is used. The amplified voltage corresponding to ***ΔT*** is brought out at the rear panel of the unit and is marked as ***ΔT OUTPUT*** for further processing in the PC based Data acquisition system.

c) 4 ½ digit DPM

The DPM at the front panel is utilised to read the linearised temperature with a resolution of 0.1°C.

2. POWER AMPLIFIER

This unit power the **furnace** designed for high temperature DTA studies on materials. The design of the Power amplifier is based on a propriety circuit with built in protection for short circuit. The output rating of the Power amplifier is 30V and 3 Amps. Input terminals are provided at the rear panel for connecting the PID signal from the D/A converter in the PC add-on card. Rare panel has socket to power the furnace. The maximum power output of 100 Watts is optimum for attaining temperatures in the range of 600°C and also for achieving linear rates of heating up to 20°C/minute.

3. FURNACE

The compact furnace has been built using non-magnetic materials with provision to house the DTA sample holder. Nichrome wire of 32 SWG is wound round an alumina tube of dimensions 27mm O.D and 23mm I.D and of length 200mm. The number of turns/inch of nichrome wire wound on the alumina tube is 18 turns/inch. The resistance of the heating coil thus wound was 13 Ω. The nominal current carrying capacity of this wire is 1Ampere. Brackets made out of Syndannio material which is a good thermal insulator are fitted at either end of alumina tube to thermally isolate it from an outer cover made out of powder coated mild steel enclosure. Nichrome wire heater is covered with K-wool along the entire length for thermal insulation. The efficiency of this heater arrangement is around 10°C/watt. The furnace is powered from the 100 watt Power Amplifier.

4. Data Acquisition Card

The PCI DAS add-on card is a high resolution 16 bit A/D converter with a front end 8 channel (differential) or 16 channel (single ended) multiplexer operating at a maximum speed of 60 kHz. The gain of the preamplifier is optimized prior to A/D conversion to enhance the accuracy of measurement. The card also has 12 bit D/A converters which are useful for close loop control of parameters such as temperature. This card with the menu driven software developed in LabView environment provides the linkage between the PC and the hardware described earlier.

5. DTA software

This menu driven software **naldta.exe** has been developed in LabView environment for acquiring and graphic display in real time of a DTA run. Significant features of this software are

a) Software selectable rate of heating from 1°C/minute to 20°C/minute

b) Real time digital filtering of the data for improved signal to noise ratio

c) Real time graphic display of ΔT versus T signal for on line observations of thermal events occurring in the sample

d) Auto scaling of the data along both X and Y axes.

The data from the run is also stored in a file named **dtadata.xls** for further analysis. Software has also been developed for numerical computation of the peak areas for estimating enthalpy changes.

6. Operating instructions for running the DTA experiment

a) Fill the sample (in the form of powder or thin filings) in the bore provided in the sample holder.
b) Fill the alumina powder which is the reference material in the other bore marked with a + sign on the outer side.
c) Insert the two Chromel-Alumel thermocouples in to the bores containing sample and the reference. Colour code for the two thermocouples are:

d) Reference : Chromel -- Red colour sleeve
Alumel -- Black colour sleeve

e) Sample : Chromel -- White colour sleeve
Alumel -- Grey colour sleeve

f) Make the connections of the thermocouple to the terminal strip provided at the top of the furnace as given below.

Terminal 1. Chromel (Reference -- Red)
Terminal 2. Alumel (Reference – Black) Joined to
Alumel (Sample -- Grey)
Terminal 3. Chromel (Sample -- White)

g) Connect the shielded wire provided with Red End connector to the **TC INPUT** terminals at the rear panel of the **DIFFERENTIAL THERMAL ANALYSER UNIT.** The other end of the wire is connected to **Terminals 1 and 2 (**Red wire to Terminal 1 and **Black wire** to Terminal 2) on the top of the **Furnace.**

h) Connect the shielded wire provided with **Black End connector** to the **ΔT INPUT** terminals at the rear panel of the DTA unit. The other end of the wire is connected to **Terminals 3 and 1** at the top of the **Furnace.**

i) Connect the **TEMP** output terminals from the DTA unit to the **Channel 0 in the Channel Selector box** using the black-shielded wire.

j) Connect the **ΔT Output** at the rear panel of the DTA unit to the **Channel 1 Channel Selector box** using the yellow-shielded wire.

k) Connect the **D/A output** from the **Channel Selector box** to the **PID OUTPUT** terminals at the rear panel of the **POWER AMPLIFIER** unit. **Purple wire** in the end connector should go to the **RED** terminal of the **PID OUTPUT.**

l) Switch on the computer and select the **naldta.exe file**

The menu driven software will prompt you to enter the necessary details for running the DTA experiment.

The performance of the system is demonstrated by some typical data on KNO_3 and $CuSO_4.5H_2O$. Figure 2 gives the data on the linear rate of heating achieved in this system. The PID parameters can be easily altered in the software to achieve the optimum linear rate of heating. It is worth mentioning that to achieve this linear rate of heating, sufficient care has to be taken in positioning the sample holder inside the furnace.

Figure 3 gives typical data on a DTA run carried out on KNO_3 . The DTA signals observed near 129°C and 340°C corresponding to a structural transition and melting transition respectively conform well with the published literature.

Figure 4 gives the results of a DTA run carried out on $CuSO_4.5H_2O$ in an open container. The first three peaks observed in the DTA plot are due to the following reactions:

$CuSO_4.5H_2O \rightarrow CuSO_4.3H_2O + 2\ H_2O\ (L)$ ----- (a)

$2\ H_2O\ (L) \rightarrow 2\ H_2O\ (g)$ ----- (b)

$CuSO_4.3H_2O \rightarrow CuSO_4.\ H_2O + 2\ H_2O\ (g)$ ----- (c)

These results are in conformity with that reported in the literature.

Maximum Power output : 100 watts

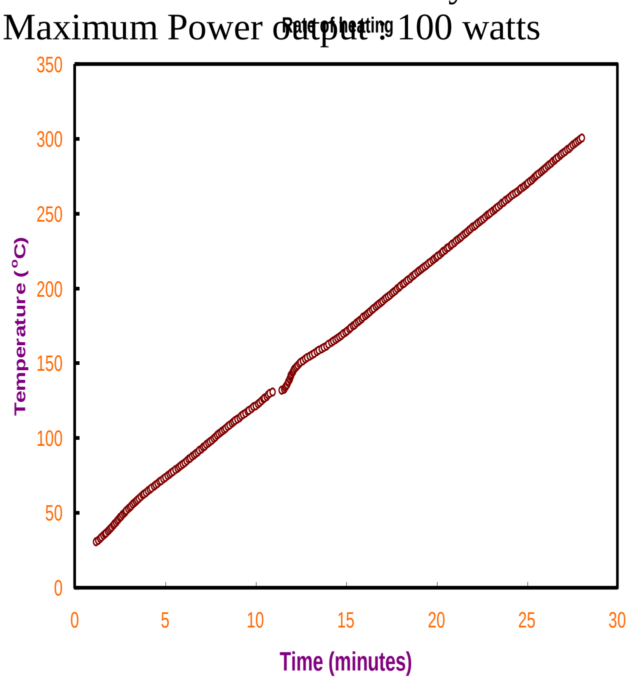


Figure II.11.2.2
Linear rate of heating

Sample: KNO_3 Reference: Alumina Heating Rate: 10°C/min

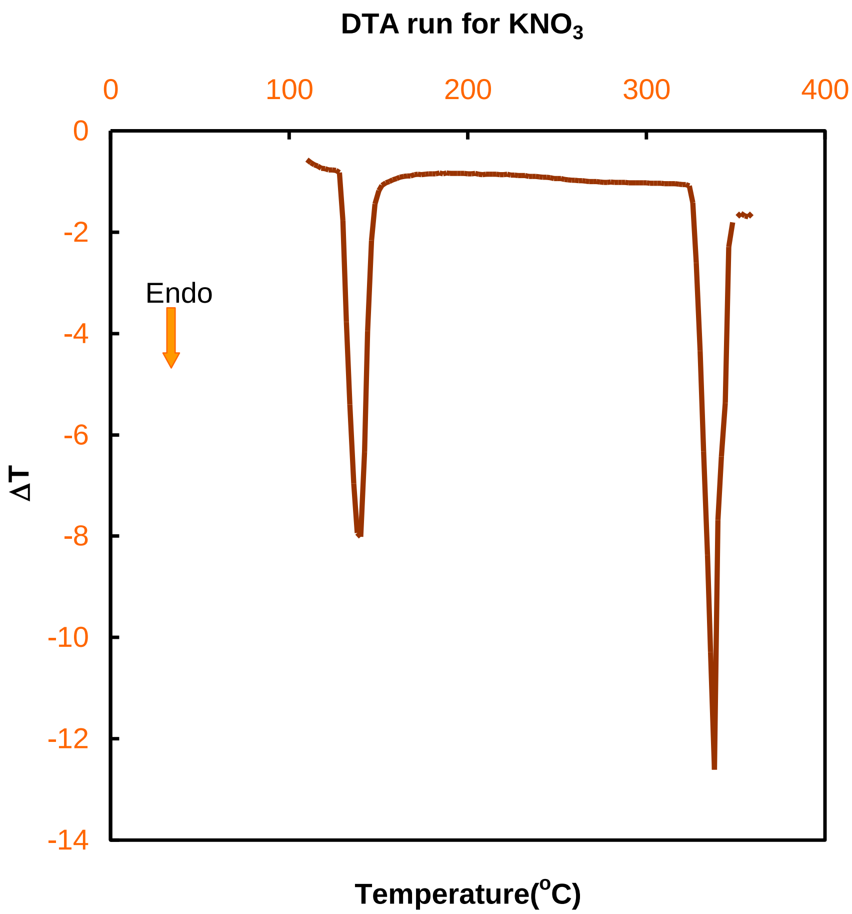


Figure II.11.2.3: DTA of KNO_3

DTA signals are observed near 129°C and 340°C, corresponding to a structural transition and melting transition respectively.

Sample: $CuSO_4.5H_2O$ Reference: Alumina
Heating Rate: 10°C/minute

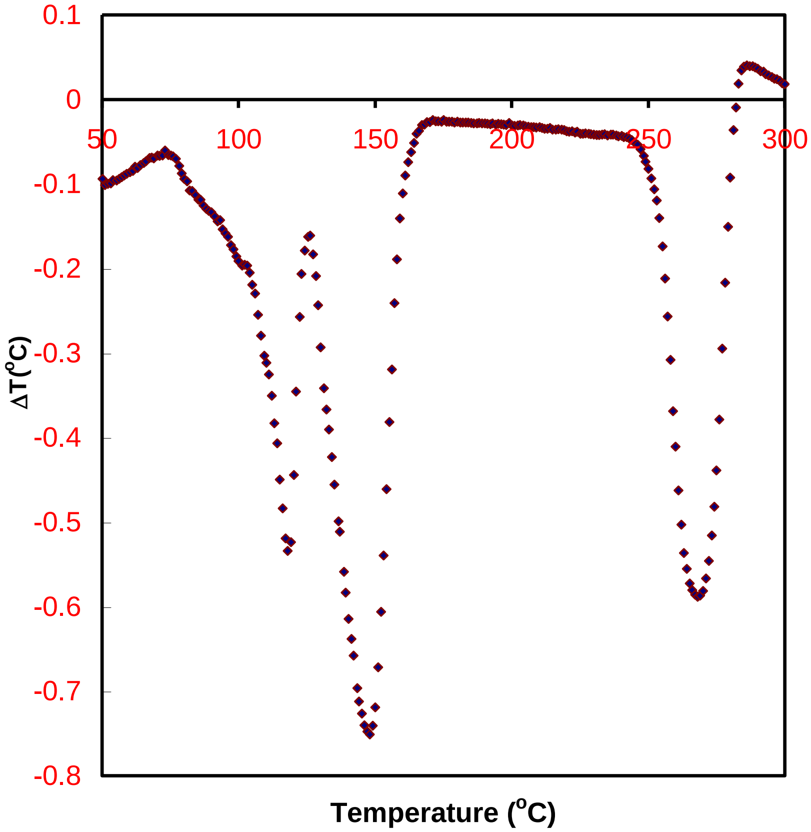


Figure II.11.2.4. DTA of $CuSO_4.5H_2O$

II.12.0 MISCELLANEOUS EXPERIMENTS

II.12.1 PERCOLATION THRESHOLD AND TEMPERATURE DEPENDENCE OF RESISTANCE IN COMPOSITES

1. INTRODUCTION:

One can make a composite of an insulator and a conducting material both of which are granular. One can mix fine grained powders of these materials in different proportions by volume and make pellets out of them in a hydraulic press. Whether the resultant material is insulating or conducting depends on the fractional volume occupied by the conducting material. When this fractional volume is below a threshold p_c, the material is insulating and when it is above this threshold the material is conducting. Near and above the threshold concentration the conductivity will vary as $(p-p_c)^t$ where t is a critical exponent.

2. PERCOLATION

One can make a lattice model for percolation. Consider a square lattice of points.

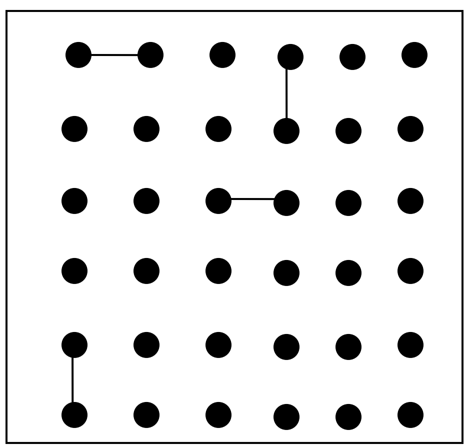


Figure II.12.1.1. A Two dimensional lattice with two adjacent points connected randomly by four conducting links.

Suppose we connect two adjacent lattice points **randomly** with a conducting link (shown by a line). There are 4 links (n) in the above figure and 36 lattice points (N). These conducting links will not make the lattice conduct from left to right. We increase the number of links. Above a critical number of links one may have a conducting path from left to right. This is shown in figure II.12.1.2

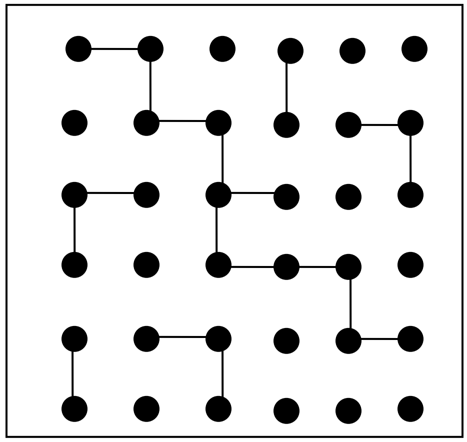


Figure II.12.1.2: Above a critical number of bonds one may have a conducting path from left to right.

The ratio of this critical number n_C/N, in the limit N tending to infinity, is called the percolation threshold p_c. If we have a one dimensional lattice then p_c must be 1 since any break in the chain will make the chain insulating. In higher dimensions p_c will be less than 1.

In a continuum percolation model we replace $p = n/N$ by volume fraction Φ.

If R_N is defined as the probability of spanning a system of size N, it will vary as shown in Figure II.12.1.3.

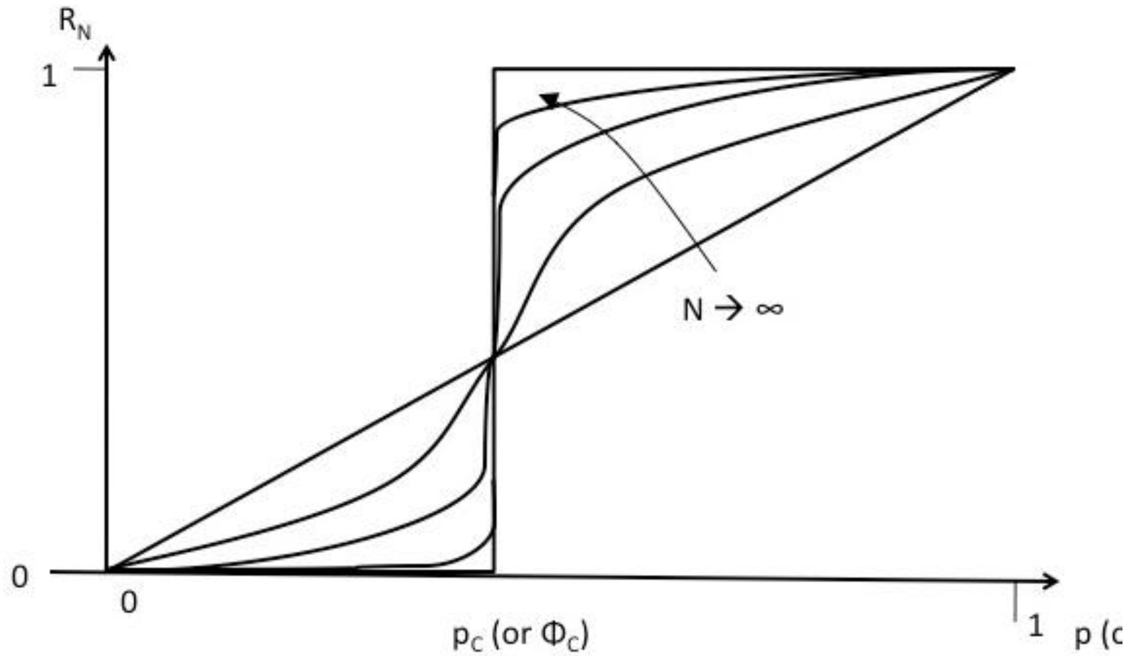


Figure II.12.1.33: Variation of R_N as p = n/N is varied

.

In the lattice model the value of p_c will depend on (1) the number of dimensions, (2) the type of lattice and (3) the coordination number for a given site.

Table II.12.1.1
Percolation Thresholds
for
Different types of lattices
(After Torquato)

LATTICE	DIMENSION d	Coordination No. z	p_C OR Φ
	1	2	1
Honeycomb	2	3	0.697
Square	2	4	0.5927
Triangular	2	6	0.657
Overlapping discs	2	-	0.676
Simple cubic	3	6	0.3116
BCC	3	8	0.2464
FCC	3	12	0.198
Overlapping spheres	3	-	0.2895

From the above table we see that p_c decreases as the number of dimensions increases and as the coordination number of a given site increases. p_c also depends on the shape of overlapping objects.

We may define a quantity p_N which is the expected size of the largest cluster divided by N. When $p<p_c$ the value of p_N tends to zero as N tends to infinity. When p tends to 1, p_N will tend to p. So p_N behaves like an order parameter in a continuous phase transition. Near the critical threshold p_N will vary as $(p-p_c)^\beta$. β is a critical exponent believed to be universal.

Suppose we take two lattice points at **r** and **r'** not in the largest cluster. Probability that the two pints are connected will vary as

$$\exp[-(|\mathbf{r}-\mathbf{r}'|/\xi)] \qquad \text{(II.12.1.1)}$$

ξ is called the correlation length. Near p_C the correlation length will vary as

$$(p-p_c)^\nu \qquad \text{(II.12.1.2)}$$

As p_c is approached the correlation length becomes large.

The overall electrical conductivity of the material will vary with p or Φ as shown in Figure II.2.1.4.

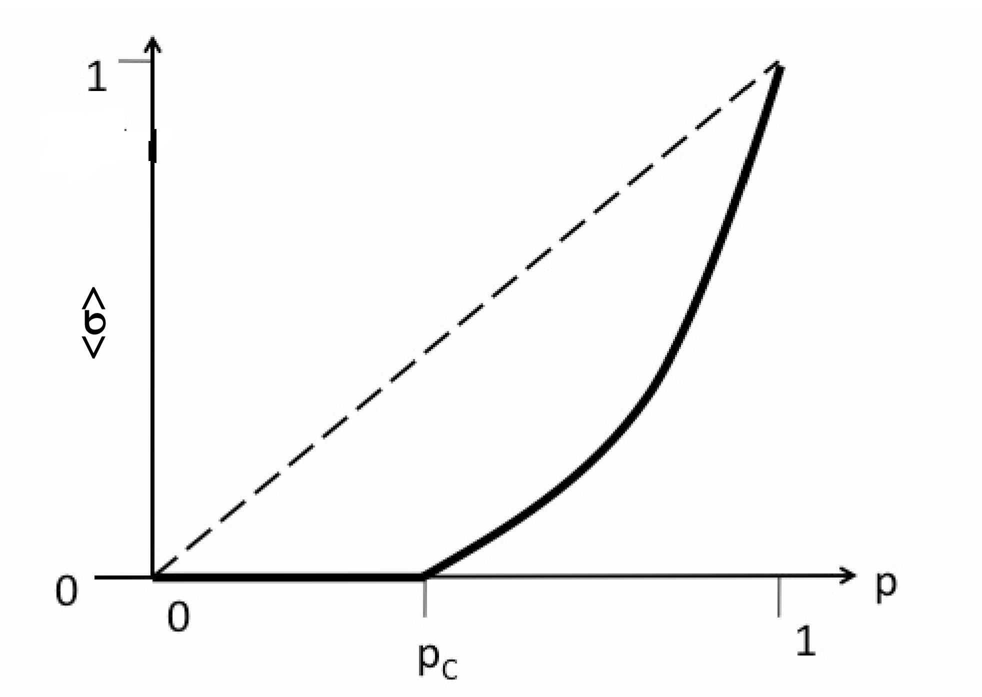


Figure II.2.1.4: Variation of <σ> as a function of p

We see that the conductivity starts increasing from p_c. Near p_c the conductivity will vary as

$$\langle\sigma\rangle \sim (p-p_c)^t \qquad \text{(II.12.1.3)}$$

The values of the critical exponents for two and three dimensions are given in Table II.12.1.2.

Table II.12.1.2

Exponent	Dimension 2	Dimension 3
β	5/36	0.41
ν	1.33	0.88
t	0.33	2

How will the electrical conductivity in such a composite material vary with temperature?

There are two models for this. They are briefly considered below.

3. TEMPERATURE VARIATION OF ELECTRICAL CONDUCTIVITY

ACTIVATED CONDUCTION MODEL:

In this model the electron hops from one site to another. Let us assume we have two

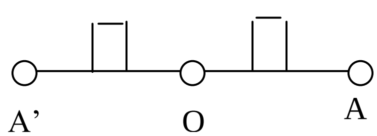


Figure II.12.1.5

sites A and A' on either side of the origin O at equal distance from O. An electron has to cross a potential barrier in hopping from O to A or O to A'. The potential barrier on either side is shown in the figure. This barrier has a height E_a. At a temperature T (absolute scale) the probability for an electron to cross the barrier and go from O to either A or A' is proportional to $\exp(-E_a/kT)$. Equal number of electrons will jump for unit time from O to A as will jump in the reverse direction O to A'. Net current will be zero.

If an electrical field E is applied parallel to the line, the potential barrier to the left will be lowered by eV (remember the electronic charge is negative and e is its magnitude) and the barrier on the right will be raised by eV since the potential at A' will be +V and the potential at A will be –V relative to the potential at the origin. So the probability for an electron to jump from O to A' will now be proportional to

$$\exp\text{-}[(E_a - eV) / kT]$$

while the probability of hopping from O to A will now be proportional to

$$\exp\text{-}[(E_a + eV) / kT] \;.$$

If eV is small compared to E_a ie for small electric fields the difference in probability will be proportional to (eV/kT) exp ($-E_a/kT$). This results in a current in the direction of the field and proportional to the field. So the electrical conductivity σ in such a material will be proportional to (1/T) exp($-E_a/kT$) and the resistance of the material will vary as

$$R \propto T \exp (E_a/kT) \qquad \text{(II.12.1.4)}$$

A plot of [(ln(R) – ln(T)] versus 1/T will give a straight line with slope E_a/k.

MOTT'S VARIABLE RANGE HOPPING MODEL:

In a disordered material the potential barrier will depend on the distance between two centers between which the electron hops. The electron is localized at center O. Its wave function will decrease as exp ($-\alpha r$) where α is the inverse of the localization length. So the probability that the electron will hop a distance r will be proportional to exp($-2\alpha r$). In addition the probability will depend on exp ($-E_a(r)/kT$). So the probability that an electron hops a distance r is proportional to

$$\exp(-(2\alpha r+E_a(r)/kT)) \qquad \text{(II.12.1.5)}$$

Mott used the following argument to get the dependence of Ea(r) on r. Consider a sphere of radius r. The number density of states (i.e. the number of energy levels per unit energy difference per unit volume) is N(0). The volume of the sphere is $(4\pi/3)\ r^3$. The number of energy levels per unit energy interval in this volume is

$$n = N_0\ (4\pi/3)r^3. \qquad \text{(II.12.1.6)}$$

The average spacing of the energy levels is

$$\Delta E = 1/n = (3/4\pi N_0 r^3) = \beta/r^3 \qquad \text{(II.12.1.7)}$$

ΔE takes the place of E_a (r) in the formula for electric conduction. We see that the localization of the electron will favour short hops while ΔE will favour long hops. So there is range r of hopping for which the hopping probability becomes a maximum.

This value of r can be obtained by maximizing

$$\exp(-(2\alpha r+ \beta/kTr^3) \qquad \text{(II.12.1.8)}$$

The maximum occurs when

$$r^4 = (3\beta/2\alpha kT) \qquad \text{(II.12.1.9)}$$

Putting this value of r in the expression for the probability, the maximum probability is

$$\exp - (T_0/T)^{1/4} \qquad \text{(II.12.1.10)}$$

where

$$T_0 = (8/3)^4(3\alpha^3\beta/k) \qquad \text{(II.12.1.11)}$$

So conductivity will increase with temperature as (1/T) exp-$(T_0/T)^{1/4}$ and resistance will decrease as Texp$(T_0/T)^{1/4}$. T_0 will be a constant for a given specimen.

The expression for conductivity on the VRH model can be derived more rigorously.

Unless one makes a high precision resistivity measurement over a wide temperature range, it will be difficult to decide whether the temperature variation of resistance follows a simple activation model or the variable energy hopping model.

4. EXPERIMENTAL PROCEDURE

Six pellets of mixture of graphite and white cement in different proportions by volume were prepared: Sample 5 (5% graphite, 95% white cement), sample 6 (6% graphite), sample7 (7% graphite), sample 8 (8% graphite), sample 9 (9% graphite) and sample 10 (10% graphite). The samples were prepared by Prof.M.S.R.Rao of the Department of Physics, IIT, Madras. The samples are mounted on either side of a rectangular copper strip of 2.5 cm width and 5 cm length. There are three samples on each side. A Pt 100 Platinum resistor is fixed on the strip to measure its temperature.

The resistance of sample 5 is in excess of 50megohms. As the volume fraction of graphite increases from sample 5tosample 8, the resistance decreases to a value of 2.2 k Ohms. So a two probe constant voltage method is used to measure the resistance of these samples. Each sample is connected to with a resistance in series. The samples are hung inside a furnace while the resistances are at the terminal block at room temperature. The samples with their series resistances are connected in parallel to a 9 V battery.

Samples 9 and 10 have resistances in the ohms range. A four probe method is used. A constant current source is connected to these samples in series so that the same current flows through the samples. Two voltage leads from each sample are taken out to measure the voltage across the sample on a DMM in the DC 200 mV range.

The terminal box is a rectangular box on the top of the furnace. The top view of the terminal box is shown in Figure II.12.1.5 and a front view in Figure II.12.1.6.

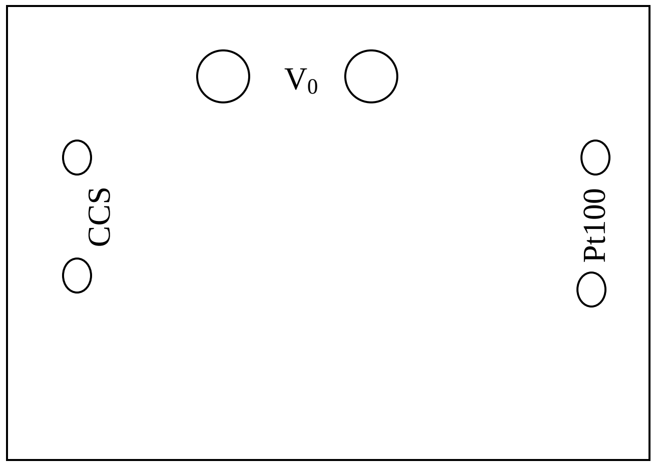


Figure II.12.1.5: Top view of the terminal box

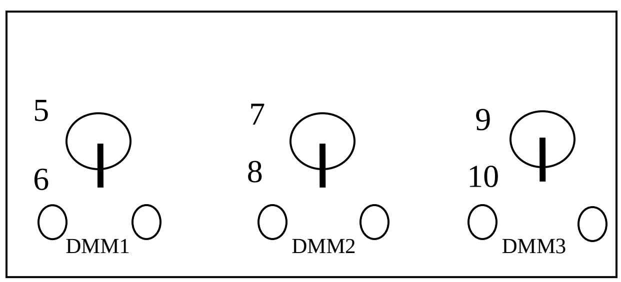


Figure 6: Front view of the terminal box

On top of the box there are two big banana terminals marked V_0. A 9 V battery is connected to these terminals. There are two small banana terminals to the left marked CCS. A constant current source is connected to these terminals. There is another pair of banana terminals to the right marked Pt100. The leads from the temperature controller are connected to these terminals.

On the front side of the box there are three DPDT switches which can be toggled up or down. When the first switch is toggled up, it connects the potential across r5, the resistance connected to sample 5, to DMM1. When it is toggled down it connects the voltage across r6, the resistance connected to sample 6, to DMM1. Both these voltages are measured on DMM1 connected to the small banana terminals below the DPDT switch. Similarly the second DPDT switch connects the voltage across r_7 connected across sample 7 to DMM2 when it is toggled up. When it is toggled down it connects the voltage across resistance r_8 connected to sample 8 to DMM2.. In all these cases the resistance of sample j (j= 5 to 8) is calculated from the voltage v_j measured on the DMM using the formula

$$R_j = (V_0/v_j - 1)\, r_j. \qquad \text{(II.12.1.12)}$$

V_0 is the voltage of the battery. The resistances r_j have the following values:

$r_5 = 470$ k
$r_6 = 47$ k
$r_7 = 10$ k
$r_8 = 220$ Ohms

The voltage leads of samples 9 and 10 are connected to the third DPDT switch. When the switch is toggled up it connects the voltage across the sample 9 to DMM3. When it is toggled down it connects the voltage across sample 10 to DMM 3. The voltages are measured on DMM3 in the DC 200 mV range .The resistance R_k of sample k (k=9 or 10) is given by

$$R_k = v_k/I \qquad \text{(II.12.1.13)}$$

Here v_k is the voltage across voltage leads of sample k measured on DMM3 and I is the current from the constant current source which is set at approximately 10 mA.

The furnace opening is 5 cm in diameter. The strip on which the samples are mounted hangs from the terminal box so that it is near the center of the furnace. The furnace is connected to a regulated power supply. The voltage on the power supply is set at 15.5 V so that the temperature of the furnace increases at the rate of about 1^0 C per minute. When the temperature of the furnace reads 30^0,35^0,40^0,…. the voltages v_6,v_8 and v_{10} are read on the DMMs with the three toggle switches in the down position. When the temperature is 32.5^0. 37.5^0, … the voltages v_5, v_7, and v_9 are read on the multimeters with the three toggle switches in the UP position. Readings are taken till the temperature of the furnace reaches 130^0. As the temperature of the furnace increases the voltage of the DC power supply is gradually increased to maintain the rate of heating. The voltage at the end of the experiment is around 20.5V.

A sample set of readings are shown in the following tables.

Table II.12.1.3(a)

V_0 = 8600 mV
r_5 = 470 k
Sample 5

Temp C	Temp K	1/TK	V mV	R	lnR	lnR-lnT
27.5	300.5	0.003328	28	1.44E+08	18.78454	13.07909
32.5	305.5	0.003273	22	1.83E+08	19.0264	13.30445
37.5	310.5	0.003221	17	2.37E+08	19.28481	13.54663
42.5	315.5	0.00317	14	2.88E+08	19.47932	13.72516
47.5	320.5	0.00312	13	3.1E+08	19.55354	13.78366
52.5	325.5	0.003072	12	3.36E+08	19.6337	13.84834
57.5	330.5	0.003026	10	4.04E+08	19.81626	14.01565
62.5	335.5	0.002981	9	4.49E+08	19.92173	14.10611
67.5	340.5	0.002937	8	5.05E+08	20.03963	14.20922
77.5	350.5	0.002853	13	3.1E+08	19.55354	13.69418
82.5	355.5	0.002813	15	2.69E+08	19.41021	13.53668
87.5	360.5	0.002774	18	2.24E+08	19.22754	13.34005
92.5	365.5	0.002736	21	1.92E+08	19.07304	13.17177
97.5	370.5	0.002699	25	1.61E+08	18.89822	12.98336
102.5	375.5	0.002663	28	1.44E+08	18.78454	12.85628
107.5	380.5	0.002628	31	1.3E+08	18.68241	12.74092
112.5	385.5	0.002594	33	1.22E+08	18.61965	12.66511
117.5	390.5	0.002561	34	1.18E+08	18.58968	12.62226
122.5	395.5	0.002528	33	1.22E+08	18.61965	12.6395
127.5	400.5	0.002497	32	1.26E+08	18.65054	12.65783

Table II.12.1.3(b)
V_0 = 8600 mV
r_6 = 47 k
Sample 6

Temp C	Temp K	1/TK	V mV	R	lnR	lnR-lnT
25	298	0.003356	1191	2.92E+05	12.58581	6.888712
30	303	0.0033	1202	2.89E+05	12.57513	6.861393
35	308	0.003247	1204	2.89E+05	12.57319	6.843093
40	313	0.003195	1191	2.92E+05	12.58581	6.839602
45	318	0.003145	1187	2.94E+05	12.58971	6.827658
50	323	0.003096	1187	2.94E+05	12.58971	6.812057
55	328	0.003049	1191	2.92E+05	12.58581	6.792791
60	333	0.003003	1176	2.97E+05	12.6005	6.79236
65	338	0.002959	1170	2.98E+05	12.60642	6.783379
70	343	0.002915	1164	3.00E+05	12.61237	6.774643
75	348	0.002874	1113	3.16E+05	12.66401	6.81181
80	353	0.002833	1074	3.29E+05	12.70488	6.838409
85	358	0.002793	1104	3.19E+05	12.67333	6.792799
90	363	0.002755	1125	3.12E+05	12.65168	6.757281
95	368	0.002717	1100	3.20E+05	12.6775	6.769413
100	373	0.002681	1097	3.21E+05	12.68063	6.759048
105	378	0.002646	1084	3.26E+05	12.69428	6.759385
110	383	0.002611	1065	3.33E+05	12.71449	6.766452
115	388	0.002577	1039	3.42E+05	12.74265	6.781642
120	393	0.002545	1044	3.40E+05	12.73719	6.763376
125	398	0.002513	1055	3.36E+05	12.72525	6.738795

Table II.12.1.3(c)
V_0 = 8600 mV
R_7 = 10 k
Sample 7

Temp C	Temp K	1/TK	V mV	R	lnR	lnR-ln(T)
27.5	300.5	0.003328	1085	69262.67	11.14566	5.440214
32.5	305.5	0.003273	1083	69409.05	11.14777	5.425823
37.5	310.5	0.003221	1078	69777.37	11.15306	5.414881
42.5	315.5	0.00317	1086	69189.69	11.14461	5.390448
47.5	320.5	0.00312	1075	70000	11.15625	5.386368
52.5	325.5	0.003072	1082	69482.44	11.14883	5.363467
57.5	330.5	0.003026	1081	69555.97	11.14989	5.34928
62.5	335.5	0.002981	1078	69777.37	11.15306	5.337443
67.5	340.5	0.002937	1081	69555.97	11.14989	5.319472
77.5	350.5	0.002853	1087	69116.84	11.14355	5.284193
82.5	355.5	0.002813	1090	68899.08	11.1404	5.266873
87.5	360.5	0.002774	1023	74066.47	11.21272	5.325226
92.5	365.5	0.002736	1108	67617.33	11.12162	5.220353
97.5	370.5	0.002699	1109	67547.34	11.12058	5.205731
102.5	375.5	0.002663	1127	66308.78	11.10208	5.173819
107.5	380.5	0.002628	998	76172.34	11.24075	5.299268
112.5	385.5	0.002594	1115	67130.04	11.11439	5.159846
117.5	390.5	0.002561	1113	67268.64	11.11645	5.149022
122.5	395.5	0.002528	1151	64717.64	11.07779	5.097638
127.5	400.5	0.002497	1139	65504.83	11.08988	5.097165

Table II.12.1.3(d)

V_0 = 8600 mV
r_8 = 220 Ohms

Sample 8

Temp C	Temp K	1/TK	V mV	R	lnR	lnR-lnT
25	298	0.003356	449	3993.808	8.292501	2.595407
30	303	0.0033	453	3956.6	8.28314	2.569408
35	308	0.003247	457	3920.044	8.273858	2.543758
40	313	0.003195	458	3911.004	8.271549	2.525346
45	318	0.003145	457	3920.044	8.273858	2.511807
50	323	0.003096	454	3947.401	8.280813	2.50316
55	328	0.003049	451	3975.122	8.287811	2.494797
60	333	0.003003	449	3993.808	8.292501	2.484358
65	338	0.002959	446	4022.152	8.299572	2.476527
70	343	0.002915	445	4031.685	8.30194	2.464209
75	348	0.002874	445	4031.685	8.30194	2.449737
80	353	0.002833	444	4041.261	8.304312	2.437844
85	358	0.002793	441	4070.249	8.31146	2.430927
90	363	0.002755	437	4109.519	8.321061	2.426659
95	368	0.002717	437	4109.519	8.321061	2.412978
100	373	0.002681	435	4129.425	8.325894	2.404315
105	378	0.002646	449	3993.808	8.292501	2.357606
110	383	0.002611	457	3920.044	8.273858	2.325823
115	388	0.002577	452	3965.841	8.285473	2.324468
120	393	0.002545	451	3975.122	8.287811	2.314001
125	398	0.002513	450	3984.444	8.290153	2.303701

Table II.12.1.3 (e)
Current I = 10.7 mA

Sample 9

T C	Temp K	1/TK	V mV	R	lnR	lnR-lnT	1/T^0.25
27.5	300.5	0.003328	112.5	10.51402	2.352709	3.35274	0.2401811
37.5	310.5	0.003221	111.4	10.41121	2.342884	-3.3953	0.2382235
42.5	315.5	0.00317	110.9	10.36449	2.338385	-3.41577	0.237274
47.5	320.5	0.00312	110.2	10.29907	2.332053	-3.43783	0.2363431
52.5	325.5	0.003072	109.4	10.2243	2.324767	-3.4606	0.2354302
57.5	330.5	0.003026	108.5	10.14019	2.316506	-3.4841	0.2345347
62.5	335.5	0.002981	107.3	10.02804	2.305385	-3.51024	0.233656
67.5	340.5	0.002937	105.9	9.897196	2.292252	-3.53816	0.2327934
77.5	350.5	0.002853	103.8	9.700935	2.272222	-3.58714	0.2311149
82.5	355.5	0.002813	102.3	9.560748	2.257666	-3.61586	0.230298
87.5	360.5	0.002774	100.6	9.401869	2.240909	-3.64658	0.2294952
92.5	365.5	0.002736	99.1	9.261682	2.225886	-3.67538	0.2287063
97.5	370.5	0.002699	97.7	9.130841	2.211658	-3.7032	0.2279308
102.5	375.5	0.002663	95.8	8.953271	2.192019	-3.73624	0.2271682
107.5	380.5	0.002628	94	8.785047	2.173051	-3.76844	0.2264182
112.5	385.5	0.002594	92.1	8.607477	2.152631	-3.80191	0.2256804
117.5	390.5	0.002561	90.1	8.420561	2.130676	-3.83675	0.2249545
122.5	395.5	0.002528	88.4	8.261682	2.111628	-3.86852	0.2242402
127.5	400.5	0.002497	87.1	8.140187	2.096813	-3.8959	0.223537

Table II.12.1.3(f)
Current 10.7 mA
Sample 10

Temp C	Temp K	1/T K	V mV	R	lnR	lnR-lnT	1/T^0.25
25	298	0.003356	50	4.672897	1.541779	-4.15531	0.240683
30	303	0.0033	49.9	4.663551	1.539777	-4.17396	0.239684
35	308	0.003247	49.6	4.635514	1.533747	-4.19635	0.238705
40	313	0.003195	49.4	4.616822	1.529707	-4.2165	0.237746
45	318	0.003145	49.1	4.588785	1.523615	-4.23844	0.236806
50	323	0.003096	48.8	4.560748	1.517487	-4.26017	0.235884
55	328	0.003049	48.5	4.53271	1.51132	-4.28169	0.23498
60	333	0.003003	48.1	4.495327	1.503038	-4.3051	0.234093
65	338	0.002959	47.7	4.457944	1.494688	-4.32836	0.233223
70	343	0.002915	47.2	4.411215	1.48415	-4.35358	0.232368
75	348	0.002874	46.8	4.373832	1.475639	-4.37656	0.231529
80	353	0.002833	46.4	4.336449	1.467056	-4.39941	0.230705
85	358	0.002793	45.8	4.280374	1.45404	-4.42649	0.229895
90	363	0.002755	45.2	4.224299	1.440853	-4.45355	0.229099
95	368	0.002717	44.8	4.186916	1.431964	-4.47612	0.228317
100	373	0.002681	44.2	4.130841	1.418481	-4.5031	0.227548
105	378	0.002646	43.6	4.074766	1.404813	-4.53008	0.226792
110	383	0.002611	43.1	4.028037	1.393279	-4.55476	0.226048
115	388	0.002577	42.6	3.981308	1.381611	-4.57939	0.225316
120	393	0.002545	42.1	3.934579	1.369804	4.60401	0.224596
125	398	0.002513	41.7	3.897196	1.360257	-4.62619	0.223887

In all tables II.12.1.3 (a) to (f), the first column gives the temperature in degrees Celsius, the second in degrees Kelvin. The third column gives the value of 1/T (in K). The fourth column gives the measured voltage in millivolts. In tables 3(a) to 3(d) the fifth column gives the resistance of the sample calculated from equation (II.12.1.12). In Tables 3(e) and (f) the fourth column is calculated from equation (II.12.1.13).

5. ANALYSIS OF DATA

In Table II.12.1.4 the resistance values at 27.5 or 25 C are collected against the concentration p (5 to 10% for the samples 5 to10).

Table II.12.1.4

P	R	1/R	(p-pc)	$(p-p_c)^2$
5	1.44E+08	6.94E-09		
6	2.92E+05	3.42E-06		
7	6.93E+04	1.44E-05	0.7	0.49
8	3.99E+03	2.51E-04	1.7	2.89
9	1.05E+01	9.51E-02	2.7	7.29
10	4.67E+00	2.14E-01	3.7	13.69

Figure II.12.1.7 shows a plot of 1/R against p. This is fitted to a second degree polynomial

$$1/R = A+Bp+Cp^2$$

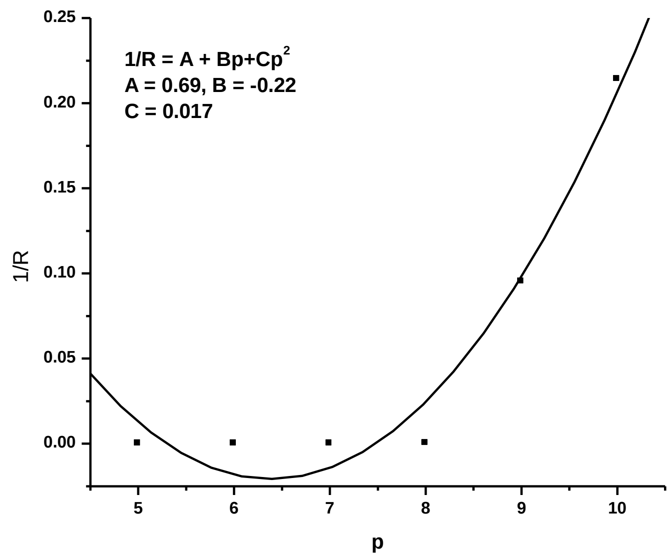


Figure II.12.1.7: A plot of 1/R against concentration p.
Fit is to a second degree polynomial

The values of the coefficients with errors are tabulated below:

Coefficient	Value	Error
A	0.689	0.189
B	-0.222	0.053
C	0.017	0.003

According to theory, above p_c 1/R should be proportional to $(p-p_c)^2$ i.e.

$$1/R = G(p-p_c)^2 = Gp_c^2 - 2Gp_cp + Gp^2$$

So $$A = Gp_c^2$$
$$B = -2Gp_c$$

Or the critical concentration $p_c = (-2A/B)$.. Using the values of A and B above

$$P_c = -2x0.689/(-0.222) = 6.3$$

We take the four readings for p = 7 to 10 and make another table with $(p-p_c)$against 1/R. To save space $(p-p_c)$ is given in column 4 of Table II.12.1.4. If we replot 1/R against $(p-p_C)^2$ for the concentrations 7 to 10 we get the figure 8. We get a reasonable fit to a linear relation between 1/R and $(p-p_c)^2$ as required by theory.

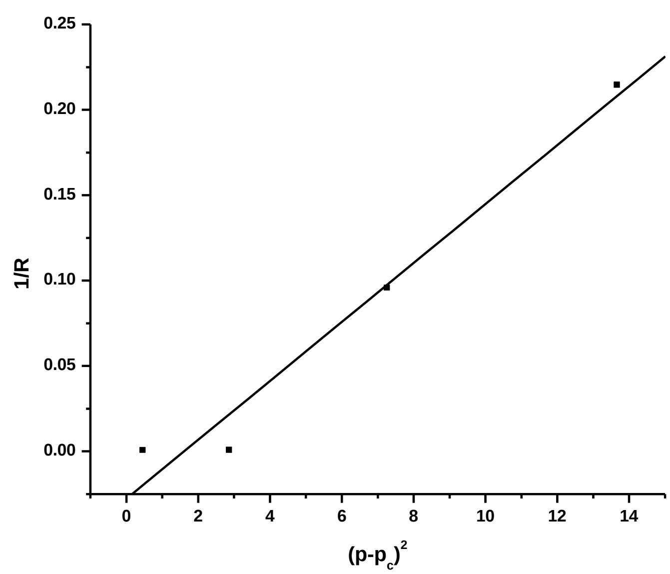


Figure II.12.1.8: Plot of 1/R vs. $(p-p_c)^2$ for samples 7 to 10

6. TEMPERATURE VARIATION OF RESISTIVITY:

Figure II.12.1.9 shows a plot of lnR-lnT against 1/T for samples 5 to 8.

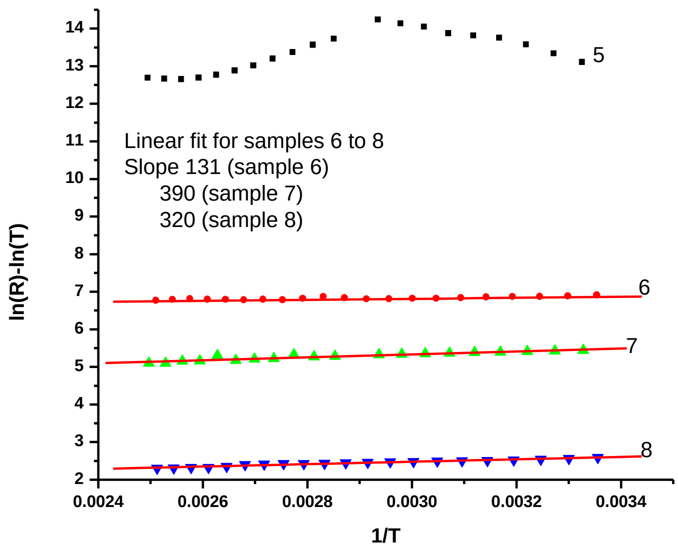


Figure II.12.1.9: Plot of lnR-lnT against 1/T
for samples 5 to 8.

We see that for sample 5, the resistance increases with temperature and then decreases. We have to verify whether such a behavior persists over a long period of time. In an earlier study we found a similar behavior which changed after a long time into the usual pattern of resistance decreasing with increasing temperature after several heating and cooling cycles.

For the other three samples one sees a linear fit to the three curves with a positive slope in agreement with the activated hopping model.

Figure II.1.1.10 shows a plot of ln(R)-ln(T) vs 1/T for samples 9 and 10.

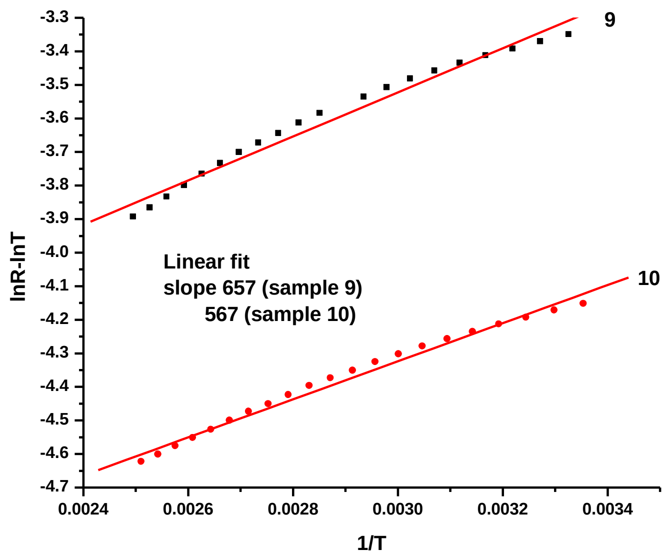


Figure 10: Plot of ln(R)-ln(T) vs. 1/T
For samples 9 and 10.

The curves were fitted with a straight line as required by an activated hopping model. But experiment indicates a distinct curvature of the curves. The activated hopping model only fits the data approximately. Here again we have to see what the effect of repeated heating and cooling will be.

We collect below the slopes of the linear fit for all samples for 6 to 10 and the calculated activation energies.

TABLE II.12.1.5
Activation energies
for hopping model

p	slope	Ea eV
6	131	0.011
7	390	0.034
8	320	0.028
9	657	0.057
10	567	0.049

On an earlier sample of 10% , the activation energy was measured as 0.045 eV which is in agreement with the value for the 10% sample above. While a systematic variation of the activation energy with concentration is not seen, we may say that in general the activation energy appears to increase with concentration.

Figure 11 shows a plot of lnR-lnT against $1/T^{1/4}$ for samples 9 and 10.

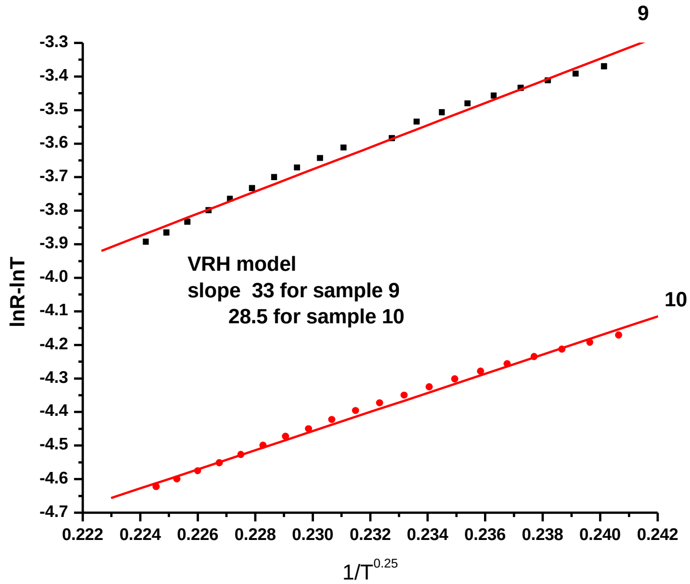


Figure 11 Plot of lnR-lnT against $1/T^{1/4}$

The fit to the VRH model is also approximate as for the Activated hopping model. The slopes of the lines are 33 for sample 9 and 28.5 for sample 10.

Conclusion:

The variation of conductivity near the percolation threshold as $(p-p_C)^2$ is found to be valid for the specimens 7,8,9 and 10 with a p_c about 6.3.

Except for sample 5, the other samples appear to follow the Activated hopping model. The activation energy appears to increase with concentration of graphite.

Both activated hopping and variable range hopping model fit the data on samples 9 and 10 to the same level of accuracy. We need more precise resistivity measurement and a larger range of temperature to distinguish between the two.

These readings are taken on pristine samples. The results may change somewhat after a few cycles of heating and cooling.

II.12.2 METAL-INSULATOR TRANSITION IN A THIN FILM OF STRONTIUM DOPED LANTHANUM MANGENITE

1. INTRODUCTION

Lanthanum mangenite has the perovskite structure. In a stoichiometric compound all Lanthanum atoms are in 3+ state, Manganese atoms are also in 3+ state and oxygen atoms are in the 2- state. If we replace a fraction x of Lanthanum atoms with the divalent Strontium or calcium atoms, an equal fraction of manganese atoms will go into 4+ states to maintain charge neutrality. These doped compounds exhibit a metal-insulator phase transition at a transition temperature which depends on the fraction x of Lanthanum atoms replaced. Above the transition temperatures the Mn^{4+} and Mn^{3+} atoms are distributed randomly. In this phase the resistance decreases as the temperature is increased. This behavior is that of a semiconductor. Below the transition temperature Mn^{3+} and Mn^{4+} atoms are arranged in an orderly periodic fashion. This phenomenon is called **charge ordering**. In this state the resistance increases with increasing temperature. This is called a metallic state. The resistance of the sample, when it is cooled from above the transition temperature exhibits a peak at the transition temperature. We say charge ordering results in a transition from the insulating to the metallic state. This transition is also accompanied by a change from the paramagnetic to ferromagnetic behavior at the transition temperature.

In this experiment we study the resistance of a thin film of $La_{1-x}Sr_xMnO_3$ with a Sr concentration of nearly 0.4. This film was prepared and supplied by Dr. V. Ganesan of UGC-DAE CSR in Indore.

2. EXPERIMENTAL SET UP

The normal boiling point of liquid nitrogen is 77 K. One can reach a temperature about 100 K by using a dipstick cryostat to cool the sample with liquid nitrogen.

The dipstick cryostat used for this purpose is shown in Figure II.12.2.1.

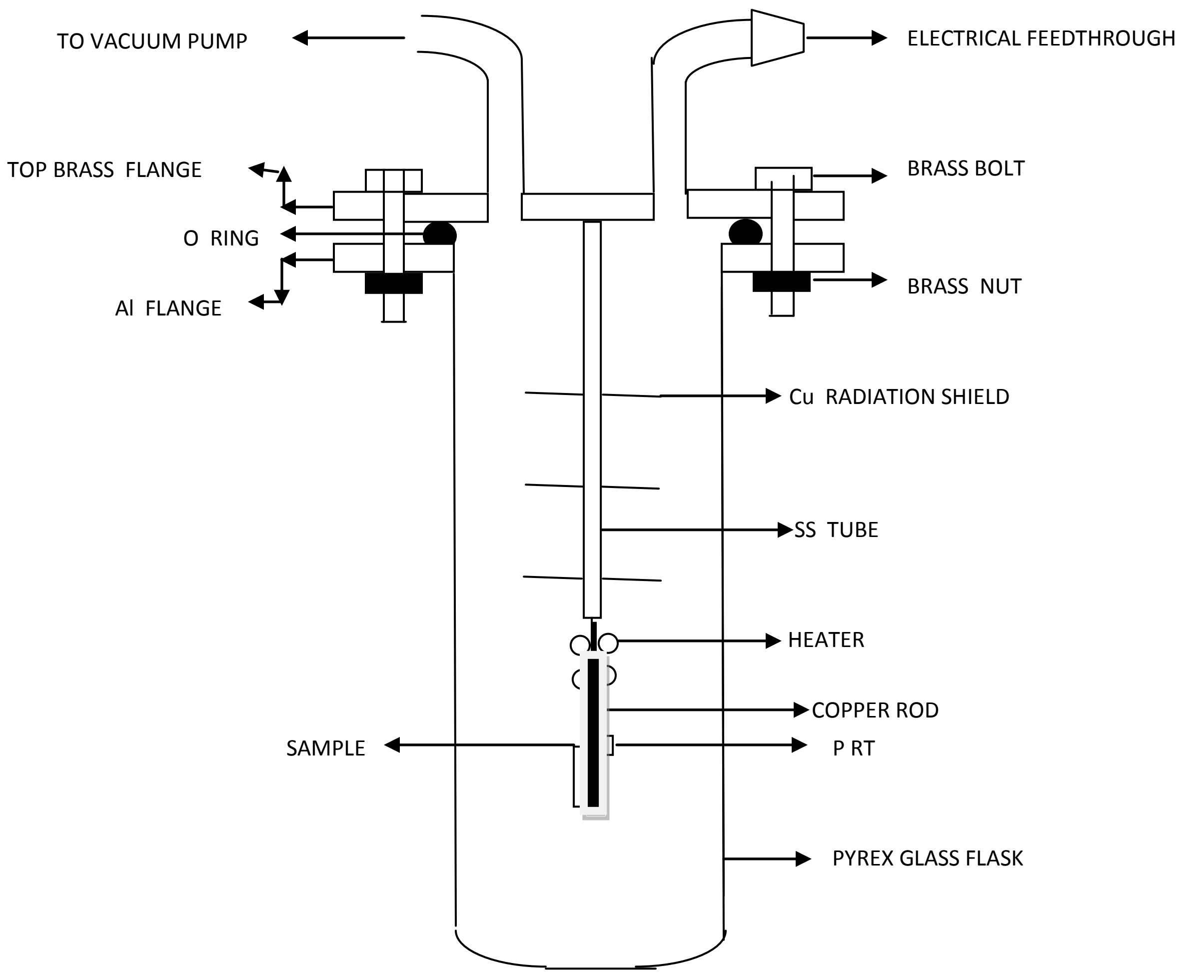


Figure II.12.2.1: Schematic diagram of a dipstick cryostat

.

The cryostat consists of an external Pyrex glass flask to which an Aluminium flange is attached at the top end. From the top brass flange a stainless steel (SS) tube of about 2 to 3 mm diameter hangs inside the vessel. The top flange is fixed to the bottom flange by brass bolts and nuts with a neoprene O ring in between the top and bottom flanges. At the bottom end of the SS tube a copper rod is brazed. The sample in the form of a rectangular pellet is fixed to the bottom end of the copper rod, which is flattened. A heater of a few watts is fixed to the copper rod at its top end. A Platinum resistance thermometer (PRT) is also fixed on the other side of the bottom portion of the copper rod carrying the sample. A number of copper radiation shields are brazed to the SS tube. Electrical leads from the PRT, current and voltage leads to the sample and leads to the heater are brought out through the electrical feed through. A rotary vacuum pump is connected to evacuate the glass flask.

The sample has a resistance of the order of several kilohms. So it is enough to use a two probe measurement for the resistance. Copper leads are attached, using silver paint, to the thin film sample on a glass substrate. The substrate is fixed to the copper block with superglue. Thin copper leads are taken from the silver painted contacts to the electrical feed-through.

3. EXPERIMENTAL PROCEDURE

The roughing pump is first switched on. Within a few minutes the vacuum inside the glass tube will reach a value of 0.02 millibar. Then the glass tube is slowly inserted into the neck of a 20 liter liquid nitrogen storage dewar. The temperature indicator connected to the Pt 100 resistor shows the temperature of the copper block. This temperature will start falling. The depth of immersion of the tube is increased slowly till the flange of the glass tube rests on the neck of the dewar. The dewar will cool to a temperature of -170 C (about 103 K) in one to one and a half hours.

Insert thermocole blocks between the flange of the glass tube and the neck of the dewar to raise the cryostat above the level of liquid nitrogen.. Connect the regulated DC power supply to the heater leads. Connect the leads of the Pt 100thermometer to a temperature indicator. Connect a DMM in the 200k range to the electrical leads from the thin film sample. The voltage on the power supply is kept at 2.5V. The temperature of the copper block will start increasing. The rate of increase is kept between 1 and 2 degrees centigrade per minute. The resistance of the sample, read on the DMM, is noted for every five degree rise in temperature. If the rate of rise of temperature decreases, the voltage on the power supply is raised slightly. **Never raise the DC power supply voltage beyond 4 V.** There is a danger of the heater resistance burning out if the voltage is exceeded beyond this value. It will be possible to come up to +20 C raising the level of flange above the neck of the dewar by inserting thermcole pads and also by raising the power supply voltage.

.

A sample set of data is given in Table II.12.2.1.

TABLE II.12.2.1

METALL-INSULATOR TRANSITION IN
A THIN LSMO FILM

T C	T K	R kOhms	T C	T K	R kOhms
-165	108	9.12	-70	203	12.46
-160	113	9.64	-57.7	215.3	11.06
-155	118	10.54	-55	218	9.74
-150	123	11.28	-50	223	8.68
-145	128	12.22	-45	228	7.74
-140	133	13.46	-40	233	6.86
-134.6	138.4	15.06	-35	238	6.18
-130	143	17.42	-30	243	5.8
-125	148	20	-25	248	5.32
-120	153	22.8	-20	253	4.86
-115	158	25	-14.4	258.6	4.38
-110	163	27.2	-10	263	3.92
-104.5	168.5	30.18	-5	268	3.68
-100	173	27.96	0	273	3.37
-95	178	25.24	5	278	3.11
-90	183	22.08	10	283	3.02
-85	188	20.12	15	288	2.96
-79.2	193.8	17.6	20	293	2.9
-75	198	13.32	25	298	2.86

A plot of variation of resistance with absolute temperature is shown in Figure II.12.2.1.

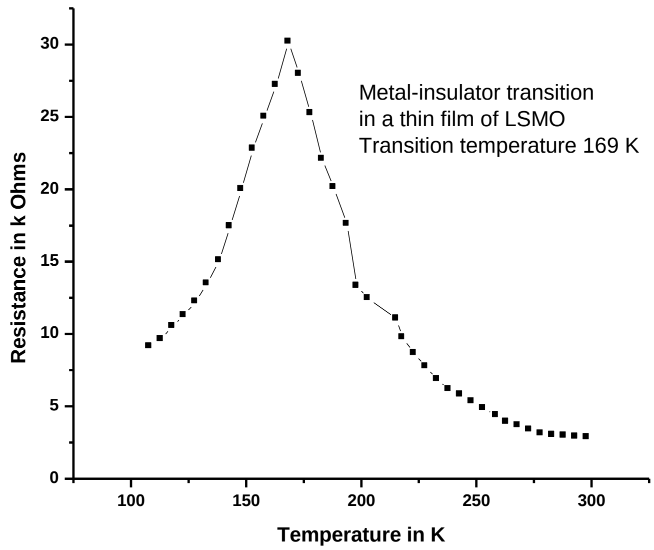


Figure II.12.2.1 Metal-insulator transition in a thin film of LSMO

A sharp transition is seen around 169 K.

II.13. OPTICS EXPERIMENTS

WITH

EQUIPMENT FROM HOLMARC CO.

II.13.1 DIFFRACTION EXPERIMENTS

1. INTRODUCTION:

The Holmarc Company sells a set-up for performing diffraction experiments. The set-up consists of a one meter long optic bench with holders, a laser diode source (power 3.5 mW), a detector (photodiode or transistor) mounted on a translation stage with a current measuring device and diffracting objects such as single slits (50 and 100 microns width), double slits, a grating and a pin hole.

In this experiment we will measure the intensity of diffraction as a function of the transverse position of the detector for (a) a single slit and (b) a double slit and fit it to theoretical formulae for Fraunhofer diffraction. We will also measure the intensity as a function of the transverse position of the detector for a diffraction grating and calculate the number of lines per cm of the grating.

Diffraction phenomena can be classified as belonging to Fraunhofer or Fresnel types. In the former the incident wave front and diffracted wave fronts are plane ie. the source sends a parallel beam of light and the screen is at an infinite distance from the diffracting object. In the Fresnel type the source and/or the detector are at finite distances from the diffracting object. Theoretically it is easier to calculate the diffracted intensity at an angle θ to the incident light in Fraunhofer diffraction.

If we use a laser source, the laser beam consists of parallel rays to a very good approximation. So the incident wave front is planar. If the detector is at a distance large compared to the width of the diffracting object we may apply the Fraunhofer diffraction formulae to calculate the diffracted intensity as a function of angle θ.

2. DIFFRACTION AT A SINGLE SLIT

For a single slit of width w, the intensity as a function of angle θ should vary as

$$I = I(0) \{ \sin(\pi w \sin(\theta)/\lambda)/(\pi w \sin(\theta)/\lambda)\}^2 \qquad \text{(II.13.1.1)}$$

I(0) is the intensity of the direct light reaching the detector. When θ approaches zero (ie. for the direct beam), the quantity in the flower bracket approaches unity. As θ increases and reaches a value such that $w \sin\theta = \lambda$, the rays coming from the edges of the slit have a path difference of λ. We may divide the slit into two halves such that the rays coming from corresponding points in the two halves have a path difference of $\lambda/2$ and destructively interfere to give zero intensity. A second peak in intensity will appear at a value of θ such that $w \sin(\theta)/\lambda \approx 3/2$. However the intensity of this peak will be $1/(3\pi/2)^2$ (about 5%) of the intensity of the direct beam. The diffraction pattern will show a principal maximum at $\theta = 0$, and a series of subsidiary maxima at larger angles, the maxima decreasing progressively in intensity. On either side of the principal maximum there is a minimum when $w \sin(\theta) = \lambda$. The shadow of the slit at the detector is not sharp and does not have the same width w as the slit.

3. DIFFRACTION AT A DOUBLE SLIT

If we have two parallel slits, each of width w and spaced a distance d apart, the diffraction pattern is more complicated. Each slit produces an intensity distribution given by the formula (II.13.1.1). However light from corresponding points in the two slits may interfere. The path difference between two such parallel rays is $d \sin(\theta)$. The result of interference is to modulate the intensity due to a single slit by a factor

$$(1+\mathrm{Cos}(2\pi d \sin(\theta)/\lambda))^2 \qquad \text{(II.13.1.2)}$$

So the intensity $I(\theta)$ for diffraction at a double slit is given by

$$I(\theta) = I(0) \{ \sin(\pi w \sin(\theta)/\lambda)/(\pi w \sin(\theta)/\lambda)\}^2 (1+\mathrm{Cos}(2\pi d \sin(\theta)/\lambda))^2 \qquad \text{(II.13.1.3)}$$

Whenever $d \sin(\theta) = n \lambda$ (n is an integer) , we will get a maximum in intensity and whenever $d \sin(\theta) = (n+1/2) \lambda$ we get zero intensity. Thus we will get nearly equally spaced bright and dark fringes, but the intensity of the bright fringes will keep decreasing as the angle θ increases.

3. DIFFRACTION AT A GRATING OF *N* SLITS

Let us have a series of N equidistant slits with a spacing d. Each slit has a width w. . Then the diffracted rays from corresponding points in adjacent slits will have a path difference $d\sin(\theta)$. If this is equal to $n \lambda$ then rays from corresponding points in all the slits will be in phase. We will get an intensity $N^2 I(\theta)$ where $I(\theta)$ is the intensity at an angle θ due to a single slit. We will get many bright peaks in the diffraction corresponding to positive and negative integral values of n. The intensity of diffraction at an angle θ will now be

$$I(\theta) = I(0) \{ \sin(\pi w \sin(\theta)/\lambda)/(\pi w \sin(\theta)/\lambda)\}^2 \{\sin(N\pi d \sin(\theta)/\lambda)/\sin(\pi d \sin(\theta)/\lambda)\}^2 \qquad \text{(II.13.1.4)}$$

We will have subsidiary minima and maxima. However the subsidiary maxima will have an intensity of the order of $1/N^2$ of the principal maxima and so will not be visible. The first minimum, neighbouring a maximum of order n, will occur when $d \sin(\theta)/\lambda = 1/nN$. As the total number of lines, N, (i.e. as the width of the grating increases) the principal maxima become narrower and narrower. So we say that the resolving power of the grating (i.e. its ability to see the principal maxima of two neighbouring wavelengths as distinct) increases as the width of the grating increases and the order of diffraction increases.

5. EXPERIMENT:

There is an Optic bench of 1 m length. At one end we mount a laser diode which emits light at 650 nm wavelength. The power of the laser is only 3.5 mW. **HOWEVER DO NOT LOOK AT THE LASER LIGHT DIRECTLY. IT WILL DAMAGE YOUR EYE.** We can see the scattered light when the beam falls on a screen. At the other end of the optic bench there is a photo-detector with a very small pinhole. This detector is mounted in a holder and can be moved perpendicular to the holder with a micrometer screw. The screw has a pitch of 0.01 mm. The holder can be moved along the bench and fixed at any position. The detector height can be adjusted.

A. DIFFRACTION AT A SINGLE SLIT

A slit of 100 micron width is placed in a holder and put in front of the source and near the source. The slit should be mounted vertical as seen by the eye. Its position on the optic bench can be noted. Using the screws behind the mount for the laser source adjust the light to fall on the slit at its center. The detector is kept at a distance of about 800 mm from the slit. On the detector you will see a bright patch of light and some fainter patches. Connect the detector to the current meter and switch on the current meter. Adjust the micrometer screw to a value of about 12.5 mm. You may find the bright patch to one side of the center of the detector. Use the screws on the source mount to adjust the position of the bright patch so that the current meter (in the microampere position) shows a maximum reading.

Now move the detector to a position of 5 mm on the micrometer scale. Then move it backwards (i.e. reading on the micrometer scale should increase) in steps of one turn of the screw (ie change in distance of 0.5 mm) and note the current reading on the meter. Carry out this procedure till the micrometer scale indicates 25 mm.

A sample set of readings is given below.

TABLE II.13.1.1

DIFFRACTION AT A SINGLE SLIT

Wavelength of the light λ			6.50E-07	m
Width of slit w			1.00E-04	m
Distance of detector from slit D			800	mm
I(0)			60.6	μa
Direct beam reading on the micrometer x_0			15.75	mm
			Z	
x mm	x-x0	Det. Curr μa	$(\pi w/\lambda)*(x\text{-}x0)/D$	I(0)*(sinz/z)^2
6	-9.75	0.6	-5.8906875	0.3
6.5	-9.25	0.6	-5.588600962	0.8
7	-8.75	0.8	-5.286514423	1.5
7.5	-8.25	1.4	-4.984427885	2.3
8	-7.75	2.1	-4.682341346	2.8
8.5	-7.25	2.7	-4.380254808	2.8
9	-6.75	3	-4.078168269	2.4
9.5	-6.25	2.8	-3.776081731	1.5
10	-5.75	2.2	-3.473995192	0.5
10.5	-5.25	1.5	-3.171908654	0.0
11	-4.75	1.3	-2.869822115	0.5
11.5	-4.25	2.3	-2.567735577	2.7
12	-3.75	5.4	-2.265649038	7.0
12.5	-3.25	11	-1.9635625	13.4
13	-2.75	19	-1.661475962	21.8
13.5	-2.25	28.8	-1.359389423	31.3
14	-1.75	39.4	-1.057302885	41.1
14.5	-1.25	49	-0.755216346	49.9
15	-0.75	56.1	-0.453129808	56.6
15.5	-0.25	60.6	-0.151043269	60.1
16	0.25	60.6	0.151043269	60.1
16.5	0.75	55.2	0.453129808	56.6
17	1.25	48	0.755216346	49.9
17.5	1.75	38.5	1.057302885	41.1
18	2.25	28.6	1.359389423	31.3

TABLE I Contd

			Z	
x mm	x-x0	Det. Curr μa	(πw /λ)∗(x-x0)/D	I(0)*(sinz/z)^2
18.5	2.75	19.3	1.661475962	21.8
19	3.25	11.7	1.9635625	13.4
19.5	3.75	6.1	2.265649038	7.0
20	4.25	2.8	2.567735577	2.7
20.5	4.75	1.3	2.869822115	0.5
21	5.25	0.9	3.171908654	0.0
21.5	5.75	1	3.473995192	0.5
22	6.25	1.3	3.776081731	1.5
22.5	6.75	1.5	4.078168269	2.4
23	7.25	1.4	4.380254808	2.8
23.5	7.75	1.1	4.682341346	2.8
24	8.25	0.8	4.984427885	2.3
24.5	8.75	0.5	5.286514423	1.5
25	9.25	0.3	5.588600962	0.8
25.5	9.75	0.3	5.8906875	0.3
26	10.25	0.4	6.192774038	0.0

Column 1 gives the micrometer reading. We see that the maximum detector current occurs for x = 15.5 and 16 mm. So the centre of the principal maximum is at x_0 = (15.5+16)/2 = 15.75 mm. Column 2 gives (x-x_0). Column 3 gives the detector current in microamperes. Column 4 gives the value of (πw /λ)∗(x-x0)/D, where w is the width of the slit (100 microns), λ is the wavelength of light (650 nm) and D is the distance of the detector from the slit. As D is large compared to the micrometer traverse we may approximate the angle of diffraction θ by tan θ = (x-x0)/D. The last column gives the theoretical intensity as a function of x from equation (II.13.1.1). Here I(0) is the maximum detector current for the direct beam, which from the above table is 60.6 μa.

Figure II.13.1.1 shows the plot of the detector current (column 3) vs (x-x_0) (column 2). The continuous curve shows the plot of the values in the last column.

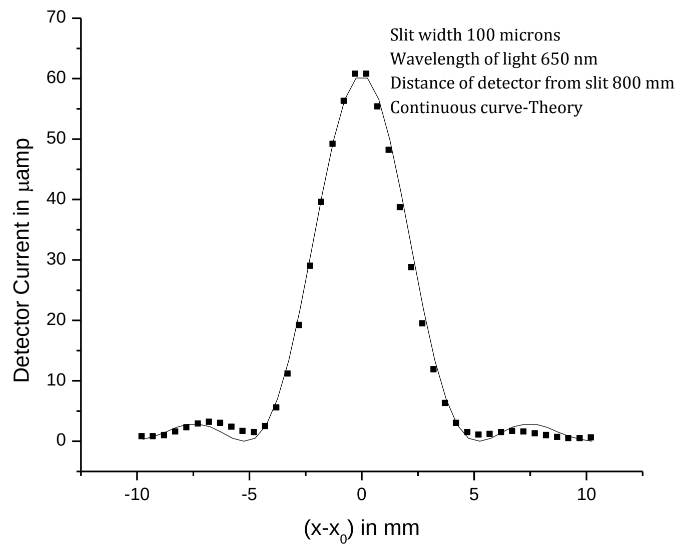


Figure II.13.1.1: Plot of Detector current vs position of for diffraction at a single slit. Continuous curve fit to equation (II.13.1.1)

We see that the fit is good.

B. DIFFRACTION AT A DOUBLE SLIT

Instead of the single slit, mount the double slit. The rest of the procedure is the same.

A sample set of readings is given in Table II.13.1.2.

TABLE II.13.1.2

DIFFRACTION FOR A DOUBLE SLIT

Distance between slits d			2.50E-04	m	
Wavelength of light		λ	6.50E-07	m	
Distance of detector n from slit D			740	mm	
Center x0=			13.75	mm	
max det current I_{max}			142.6	μa	
width of slit w			1.00E-04	m	
			z1	z2	
x mm	det curr μa	(x-x0) mm	$(2\pi d/\lambda)*$(x-x0)/D	$(\pi w/\lambda)*$(x-x0)/D	Intensity I
22	0.2	8.25	26.94285343	5.39E+00	4.35E-01
21.5	0.2	7.75	25.30995322	5.06E+00	4.84E+00
21	0.6	7.25	23.67705301	4.74E+00	1.97E+00
20.5	2	6.75	22.04415281	4.41E+00	3.29E-06
20	2.6	6.25	20.4112526	4.08E+00	1.42E+00
19.5	1	5.75	18.77835239	3.76E+00	3.35E+00
19	0.3	5.25	17.14545218	3.43E+00	1.83E-01
18.5	1.7	4.75	15.51255198	3.10E+00	2.05E-06
18	2.5	4.25	13.87965177	2.78E+00	9.31E-01
17.5	2.1	3.75	12.24675156	2.45E+00	9.20E+00
17	1.1	3.25	10.61385135	2.12E+00	2.26E+00
16.5	1	2.75	8.980951143	1.80E+00	9.85E-02
16	14.3	2.25	7.348050936	1.47E+00	3.60E+01
15.5	43.7	1.75	5.715150728	1.14E+00	7.67E+01
15.25	41.9	1.5	4.898700624	9.80E-01	3.60E+01
15	11.2	1.25	4.08225052	8.16E-01	4.79E+00
14.75	13	1	3.265800416	6.53E-01	1.83E-03
14.5	45.8	0.75	2.449350312	4.90E-01	1.74E+00
14.25	91.5	0.5	1.632900208	3.27E-01	3.03E+01
14	132.8	0.25	0.816450104	1.63E-01	1.00E+02
13.75	142.6	0	0	0.00E+00	1.43E+02
13.5	126.4	-0.25	-0.816450104	-1.63E-01	1.00E+02
13.25	83.9	-0.5	-1.632900208	-3.27E-01	3.03E+01
13	39	-0.75	-2.449350312	-4.90E-01	1.74E+00
12.75	8.6	-1	-3.265800416	-6.53E-01	1.83E-03
12.5	16.8	-1.25	-4.08225052	-8.16E-01	4.79E+00

TABLE II.13.1.2 (Contd)

			z1	z2	
x mm	det curr μa	(x-x0) mm	(2πd/λ)∗(x-x0)/D	(πw/λ)∗(x-x0)/D	Intensity I
12.25	39.3	-1.5	-4.898700624	-9.80E-01	3.60E+01
12	57.4	-1.75	-5.715150728	-1.14E+00	7.67E+01
11.75	60.2	-2	-6.531600832	-1.31E+00	7.55E+01
11.5	48.8	-2.25	-7.348050936	-1.47E+00	3.60E+01
11.25	30.9	-2.5	-8.16450104	-1.63E+00	6.42E+00
11	14.2	-2.75	-8.980951143	-1.80E+00	9.85E-02
10.75	4.3	-3	-9.797401247	-1.96E+00	3.74E-02
10.5	0.7	-3.25	-10.61385135	-2.12E+00	2.26E+00
10	1.6	-3.75	-12.24675156	-2.45E+00	9.20E+00
9.5	2.9	-4.25	-13.87965177	-2.78E+00	9.31E-01
9	3.8	-4.75	-15.51255198	-3.10E+00	2.05E-06
8.5	2.3	-5.25	-17.14545218	-3.43E+00	1.83E-01
8	0.3	-5.75	-18.77835239	-3.76E+00	3.35E+00
7.5	2.3	-6.25	-20.4112526	-4.08E+00	1.42E+00
7	5.1	-6.75	-22.04415281	5-4.41E+00	3.29E-06
6.5	3.5	-7.25	-23.67705301	-4.74E+00	1.97E+00
6	0.8	-7.75	-25.30995322	-5.06E+00	4.84E+00
5.5	0.2	-8.25	-26.94285343	-5.39E+00	4.35E-01
5	0.5	-8.75	-28.57575364	-8.572726091	5.59E-04

The first three columns do not need explanation. The four0h column gives

$$z_1 = 2\pi d \sin(\theta)/\lambda$$

and the fifth column gives $z_2 = \pi w \sin(\theta)/\lambda$

In both we have used the approximation $\sin(\theta) = \tan(\theta) = (x-x_0)/D$.

The last column gives the intensity $I((x) = (I_{max}/4)^* (1+\cos(z_1))^2 (\sin(z_2)/z_2)^2$

The values of d and w are adjusted to give the best fit. We find d = 250 μm and the width comes out to be 100 μm. The fit is good as seen in Figure II.13.1.2.

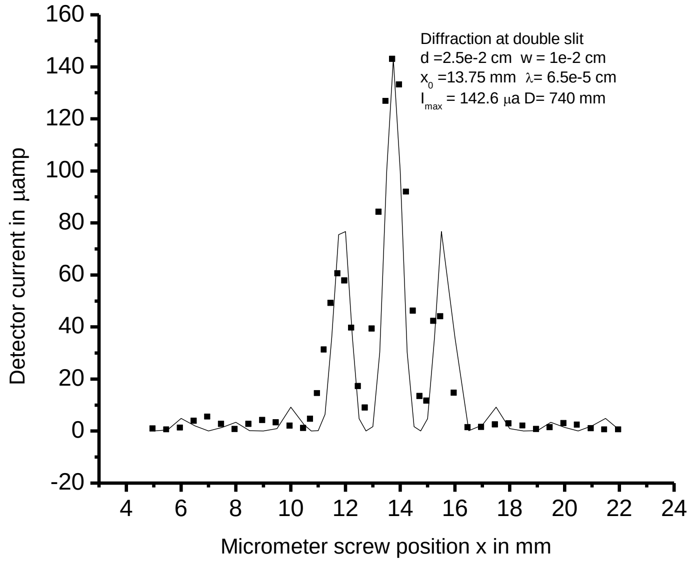


Figure II.13.1.2: Double slit diffraction pattern

C. DIFFRACTION GRATING:

The diffraction grating is mounted in place of the double slit on the optic bench. One will see a series of bright spots on the detector cover. If necessary turn the grating so that the spots are horizontal. Rest of the procedure is the same as in the other two cases.

A set of sample data is given in Table II.13.1.3.

TABLE II.13.1.3

DIFFRACTION GRATING

Wavelength of light			650	nm		
x_0			13.25	mm		
Distance between detector and grating			740	mm		
x mm	Detector current	$(x-x_0)$		x mm	Detector current	$(x-x_0)$
	in μa				in μa	
5	2.6	-8.25		10.25	6.3	-3
5.25	3.1	-8		10.5	29.9	-2.75
5.5	8.8	-7.75		10.75	70.9	-2.5
5.75	37.9	-7.5		11	56	-2.25
6	73.5	-7.25		11.25	18.2	-2
6.25	44.2	-7		11.5	5.2	-1.75
6.5	10	-6.75		11.75	3.8	-1.5
6.75	2.2	-6.5		12	4	-1.25
7	1.7	-6.25		12.25	6.3	-1
7.25	2.2	-6		12.5	55.2	-0.75
7.5	5.8	-5.75		12.75	1100	-0.5
7.75	34.9	-5.5		13	6700	-0.25
8	500	-5.25		13.25	10700	0
8.25	2300	-5		13.5	5630	0.25
8.5	3100	-4.75		13.75	700	0.5
8.75	1000	-4.5		14	60.6	0.75
9	109.5	-4.25		14.25	12.1	1
9.25	17.4	-4		14.5	7.9	1.25
9.5	4.8	-3.75		14.75	5.1	1.5
9.75	3.4	-3.5		15	6.7	1.75
10	4	-3.25		15.25	25.1	2

TABLE II.13.1.3 contd

x mm	Detector current	$(x-x_0)$		x mm	Detector current	$(x-x_0)$
	in μa				in μa	
15.5	69.4	2.25		20	14.1	6.75
15.75	65.8	2.5		20.25	64.4	7
16	22.8	2.75		20.5	91.5	7.25
16.25	6.6	3		20.75	40	7.5
16.5	3.8	3.25		21	7	7.75
16.75	4.6	3.5		21.25	2.4	8
17	3.7	3.75		21.5	2.6	8.25
17.25	11.8	4		21.75	2.8	8.5
17.5	1703	4.25		22	2.8	8.75
17.75	2000	4.5		22.25	9.8	9
18	4100	4.75		22.5	87.8	9.25
18.25	2600	5		22.75	300	9.5
18.5	400	5.25		23	200	9.75
18.75	37	5.5		23.25	52	10
19	6.8	5.75		23.5	8	10.25
19.25	4.2	6		23.75	1.7	10.5
19.5	3.4	6.25		24	1.6	10.75
19.75	4.3	6.5				

A plot of the detector current as a function of $(x-x_0)$ is shown in Figure II.13.1.3.

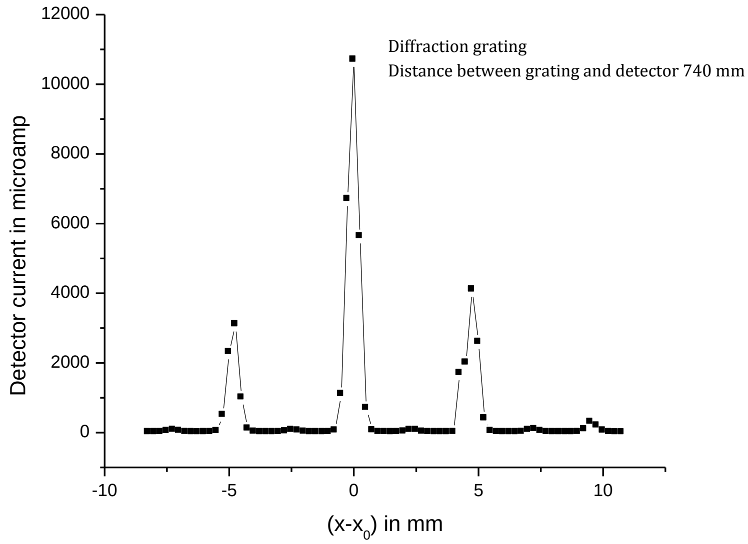


Figure II.13.1.3 Diffraction pattern for a grating

We see the n = 1 and n = -1 order in this grating. They occur when $(x\text{-}x_0) = \pm 4.73$ mm. Using the equation

$$d \sin(\theta) = \lambda$$

and $\sin(\theta) = (x\text{-}x_0)/D$ we find d = 0.010 cm. The manufacturer gives a value for d = 0.01 cm.

II.13.2 DISPERSIVE AND RESOLVING POWERS

II.13.2.1 DISPERSIVE POWER AND THE RESOLVING POWER OF A PRISM

1. INTRODUCTION:

The refractive index n of the material of a prism varies with the wavelength λ of the light. If the wave length of light is much longer than the absorption wavelength of the material, the refractive index varies as

$$n = A + B/\lambda^2 \qquad \text{(II.13.2.1.1)}$$

where A and B are material constants. This is called the Cauchy dispersion formula. The dispersive power of a prism is the rate of change of refractive index with wavelength. Differentiating the Cauchy's formula with respect to λ we get

$$\text{Dispersive power} = dn/d\lambda = -2B/\lambda^3 \qquad \text{(II.13.2.1.2)}$$

Note that the dispersive power is negative ie the refractive index increases as the wavelength decreases. Red light, which has a longer wavelength than blue light, will have a lower refractive index. This is the common behavior of all materials at wavelengths much longer compared to the absorption wave lengths of the material.

Another important property of a dispersing element like a grating or a prism is the Resolving power. Between two close wavelengths λ and $\lambda+\Delta\lambda$, a minimum separation $\Delta\lambda$ is required to see the images produced by the dispersive element at the two wave lengths to be seen as separate. The value $\lambda/\Delta\lambda$ is called the resolving power of the dispersive element. An element with a larger resolving power has to be used in the study of spectra of closely separated spectral lines.

Resolving power is a consequence of the diffraction of light. If we pass a parallel beam of light through a slit, the shadow cast on a distant screen will not be bounded by sharp edges. The intensity will be a maximum at the centre of the image and then will decrease as we move away from the center passing though maxima and minima at the edge of the image. This was seen in the earlier experiment on the diffraction of light through a slit. If we have two images of the slit at neighbouring wavelengths produced by the dispersive element, the resultant intensity we see in the telescope of the spectrometer is the sum of the two intensities of the images. If the difference in wavelength is large, the two principal maxima of the intensities in the images produced at the different wavelengths will be widely separated and we will see the two images as distinct. As the wavelength difference decreases the two principal maxima move closer to each other in the telescope. When the principal maximum of the image at wavelength λ falls on the first minimum in the diffraction pattern of the image due to $\lambda+\Delta\lambda$, the sum of the two intensities will show a single peak. Then we say that the images at the two wavelengths are not seen as distinctly two images separated by a low intensity region.

For a prism at the minimum deviation position, the resolving power

$$\lambda/\Delta\lambda = t\,|dn/d\lambda| \qquad \text{(II.13.2.1.3)}$$

where t is the length of the base of the prism.

2. **PROCEDURE:**

We use a mercury lamp which gives the following prominent lines in the spectrum:

Colour	Wavelength λ
Yellow 1	5.790 E-7 m
Yellow 2	5.770 E-7 m
Green	5.461 E-7 m
Blue	4.358 E-7 m
Violet 1	4.076 E-7 m
Violet 2	4.046 E-7 m

Switch on the mercury lamp. Mount a narrow slit on the optic bench illuminated by the lamp. On the optic bench there is a lens whose focal length is approximately 200 mm. There is a prism table and a telescope on a goniometer attached to the optic bench. Turn the telescope to a distant object (say a window with cross bars) more than 5 meters away from the telescope and focus the telescope on the object. The telescope is now set to receive parallel rays. Now place the lens on the optic bench at a distance approximately 200 mm from the slit. Look through the lens and adjust its height and the position of the lamp so that the slit appears bright at the center of the lens. Turn the telescope and fix it so that it sees directly the image of the slit. The telescope will be pointing towards the lens and you will see a bright patch of light. Now move the lens till this bright patch appears as a well focused image in the telescope. Then the lens is rendering the rays issuing from the slit to come out as a parallel beam.

Move the light source with the slit to the right or left slightly till the image of the slit appears at the crosswire of the eyepiece of the telescope. This completes the alignment of the beam.

Place the equilateral prism (angle A of the prism is 60^0) on the prism table. Adjust its height and turn it so that the rays of light from the collimating lens are incident at an angle on the first face. Look through the other face to see the images of the slit in the different coloured lights (Yellow to Violet) emerging from the prism. As the prism is turned the images move towards the right (ie in the direction in which the angle of deviation D is decreasing), stop and then move in the opposite direction. Fix the prism when the deviation is a minimum. Turn the telescope to view the different coloured images of the slit. Measure the angle of minimum deviation by noting the telescope position on the vernier scale for each colour.

The refractive index for any one colour is

$$n = \sin((A+D)/2)/\mathrm{Sin}(A/2) \qquad \text{(II.13.1.4).}$$

where A, the angle of the prism is 60^0, and D is the angle of minimum deviation for light of that colour. Calculate n for the different wavelengths and plot n vs $1/\lambda^2$.

A sample set of readings is shown below. Note the readings of the angles are converted to decimals. A reading of $30^0 12'$ in decimals would be 30.2^0. In calculating the refractive index the angles are converted to radians. 1^0 is equal to 0.01745 radians.

TABLE 1

Refractive index of the material of the prism as a function of wavelength

Dispersive power of a Prism					
1 degree =	0.017454	Radn			
Angle of Prism		60	Degrees		
Wavelength	D	(A+D)/2	Ref index	$1/\lambda^2$	$dn/d\lambda$
in nano-m	in deg	In deg	n	in m^{-2}	
579	57.25	58.625	1.707537	2.98E+12	-1.5E+05
577	57.3	58.65	1.707991	3.00E+12	-1.6E+05
546.1	57.83	58.915	1.712785	3.35E+12	-1.8E+05
435.8	60.63	60.315	1.737501	5.27E+12	-3.6E+05
407.8	62.33	61.165	1.752002	6.01E+12	-4.4E+05
404.6	62.57	61.285	1.754018	6.11E+12	-4.5E+05
A	1.664				
B	1.449E-14	m^{-2}			

The dispersive power is

$$dn/d\lambda = -2B/\lambda^3$$

This is calculated and given in the last column.

Figure II.13.2.1.1 shows a plot of n vs $1/\lambda^2$.

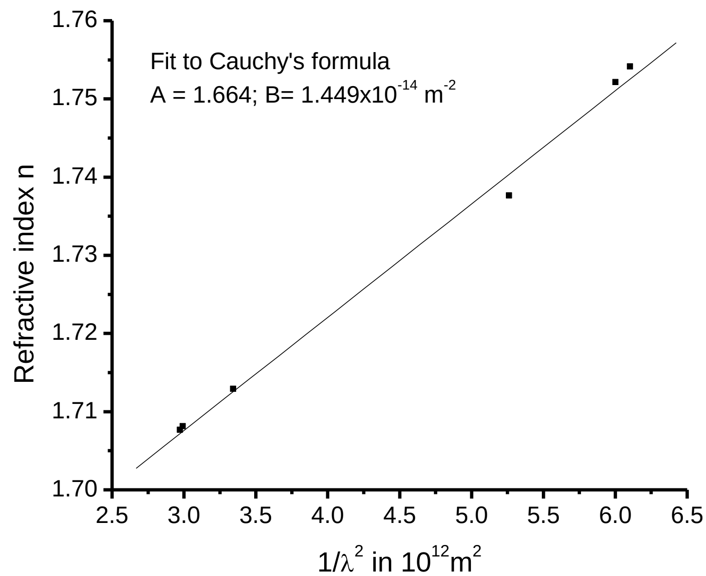


Figure II.13.2.1.1 Fit of the refractive index n to the Cauchy formula

3. RESOLVING POWER OF THE PRISM:

We put a vertical slit of adjustable width after the collimating lens on the optic bench Make the width large by turning the screw on the adjustable slit. In the telescope we see the two yellow lines in the spectrum as clearly separated. We start reducing the width of the slit by turning the screw. Each line will broaden out and at one point we **cannot** see two distinct lines separated by a region of low intensity. The two lines merge. Take the slit out and measure its width w.

Figure II.13.2.1.2 shows the two extreme rays which pass through the slit in front of the equilateral prism P and enter the objective O of the telescope. In the minimum deviation position the rays pass through the prism parallel to its base.

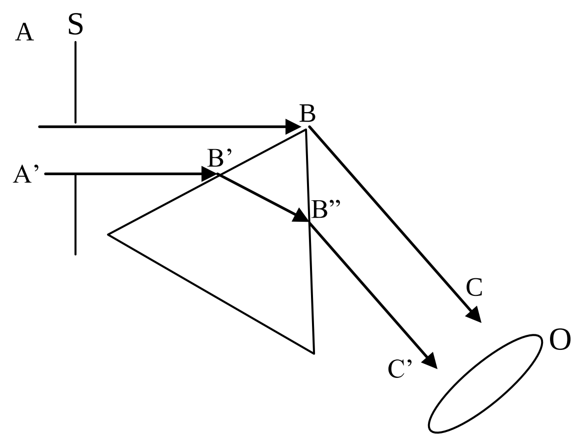


Figure II.13.2.1.2 Path of the extreme rays through the prism

Figure II.13.2.1.3 shows an expanded image of the portion ABC, A’B’B”C’

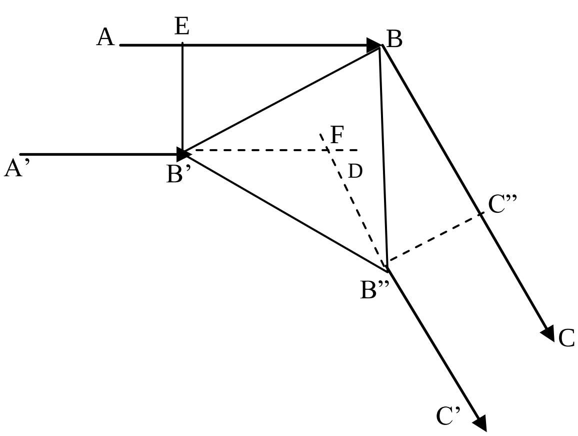


Figure II.13.2.1.3   Expanded view of the portion ABC,A’B’B”C’ of Figure II.13.2.1.2.

In this figure we produce the incident ray A’B’ forward and the emergent ray B”C’ backward to intersect at F.   The angle between the two rays is the angle of minimum deviation D.   From B’ draw a perpendicular B’E to the incident ray AB.   The width B’E is w, the width of the slit.

The triangle FB'B" is isosceles with the angle B'FB" equal to 180-D. So the angle FB'B" is D/2. Since the prism is equilateral the angle BB'B" is 60 degrees. So the angle between B'E and the face BB' is [90 - (60-D/2)] = 30+D/2. BB'E is a right angled triangle. So BB' = B'E/cos(30+D/2) or w/cos(30+D/2). Since the triangle BB"B" is equilateral, the length t of the base B'B'' = BB' = w/cos(30+D/2).

If the two yellow lines just merge for this width of the slit, the resolving power of the prism is

$$RP = t\,(dn/d\lambda)_{yellow} = [w/\cos(30+D/2)]\,(dn/d\lambda)_{yellow} \qquad (II.13.2.1.5)$$

According to theory this should be nearly equal to $\Delta\lambda/\lambda$ where $\Delta\lambda$ is the wavelength separation between the two yellow lines and λ is the mean wavelength of the yellow line.

After finding the width of the slit at which the two yellow lines just merge , the slit is placed before a traveling microscope and the width w is measured. This width came out to be 1.27 mm. From the angle of minimum deviation D for the yellow lines, one calculates the base t of the prism at the limit of resolution to be 2.6 mm. The magnitude of the dispersive power for the yellow lines is 1.53526×10^5 /m. So the resolving power is

$$RP = 2.6 \times 10^{-3} \times 153526 = 392$$

$\lambda/\Delta\lambda$ for the yellow lines of mercury is 5780/20 = 289. Thus the resolving power is nearly equal to the value of $\lambda/\Delta\lambda$.

.

II.13.2.2 RESOLVING POWER OF A GRATING

1. INTRODUCTION:

In a grating we have periodic rulings with a spacing d. The number N of such rulings per m of the grating is

$$N = 1/d \qquad (II.13.2.2.1)$$

When parallel light of wavelength λ falls on a grating, each of the transparent regions on the grating diffracts the light. The telescope collects the rays which are diffracted at an angle θ to the incident rays and brings them to a focus. This is shown in Figure II.13.2.2.1

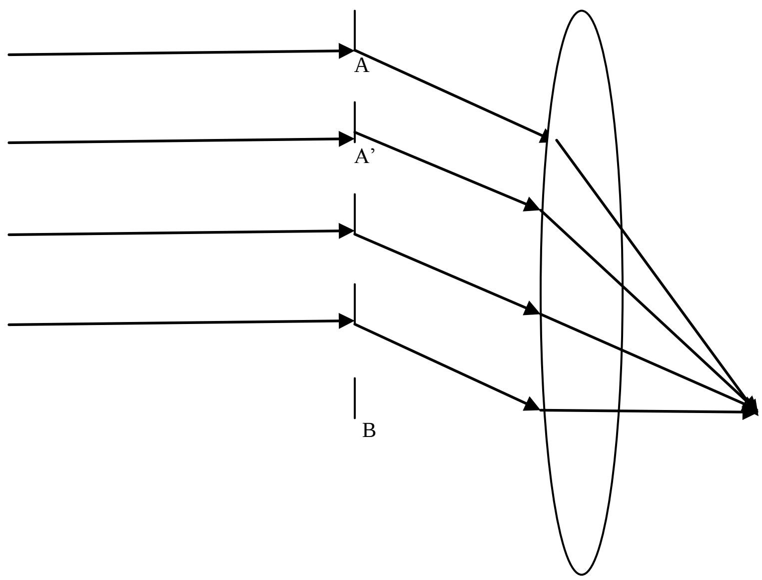


Figure II.13.2.2.1: AB is the diffraction grating. Incident parallel rays normal to the grating are diffracted through an angle θ and are brought to a focus by the objective.

The phase difference between rays diffracted from corresponding points in the adjacent slits of the grating is d sin (θ). If this is an integer n times the wavelength λ, then all the rays from corresponding points in all the transparent regions diffracted through the angle θ will be in phase and one will get maximum intensity at the focal point of the telescope. For a monochromatic beam of wavelength λ constructive interference will occur at an angle θ_1 for n = 1, at an angle $-\theta_1$ for n = -1, an angle θ_2 for n = 2, at an angle $-\theta_2$ for n =- -2, and so on. Thus we see many bright images of the slit on either side of the incident beam. n = 1 gives the first order diffraction image, n = 2 the second order diffraction image and so on. The larger the diffraction angle the higher the order. If the incident beam contains many different wavelengths each of the wavelengths produces diffraction images. We see that the shorter the wavelength of the light the smaller is the angle θ_n for the nth order diffraction beam. One should note that violet light will be diffracted at a smaller angle than red light. Order of colours from the direction of the incident beam will be from violet to red as the diffraction angle increases.

On the other hand in a prism red is deviated less than violet. So as the angle of deviation increases one goes from red to violet. The order is opposite to that in the grating.

The dispersive power of a diffraction grating is defined as

$$d\theta_n/d\lambda = n/d\cos\theta_n \qquad \text{(II.13.2.2.2)}$$

As n increases θ_n increases and $\cos\theta_n$ decreases. Thus the dispersive power of a grating increases with order. For the same order n, θ_n decreases as the wavelength λ decreases. So dispersive power of a grating decreases as the wavelength decreases – a result opposite to that in a prism.

If we have two close wavelengths, then the images of the slit formed at these two wavelengths by diffraction at a plane grating will not appear as distinct when the wavelength difference becomes smaller than a certain value $\Delta\lambda$. The resolving power of the grating is defined as

$$\lambda/\Delta\lambda$$

where $\Delta\lambda$ is the difference in wavelength for which the two images just merge with each other. Using the criterion that this will happen when the principal maximum of the diffracted image of the slit at wavelength $\lambda + \Delta\lambda$ falls on the first minimum of the diffracted image at the wavelength λ, theory leads to the result that

$$\lambda/\Delta\lambda = nN' \qquad \text{(II.13.2.2.3)}$$

where N' is the total number of lines in the grating, diffracted rays from which reach the telescope objective. Thus the resolving power increases with the order of diffraction.

In this experiment we will determine the wavelength of the different lines of mercury light by measuring the diffraction angles θ_1 in the order n = 1 and θ_{-1} in the order -1. We will also measure the resolving power of the grating for the yellow lines 5770 and 5790 A of mercury in these two orders and compare it with the theoretical result.

2. PROCEDURE:

The procedure for adjusting the lens to produce a collimated beam of rays is the same as for the case of the prism. The grating has to be set perpendicular to the incident rays. In the given set up we have received from Holmarc, the prism table keeps rotating and cannot be fixed rigidly. So we use the following procedure. Mount the grating in the given mount and adjust the grating so that it is nearly normal to the incident rays as seen by the eye. View the green line of mercury in the order n =1 by turning the telescope clockwise. Fix the cross wire on the green line and take the scale reading on the telescope table. Then rotate the telescope anticlockwise to see the green line in the n = -1 order. The angle measured from the incident beam direction will be the same if the orientation of the grating is correct. If the grating is mis-oriented the two angles will be different. Turn the prism table slightly so that the two angles become the same within 30' of arc of the telescope scale reading. Now the mis-orientation of the grating will be less than 15'of arc. After this adjustment do not touch the prism table.

Measure the angles θ_1 for the different colours starting from the yellow lines and moving towards the violet. The telescope will be turned from left to right. Proceed further and take the readings θ_{-1} for the n =-1 order staring from violet to yellow. A sample set of readings are given below.

TABLE II.13.2.2.1

Wavelengths of mercury lines using a plane grating

Diffraction Grating					
No. of lines/m		1000000			
Lines of					
Mercury	θ_1 deg	θ_{-1} deg	θ_{av} deg	λ_{calc} in m	Actual λ in m
Yellow1	35.67	35.00	35.335	5.784E-07	5.790E-07
Yellow2	35.55	34.90	35.225	5.768E-07	5.770E-07
Green	33.50	32.80	33.150	5.468E-07	5.461E-07
Blue	26.00	25.67	25.835	4.358E-07	4.358E-07
Violet	24.00	23.75	23.875	4.048E-07	4.046E-07

The wavelength of the different lines are within about 7 A from the correct result.

3. RESOLVING POWER OF THE GRATING:

Fix the telescope on the yellow lines of mercury in the first order. Mount a slit of adjustable width on the optic bench after the collimating lens. Adjust the slit width till the two yellow lines appear to merge into a single broad line. When the slit is reduced in width the intensity of the lines will go down and fixing this position requires care. Let w be the width of the slit as measured with a travelling microscope. Repeat the measurement for the yellow lines in the n = -1 order. Let the width be w'. Take the average of w and w'. Let this be w_{av}. Then the width of the grating from which rays are received by the telescope through the slit is W = w_{av}. The number N' of lines in the grating of width W is

$$N' = WN$$

Compare this with $\lambda/\Delta\lambda$.

Sample data are given below

RESOLVING POWER OF AGRATING

Ist Order to the left	w	Slit width	0.035	cm
Ist Order to the right	w'	Slit Width	0.031	cm
Average Width W of the grating through which rays pass			0.033	cm
Number of lines in the grating in this width			331	
$\lambda/\Delta\lambda$ (Theory)			289	

II.13.3 EXPERIMENT ON POLARIZATION OF LIGHT

II.13.3.1 INTRODUCTION TO POLARIZATION OF LIGHT

1. INTRODUCTION

Light is a transverse electromagnetic wave. In an isotropic medium the velocity of propagation of light is independent of the direction of propagation. We define the wave vector of a light wave as the vector **k** which has the magnitude $2\pi/\lambda$, where λ is the wavelength of the wave. The direction of **k** is the direction of propagation of the wave. If ν is the frequency of the wave**,** the phase velocity, v, of the wave is $\nu\lambda = \omega/k$, where $\omega = 2\pi\nu$. The phase velocity is the velocity with which the phase of the wave propagates. Generally light propagates as a group of waves with a slight spread in frequency. The velocity with which the group propagates is the velocity with which the energy in the group propagates. The group velocity is given by $d\omega/dk$. If the phase velocity in a material is the same at all frequencies, the medium is said to be non-dispersive. Vacuum is such a medium. In such a non-dispersive medium, the phase velocity is equal to the group velocity. For vacuum this velocity is $c = 2.998\text{x}10^8$ meters per second. All media, other than vacuum, are dispersive. In such media ω does not vary linearly with k. So the phase velocity is different from the group velocity. The refractive index of a medium for light of wavelength λ, is the ratio of the velocity of light in vacuum to the phase velocity of light (of this wavelength) in the medium. The refractive index is denoted by the symbol n and is greater than 1 except in the close vicinity of an absorption wavelength of the medium. For an isotropic medium n is independent of the direction of propagation of the light.

In a light wave there are electric displacement vector **D** and magnetic induction field vector **B** varying with position and time. **These two fields are in a plane perpendicular to k.** That is why light is called a transverse electromagnetic wave. .Associated with the electric displacement **D** is the electric field **E.** With the magnetic induction field **B** is associated the magnetic intensity field **H**. **D** and **B** are in a plane perpendicular to the propagation vector **k. D** and **B** are also mutually orthogonal. In an isotropic medium **E** is parallel to **D** and **H** is parallel to **B.** This will not be so in an anisotropic medium.

In the plane, perpendicular to the direction of propagation, we may choose two unit vectors **i** and **j** which are mutually perpendicular. Then the electric displacement vector **D** can be written as

$$\mathbf{D} = D_i\,\mathbf{i} + D_j\,\mathbf{j} \qquad \text{(II.13.3.1)}$$

If $D_j = 0$, the displacement vector is in the direction of **i** and if $D_i = 0$, it is in the direction **j**. These two states are said to be two independent states of **polarization** of the light. In general, the phase velocity of light may depend on (i) direction of propagation, (ii) direction of polarization in the given direction of propagation and (iii) the wavelength of the light.

2. ISOTROPIC, UNIAXIAL AND BIAXIAL CRYSTALS

In an **isotropic** medium, the phase velocity of light does not depend on the direction of propagation and the direction of polarization. It depends only on the wavelength of light. There is a unique refractive index n which only depends on wavelength. Examples of such materials are glass, a liquid like water, or crystals which belong to the cubic system like diamond, sodium chloride or calcium fluoride.

In a **uni-axial** material, there is a unique direction of propagation for which the refractive index does **NOT** depend on the direction of polarization. For any other direction of propagation the velocity depends on the state of polarization. Light polarized perpendicular to the direction of the unique axis will travel with a phase velocity independent of the direction of propagation. The refractive index for this state of polarization is called n_o and the corresponding light ray in the direction of propagation is called the **Ordinary ray.** For light polarized in a direction **not perpendicular to the unique axis,** the refractive index will vary with the direction of propagation. The corresponding ray is called the **Extraordinary ray.** If light is propagating perpendicular to the unique axis, the polarization of the extraordinary ray will be parallel to the unique axis and the corresponding refractive index, n_e, is called the extraordinary refractive index. For any direction of propagation inclined to the unique axis, the extraordinary refractive index will have a value between n_o and $n_{e.}$ Light propagation in a uni-axial crystal is completely determined by the two values n_o and n_e of the refractive index of the material. Crystals belonging to the tetragonal, trigonal or hexagonal crystal systems are optically uni-axial. The optic axis (ie. the direction of propagation for which the refractive index is n_o for both states of polarization) is along the four-fold, three-fold or six-fold axis of symmetry, respectively, of the crystal. Examples of uni-axial crystals are Rutile (tetragonal TiO_2), Calcite (trigonal $CaCO_3$) and quartz (hexagonal SiO_2).

In a **bi-axial** material the phase velocity in any direction of propagation is dependent on the state of polarization. There are three principal refractive indices for such a material corresponding to three mutually perpendicular directions of propagation X', Y'and Z'. These indices are labeled n_1, n_2 and n_3. Crystals belonging to orthorhombic, monoclinic and triclinic systems are in general biaxial. Light propagation in these crystals is very complicated. New phenomena like internal and external conical refraction arise in these materials. For a discussion of these phenomena one should refer to text books in Optics.

3. STATES OF POLARIZATION LIGHT

The electric displacement vector **D** at a point **r** in a light wave, propagating along a direction **k,** can be written in general as

$$\mathbf{D}(t) = (D_1\mathbf{i} + D_2 \exp(i\phi)\,\mathbf{j}) \exp(i\omega t) \qquad \text{(II.13.3.2)}$$

Here **i** and **j** are two orthogonal unit vectors in the plane perpendicular to the direction of propagation. We may now define different states of polarized light:

a) **Unpolarized light**

For un-polarized light $D_1 = D_2$ and ϕ, the phase difference between the displacements along **i** and **j,** varies randomly with time. The intensity of the light beam will be proportional to $2D^2$ where D is the value of $D_1 = D_2$.

Light from a thermal source like a sodium or mercury vapour lamp or sun light is unpolarized.

b) **Linearly polarized light**

If $\phi = 0$ (ie. if the two displacements along **i** and **j** vibrate in phase) then the light is linearly polarized. The displacement vector oscillates in the **ij** plane along a line making an angle α with the **i** axis given by

$$\mathrm{Tan}(\alpha) = D_2/D_1 \qquad \text{(II.13.3.3)}$$

Such linearly polarized light is shown in Figure II.13.3.1(a).

If $\phi = \pi$ (ie. if the two displacements along **i** and **j** vibrate 180^0 out of phase) then also the light is linearly polarized. Now the displacement vector oscillates in the **ij** plane along a line making an angle α with the **i** axis given by

$$\mathrm{Tan}(\alpha) = -\,D_2/D_1 \qquad \text{(II.13.3.3)}$$

If $D_2 = 0$, the light is polarized along the **i** axis. If D_1 is zero the light is polarized along the **j** axis. If $\phi = 0$ and $D_1 = D_2$ the light is polarized in a direction making 45^0 to the **i** axis. If $\phi = \pi$ and $D_1 = D_2$ the light beam is polarized in a direction making 135^0 to the axis **i**..

The intensity of the light beam is proportional to $(D_1^2 + D_2^2)$.

c) **Elliptically polarized light**

If ϕ is different from zero or π, and $D_1 \neq D_2 \neq 0$, then light is elliptically polarized. The tip of the displacement vector goes round an ellipse in one period. The directions of the principal axes of the ellipse are in general inclined to the unit vectors **i** and **j**. The angle of inclination and the ratio of the semi-principal axes of the ellipse will depend on ϕ, D_1 and D_2. The intensity of the light beam is proportional to the sum of the squares of the semi-minor and semi-major axes. Such an elliptically polarized light is shown in Figure II.13.3.(b).

If $\phi = \pi/2$ or $\phi = 3\pi/2$, the principal axes of the ellipse are along **i** and **j**. The ratio of the semi-principal axes is D_1/D_2. If $D_1 < D_2$ the minor axis is along **i**. If $D_1 > D_2$ the minor axis is along **j**. If $\phi = \pi/2$, the tip of vector **D** goes round the ellipse in a clockwise direction. This is shown in Figure II.13.3.1(c). If $\phi = 3\pi/2$, the tip of vector **D** goes round the ellipse in an anti-clockwise direction. This is shown in Figure II.13.3.1(d).

In these cases the intensity of the light beam is proportional to $D_1^2 + D_2^2$.

d) **Circularly polarized light**

This is a special case of elliptically polarized light when $\phi = \pi/2$ or $3\pi/2$ and $D_1 = D_2 = D$. Here the tip of the displacement vector moves on a circle of radius D. The tip goes round the circle clockwise when $\phi = \pi/2$ (Figure II.13.3.1(e)) and anti-clockwise when $\phi = 3\pi/2$ (Figure II.13.3.1(f)).

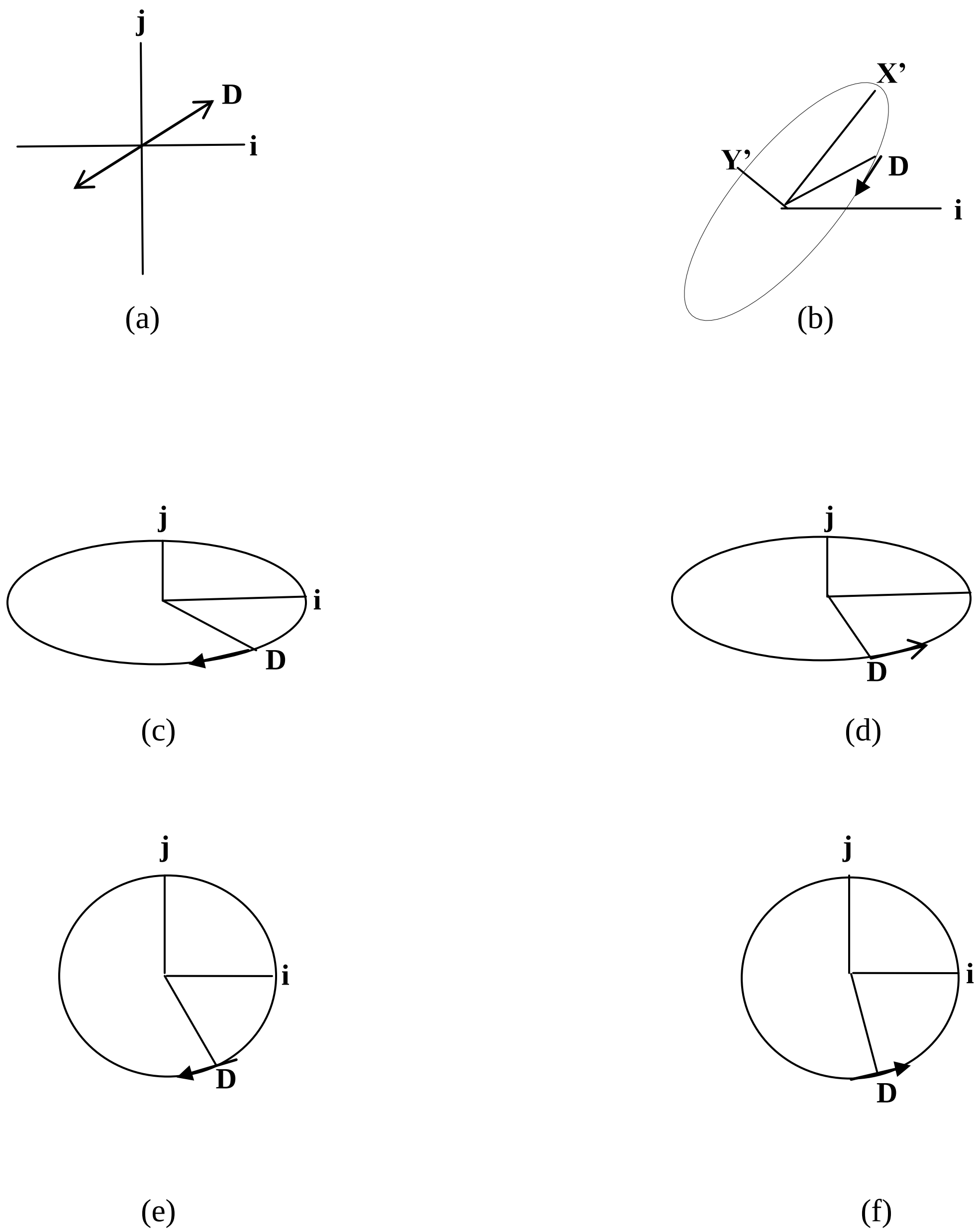


Figure II.13.3.1(a to f) Different states of polarized light

4. POLARIZER AND QUARTER WAVE PLATE

To produce different states of polarized light from unpolarized light we use devices called a Polarizer and a Quarter wave plate.

There are different types of polarizers. The least expensive polarizer is a Polaroid sheet. This is a transparent sheet coated with a material. This material will completely absorb light polarized in one direction and allow light polarized in a perpendicular direction to go through. The direction of polarization of the light which is transmitted with maximum intensity is called the axis of the polarizer. If unpolarized or polarized light falls on this Polaroid sheet, it will allow the component of the light with the displacement vector parallel to its axis to pass through. The light coming out of the sheet will be linearly polarized with the displacement vector parallel to its axis. This is how linearly polarized light is produced. If we have a second Polaroid sheet placed after the first one in the path of the light beam, no light will be transmitted if the axis of the second sheet is perpendicular to the axis of the first sheet. The two Polaroid sheets are said to be crossed.

To produce elliptically or circularly polarized light from an incident linearly polarized beam one uses a quarter wave plate. This is made of a uni-axial crystal like Quartz. A plate is cut with the optic axis parallel to the surface of the plate. If linearly polarized light falls on the plate, and the displacement vector in the incident light makes an angle β with the optic axis of the plate, the displacement vector **D** will be resolved into two components $D_1 = D\cos(\beta)$ parallel to the optic axis and $D_2 = D\sin(\beta)$ perpendicular to the optic axis. If the incident beam is normal to the plate, both components will travel normal to the plate within the plate. The first component will travel as the extraordinary ray with refractive index n_e and the second component will travel as the ordinary ray with refractive index n_0. When they come out they will have a phase difference

$$\phi = (2\pi/\lambda)(n_e - n_o)t \qquad \text{(II.13.3.4)}$$

Here t is the thickness of the plate and λ is the wavelength of light. In general the emergent light will be elliptically polarized with its principal axes at an angle to the Optic axis of the plate.

If we choose the thickness t such that ϕ is $\pm \pi/2$, the emergent light will be elliptically polarized with one of its principal axis parallel to the optic axis of the plate. The plate is called a Quarter Wave Plate (QWP) for the wavelength λ. A QWP for wavelength λ will not act as a QWP for a different wavelength λ'.

If we use a QWP for the incident wavelength λ and orient it to make an angle $\beta = \pi/4$ to the polarization direction of the incident light then the emergent light will be circularly polarized.

Thus using a polarizer and a QWP, we can produce light in different states of polarization from incident unpolarized light.

5. ANALYSIS OF POLARIZED LIGHT:

How do we analyze light to find its state of polarization? This can be done by using a Polaroid. We place the analyzer Polaroid in the path of the beam and find the intensity variation as the Polaroid is rotated.

If the intensity of the light coming out of the analyzer does not change as the analyzer is rotated, the incident beam is either unpolarized or circularly polarized. To distinguish between the alternatives we need a QWP designed for the wavelength of light used. We place this QWP in the path of the light beam before the analyzer. If the analyzer is now rotated, and the intensity is found to remain constant, the incident light is un-polarized. A circularly polarized incident beam will emerge from a QWP as a linearly polarized beam. When the analyzer is rotated the intensity will vary from a maximum to zero.

If the intensity of the light beam is viewed through an analyzer, and the intensity varies from a maximum to a minimum value as the analyzer is rotated, the incident beam is elliptically polarized. When the transmitted intensity is a maximum, the axis of the analyzer Polaroid is parallel to the major axis of the ellipse. The minor axis of the ellipse is perpendicular to this direction. The length of the major axis of the ellipse is proportional to the square root of the maximum intensity and the length of the minor axis of the ellipse is proportional to the square root of the minimum intensity. When the Polaroid is rotated through an angle θ from the setting for maximum intensity, the intensity I(θ) of the light transmitted through the polaroid varies as

$$\mathbf{I(\theta) = I_{max}\cos^2(\theta)+I_{min}\sin^2(\theta)} \qquad \text{(II.13.3.5)}$$

If the light is linearly polarized then $I_{min} = 0$. No light comes through because the axis of the analyzer is at right angles to the polarization direction of the incident light. If we turn the analyzer through an angle θ from this crossed direction, the intensity I(θ) of the transmitted light will vary as

$$\mathbf{I(\theta) = I_{max}\sin^2(\theta)} \qquad \text{(II.13.3.6)}$$

In the next chapter we describe the experiment for analysis of linear and elliptically polarized light.

III.13.3.2 EXPERIMENT FOR DETECTION OF LINEAR AND ELLIPTICALLY POLARIZED LIGHT

1. INTRODUCTION:

In the previous chapter an introduction to different states of polarized light was given. In this chapter we will describe an experiment to produce linearly and elliptically polarized light and analyze the light so produced quantitatively.

For this purpose we use a 5 mW laser diode emitting green light at 550 nm wavelength, a diode detector to measure the intensity of the light, two Polaroid sheets on a rotatable mount to serve as a polarizer and analyzer and a quarter wave plate for wavelength 589 nm. Since the laser wavelength is different from the wavelength for which the QWP is designed, the QWP will not produce a phase difference of $\pi/2$ for the components of the electric field of the laser light resolved along the fast and slow axes of the QWP. The phase difference ϕ produced by the QWP at the laser wavelength will be

$$\phi = (\pi/2)(589.3/550) = 1.683 \text{ radians}$$

All the components namely the laser, the polarizer, QWP, analyzer and the detector are mounted on an optic bench.

2. LINEARLY POLARIZED LIGHT:

We place the laser, the polarizer, the analyzer and the detector in this sequence on the optic bench. We turn the analyzer on the rotatable mount till the detector current is zero in the micro-amp range. At this position the polarizer and analyzer are crossed ie. the axis of the analyzer is at right angles to the axis of the polarizer.

From this position the analyzer is rotated in steps of 10^0 and the detector current is noted.. This is done till the analyzer is rotated through 360^0. A sample set of data is given in Table II.13.3.2.1.

Table II.13.3.2.1

Analysis of Linearly Polarized Light

Polarizer and analyzer are initially crossed.
Analyzer is rotated in steps of 10 degrees and intensity readings are taken

Crossed position of analyzer	275 Degrees
I_{max}	1800 μa

Setting	θ	I μamp	$\sin^2\theta$	Ical	Setting	θ	I μamp	$\sin^2\theta$	Ical
275	0	0.1	0.0000	0.0	85	-190	39	0.0302	54.3
265	-10	44	0.0302	54.3	75	-200	128	0.1171	210.7
255	-20	157.8	0.1170	210.6	65	-210	400	0.2501	450.2
245	-30	500	0.2500	450.0	55	-220	600	0.4133	743.9
235	-40	700	0.4132	743.8	45	-230	900	0.5870	1056.5
225	-50	1000	0.5869	1056.3	35	-240	1200	0.7501	1350.2
215	-60	1300	0.7500	1350.1	25	-250	1400	0.8831	1589.6
205	-70	1600	0.8830	1589.5	15	-260	1700	0.9699	1745.8
195	-80	1800	0.9699	1745.8	5	-270	1700	1.0000	1800.0
185	-90	1800	1.0000	1800.0	355	-280	1700	0.9698	1745.6
175	-100	1700	0.9698	1745.7	345	-290	1400	0.8829	1589.2
165	-110	1500	0.8830	1589.4	335	-300	1200	0.7498	1349.7
155	-120	1200	0.7499	1349.9	325	-310	900	0.5866	1056.0
145	-130	900	0.5867	1056.1	315	-320	600	0.4130	743.4
135	-140	600	0.4131	743.6	305	-330	300	0.2498	449.7
105	-170	21	0.0301	54.2	275	-360	0.5	0.0000	0.0
95	-180	0.9	0.0000	0.0					

The first column in the table gives the angular reading of the rotatable mount of the analyzer. The second column gives the rotation in degrees from the crossed analyzer setting. The third column gives the detector current in μ amp. The fourth column gives $\sin^2\theta$. The last column gives $I_{max} \sin^2\theta$.

Since the analyzer is in the crossed position when the transmitted intensity is zero, the light coming from the polarizer has its electric vector perpendicular to the analyzer axis. When the analyzer is turned through an angle θ the incident electric displacement vector makes an angle

(90-θ) with the axis of the analyzer. So the component of the electric displacement vector parallel to the analyzer axis is D sinθ and the intensity $I_{max} \sin^2\theta$, where I_{max} is the maximum detector current when θ= 90 ie. when the axes of the polarizer and analyzer are parallel. Figure II.13.3.2.1 shows the intensity as a function of θ, with the continuous curve showing the plot of $I_{max} \sin^2\theta$. This plot shows how the intensity varies with the angle of rotation of the analyzer for linearly polarized light.

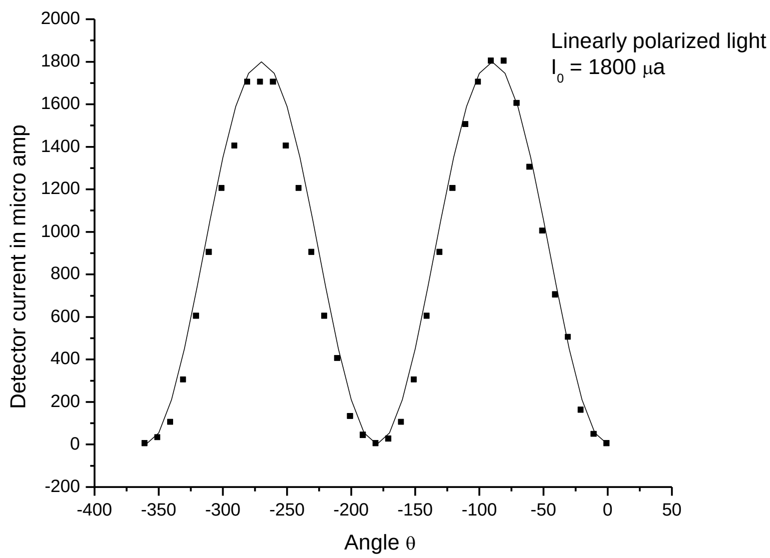


Figure II.13.3.2.1:Variation of transmitted intensity as a function of the angle of rotation from the crossed position of the analyzer for linearly polarized light.

3. Elliptically Polarized Light:

Interpose the quarter wave plate between the polarizer and analyzer in the crossed position. Rotate the quarter wave plate till the detector shows zero current. Then one axis of the QWP is parallel to the axis of the polarizer and the other axis is perpendicular to it. In this case light emerges from the QWP with its state of polarization unchanged. The QWP is now rotated through some angle ϕ from this position. The light emerging from the QWP will be elliptically polarized with its principal axes at an angle to the polarizer and analyzer axes. Now rotate the analyzer in steps of 10^0 through 360^0 and note the detector reading for each position of the analyzer. A sample set of data is given in Table II.13.3.2.2.

Table II.13.3.2.2 Analysis of elliptically polarized light

Polarizer setting			164	Degrees	
Analyzer setting in crossed position			172	Degrees	
QW plate setting to get null intensity			225	Degrees	
QW plate turned to			235	Degrees	
Θ_0			270	Degrees	
I_{max}			12.7	mA	
Imin			0.2	mA	
an.set	$(\Theta-\Theta_0)$	detCurr I	$I_{max}\cos^2(\Theta-\Theta_0)$	$I_{min}\sin^2(\Theta-\Theta_0)$	$I(\Theta)$ cal
172	-98	0.9	0.246192862	0.196122947	0.442316
182	-88	0.2	0.015421823	0.199757137	0.215179
192	-78	0.3	0.548746084	0.191358329	0.740104
202	-68	1	1.781834517	0.171939614	1.953774
212	-58	2.3	3.565948397	0.143843332	3.709792
222	-48	4.3	5.685882704	0.11045854	5.796341
232	-38	7.5	7.885924777	0.075812208	7.961737
242	-28	9.1	9.900699122	0.044083478	9.944783
252	-18	10.7	11.48717778	0.019099563	11.50628
262	-8	12.7	12.45399507	0.003874093	12.45787
272	2	12.1	12.68453066	0.000243612	12.68477
282	12	11.9	12.15097669	0.008646036	12.15962
292	22	10.9	10.91769198	0.028067843	10.94576
302	32	9.3	9.133438944	0.056166316	9.189605
312	42	7.6	7.01343938	0.089552136	7.102992
322	52	5.3	4.813413823	0.124198208	4.937612
332	62	3.7	2.798735772	0.155925421	2.954661
342	72	1.8	1.212421574	0.180906747	1.393328
362	92	0.2	0.015516925	0.199755639	0.215273
372	102	0.4	0.549300606	0.191349597	0.74065
382	112	1.1	1.78278157	0.1719247	1.954706
392	122	2.7	3.567173746	0.143824036	3.710998

Table II.13.3.2.2 (Contd)

an.set	$(\Theta - \Theta_0)$	detCurr I	$\text{Imax}\cos^2(\Theta - \Theta_0)$	$\text{Imin}\sin^2(\Theta - \Theta_0)$	$I(\Theta)$ cal
402	132	4.7	5.687238543	0.110437188	5.797676
412	142	6.5	7.88724756	0.075791377	7.963039
422	152	8.1	9.901829292	0.04406568	9.945895
432	162	9.2	11.48797901	0.019086945	11.50707
442	172	11.1	12.45437072	0.003868178	12.45824
452	182	10.7	12.68443541	0.000245112	12.68468
462	192	10.4	12.15042203	0.008654771	12.15908
472	202	9.8	10.91674482	0.028082759	10.94483
482	212	8.5	9.132213532	0.056185614	9.188399
492	222	6.8	7.012083527	0.089573488	7.101657
502	232	4.6	4.812091075	0.124219038	4.93631
512	242	3	2.797605684	0.155943218	2.953549
522	252	1.5	1.21162046	0.180919363	1.39254
532	262	0.6	0.245441566	0.196134778	0.441576

Column 1 gives the setting of the analyzer. Column 3 gives the detector current in μa. From the readings we note that the maximum detector current I_{max} is 12.7 mamp, and the minimum detector current I_{min} is 0.2 mamp. The maximum current occurs for the analyzer setting around 265^0. When the detector current is a maximum, the axis of the analyzer is parallel to the major axis of the elliptically polarized light. Call this setting Θ_0. At a setting Θ of the analyzer, the major axis of the ellipse makes an angle $(\Theta - \Theta_0)$ with the analyzer axis. This is given in column 2. The intensity of the light passing through the analyzer should vary as

$$I(\Theta) = I_{max}\cos^2(\Theta - \Theta_0) + I_{min}\sin^2(\Theta - \Theta_0)$$

Columns 4,5and 6 give the values of $I_{max}\cos^2(\Theta - \Theta_0)$, $I_{min}\sin^2(\Theta - \Theta_0)$ and $I(\Theta)$ using the values of I_{max}, I_{min} and Θ_0 given above the table. Figure II.13.3.2.2 compares the measured intensity $I(\Theta)$ with the value calculated from the above formula.

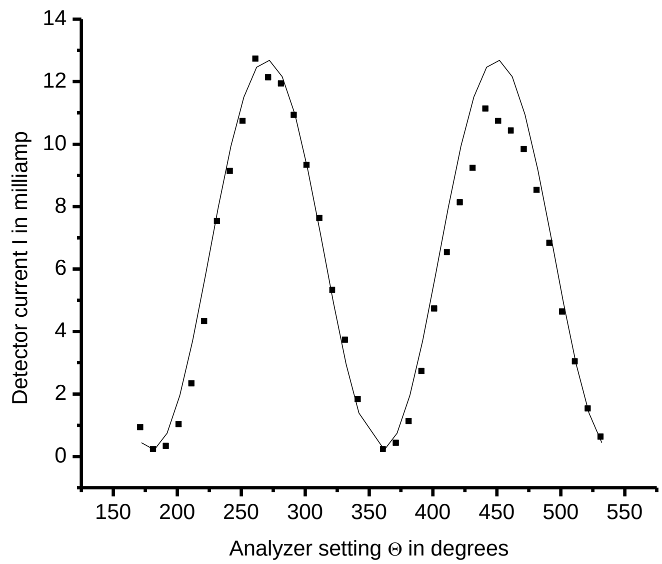


Figure II.13.3.2.2: Comparison of the transmitted intensity as a function of the setting Θ of the analyzer for an elliptically polarized light. The parameters of the fit are $\Theta_0 = 270^0$, I_{max}= 12.7 ma; I_{min} = 0.2 ma.

Thus the major axis of the ellipse is parallel to the axis of the analyzer set at 270^0. The ratio of the semi-minor to the semi-major axis of the ellipse is $b/a = (I_{min}/I_{max})^{0.5} = (0.2/12.7)^{1/2} = 0.125$.

We repeat the experiment by setting the QWP at 240^0, 250^0 and 275^0. The values of Θ_0 and (b/a) are given in Table II.13.3.2.3 for the different settings of the quarter wave plate.

Table II.13.3.2.3

QWP SETTING	Θ_0	b/a
235	270	0.125
240	282	0.198
250	302	0.327
275	365	0.415

The parametric equation for an ellipse referred to its principal axes is

$$X' = a \cos(\eta)$$

$$Y' = b \sin(\eta) \qquad \text{(II.13.3.2.1)}$$

If X’ and Y’ are calculated for all values of η from 0 to 2π and the points (X’,Y’) are plotted, they lie on an ellipse with semi-principal axes a and b. Θ_0-β gives the angle which X’ makes with the fixed X axes. β is the setting of the analyzer when it is crossed with the polarizer. $\beta = 172^0$ in this case. We can get (X,Y) from (X’,Y’) using the transformation equations

$$X = X' \cos(\Theta_0-\beta) - Y' \sin(\Theta_0-\beta)$$

$$Y = X' \sin(\Theta_0-\beta) + Y' \cos(\Theta_0-\beta) \qquad \text{(II.13.3.2.2)}$$

The ellipses plotted thus are shown in Figures II.13.3.2.3 (a) to (d).

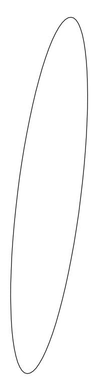


Figure II.13.3.2.3 (a)

$\Theta_0 = 235^0$

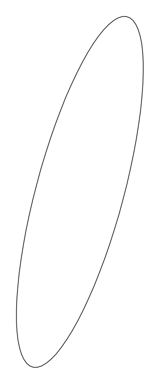


Figure II.13.3.2.3 (b)

$\Theta_0 = 240^0$

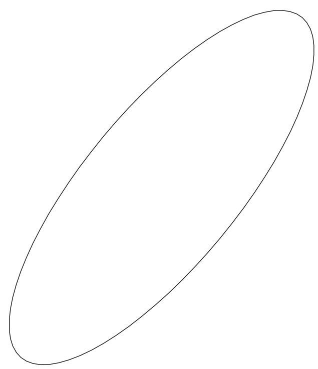


Figure II.13.3.2.3 (c)

$\Theta_0 = 250^0$

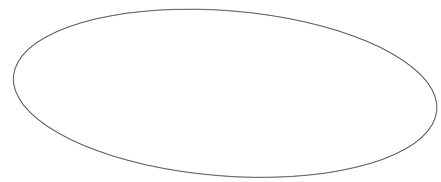


Figure II.13.3.2.3 (d)

$\Theta_0 = 275^0$

These pictures show that as the QWP settings are changed the major axis of the ellipse rotates clockwise and the ratio (b/a) increases.

II.14.1 e by Shot Noise

1. Introduction:

When electrons are generated by photo- or thermionic emission the electrons come out at random. The random generation of electrons follows Poisson' distribution. If on an average $<N>$ electrons are generated in one second, there are mean square fluctuations about the average value. The root mean square fluctuation $(<(N-<N>)^2>)^{1/2}$ is $(<N>)^{1/2}$ for this distribution. Since the generation is random, the power spectrum S(f) of the fluctuation is independent of the frequency f, giving rise to a WHITE NOISE called Shot noise.

If these electrons pass through a resistance R, then there is an average current

$$<I> = <N> e \qquad \text{(II.14.1.1)}$$

producing an average voltage $<V> = R<I>$. The voltage across R fluctuates with time. The power spectrum for the mean square voltage fluctuation $S_v(f)$ is

$$S_v(f) = 2eR^2 <I> \qquad \text{(II.14.1.2)}$$

If we have an amplifier with gain G(f), then the mean square fluctuation of the amplified voltage is

$$v_{rms}^2 = 2eR^2<I> \int G^2(f)\, df \qquad \text{(II.14.1.3)}$$

over the bandwidth of the amplifier. If we denote the integral by G^2BW, and replace R by $<V>/<I>$, we get

$$v_{rms}^2 = 2e(<V>^2/<I>)\, G^2BW \qquad \text{(II.14.1.4)}$$

For a given $<I>$, varying R will give v_{rms} proportional to $<V>$. The slope of $v_{rms}/<V>$ is α ωhich will be a function of $<I>$ for an amplifier with a given G^2BW. A plot of α^2 against $1/<I>$ will be a straight line with a slope β given by

$$\beta = 2eG^2BW$$

If we measure G^2BW we can get the value of the electronic charge e. This is the principle of the experiment.

2. Noise Detector Circuit

The block diagram of the Noise detector circuit is shown below

The input to the detector may be a signal from a signal generator for calibration or the fluctuating voltage across the resistor. This is amplified 200 times in a preamplifier. There are three stages of amplification (2, 10 and 10). The output from the pre-amplifier passes through a band-pass filter. This

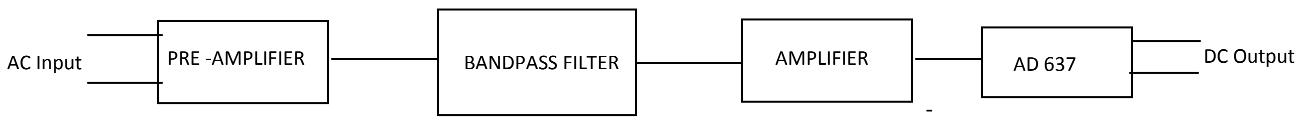


Figure II.14.1.1: Block diagram of the Shot Noise detector

is a Butterworth filter consisting of two stages of an active high-pass filter followed by two stages of an active low-pass filter. If there are two stages and each produces an attenuation ε at a frequency f, the total attenuation of the two stages will be ε^2 leading to sharper rise of the transmission coefficient at the low frequency end and sharper decline at the high frequency end of the pass band.

The signal then goes through a final amplifier which amplifies by a factor of 5 before entering the chip AD 637. This chip will give a DC output which is the true RMS vale of the AC input *whatever be the waveform of the input.* This output is measured on a DMM.

3. Front Panel diagram of the noise detector circuit:

The lay out of the noise detector circuit is shown schematically in Figure II.14.1.2.

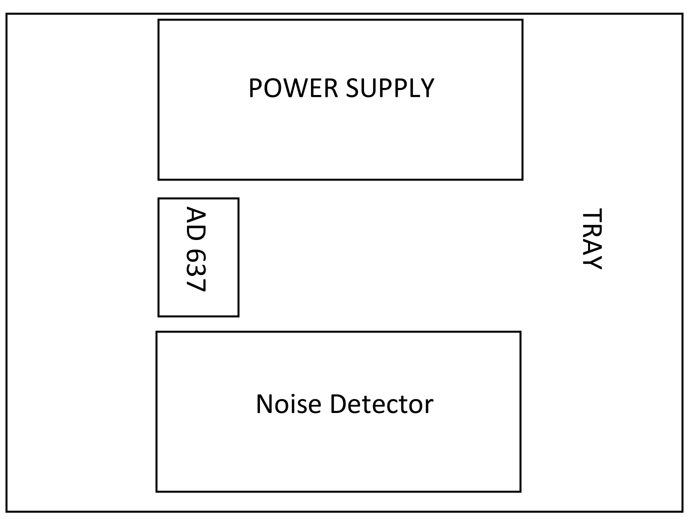


Figure II.14.1.2: Lay out of the Noise Detector

In a tray three boxes are mounted. In the rear is the split power supply which provides the power to the noise detector and the AD 637 box. The power connections are made through cables with banana plug terminations. The noise detector box is mounted in the front and the box containing AD 637 is mounted between the power supply and noise detector.

The front panel of the noise detector is shown in Figure II.14.1.3.

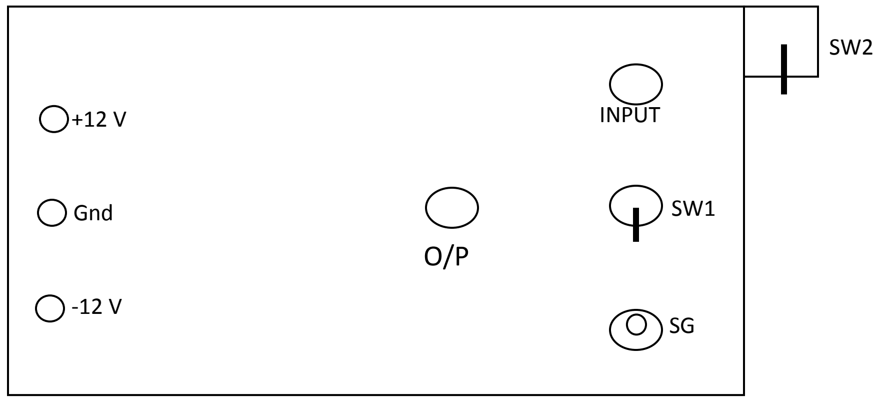


Figure II.14.1.3: Front Panel of Noise Detector Circuit

On the left of the front panel are three banana terminals (Red, Green and Black) marked +12 V, Gnd and -12 V respectively. These are connected to the corresponding banana terminals on the split power supply in the rear of the tray. Slightly to the right of the center is the B & C connector marked O/P. The amplified AC output of the noise detector appears at this B&C connector. This has to be connected to the input of the AD 637 box. There is a B&C connector on the right top of the box marked INPUT. This is to be connected to the corresponding B&C connector on the Noise source to be described below. There is a DPDT switch SW1 below the B&C connector. When calibrating the noise detector this switch is put down. When measuring the external noise this switch is put up. There is a RCA socket below SW1. During calibration the signal generator output is connected to this RCA terminal. Inside the box there is a potential divider which feeds $1/1000^{th}$ of the input voltage of the signal generator to the broad band amplifier. There is a second DPDT switch SW2 attached to the top right corner of the front panel. During calibration, the switch should be put down to read the RMS input from the signal generator. It is put up when the amplified output of 1/1000 th of the signal generator voltage is to be read. During measurement of External noise this switch is in the UP position.

The front panel of the AD 637 box is shown in Figure II.14.1.4.

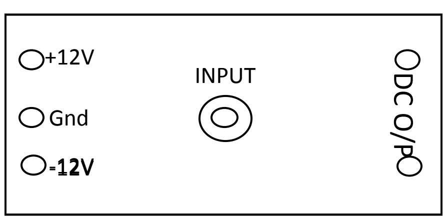


Figure II.14.1.4: Front panel of AD 637 box

On the left there are three small banana plugs (Red, Green and Black) marked +12V, Gnd and -12V respectively. They are connected to the split power supply terminals. In the center there is a RCA socket marked INPUT. This should be connected to the B&C socket marked O/P at the center of the front panel of the Noise detector shown in Figure II.14.1.3. On the right are two small banana terminals marked DC O/P. These are connected to measure the amplified rms voltage from the output of the noise detector circuit.

4. Calibration

We have to measure the gain of the system as a function of frequency when we give a sinusoidal AC input. The AC output of a signal generator producing sine waves is connected to the RCA socket marked SG on the front panel of the noise detector circuit (Figure II.14.1.3). This signal is attenuated by a potential divider by a factor of 1000. A 1 volt AC signal from the signal generator will then generate an AC input of 1 mV. The rms value of the signal generator input is measured on the DC multimeter connected to the AD637 box when the switch SW2 is put in the DOWN position. When the switch SW2 is pushed up, the DC multimeter connected to the banana terminals on the AD 637 box will measure the rms amplified output of the noise detector circuit when it amplifies 1/1000 th part of the signal generator input. The ratio of the rms output to 1/1000 of the input value gives the gain G of the broad band amplifier. Thus one may measure the gain as a function of frequency. By measuring the AC signal from the Signal generator and the output, after amplification, on the same meter connected to the AD 637 output, we avoid any differences that may arise using two different meters for measuring the AC signal from the SG and the output.

The following table gives the frequency, V_{AC} from the signal generator in Volts and v_{rms} at the output.

$$G = (v_{rms}/V_{AC})*1000$$

The first column of the table gives the frequency of the signal generator output. This is measured by connecting a Meco 801 multimeter to the output terminals of the signal generator and arranging the dial switch on the multimeter to the position Hertz. The second column of the table gives the rms input voltage from the signal generator given at the input of the noise detector. For this measurement Switch SW2 should be down. The third column gives the rms voltage of the amplified output of $1/1000^{th}$ of the signal generator input signal. For this measurement switch SW2 must be up. The fourth column reads the gain G calculated from the formula above. The fifth column gives G^2, the square of the gain.

Table I

G^2 as a function of frequency

f kHz	V in Volts	vrms V	G	G^2
0.5	1.031	0.08	7.76E+01	6.02E+03
1	1.014	0.365	3.60E+02	1.30E+05
1.5	1.012	0.88	8.70E+02	7.56E+05
2	1.007	1.41	1.40E+03	1.96E+06
2.5	1.002	1.806	1.80E+03	3.25E+06
3	1	2.107	2.11E+03	4.44E+06
3.5	0.999	2.302	2.30E+03	5.31E+06
4	0.999	2.427	2.43E+03	5.90E+06
4.5	0.998	2.474	2.48E+03	6.15E+06
5	0.998	2.491	2.50E+03	6.23E+06
5.5	0.997	2.468	2.48E+03	6.13E+06
6	0.996	2.419	2.43E+03	5.90E+06
6.5	0.995	2.363	2.37E+03	5.64E+06
7	0.993	2.298	2.31E+03	5.36E+06
7.5	0.994	2.228	2.24E+03	5.02E+06
8	0.993	2.157	2.17E+03	4.72E+06
8.5	0.993	2.091	2.11E+03	4.43E+06
9	0.992	2.027	2.04E+03	4.18E+06
9.5	0.992	1.971	1.99E+03	3.95E+06
10	0.999	1.902	1.90E+03	3.62E+06
10.5	0.998	1.849	1.85E+03	3.43E+06
11	1.004	1.809	1.80E+03	3.25E+06
11.5	1.01	1.776	1.76E+03	3.09E+06
12	1.013	1.732	1.71E+03	2.92E+06
12.5	1	1.675	1.68E+03	2.81E+06
13	1.001	1.639	1.64E+03	2.68E+06
13.5	1.001	1.615	1.61E+03	2.60E+06
14	1.003	1.577	1.57E+03	2.47E+06
14.5	1.005	1.558	1.55E+03	2.40E+06
15	1.005	1.532	1.52E+03	2.32E+06
15.5	1.012	1.515	1.50E+03	2.24E+06
16	1.016	1.491	1.47E+03	2.15E+06
16.5	1.016	1.477	1.45E+03	2.11E+06
17	1.016	1.463	1.44E+03	2.07E+06
17.5	1.016	1.445	1.42E+03	2.02E+06

18	1.014	1.426	1.41E+03	1.98E+06
18.5	1.057	1.494	1.41E+03	2.00E+06
19	1.052	1.463	1.39E+03	1.93E+06
19.5	1.047	1.436	1.37E+03	1.88E+06
20	1.042	1.407	1.35E+03	1.82E+06
21	1.04	1.367	1.31E+03	1.73E+06
22	1.036	1.321	1.28E+03	1.63E+06
23	1.036	1.274	1.23E+03	1.51E+06
24	1.041	1.233	1.18E+03	1.40E+06
25	1.038	1.19	1.15E+03	1.31E+06
26	1.036	1.147	1.11E+03	1.23E+06
27	1.034	1.105	1.07E+03	1.14E+06
28	1.036	1.071	1.03E+03	1.07E+06
29	1.034	1.028	9.94E+02	9.88E+05
30	1.032	0.987	9.56E+02	9.15E+05
32	0.997	0.89	8.93E+02	7.97E+05
34	0.995	0.8	8.04E+02	6.46E+05
36	0.994	0.721	7.25E+02	5.26E+05
38	0.989	0.642	6.49E+02	4.21E+05
40	0.987	0.561	5.68E+02	3.23E+05
42	0.985	0.489	4.96E+02	2.46E+05
44	0.984	0.428	4.35E+02	1.89E+05
46	0.982	0.376	3.83E+02	1.47E+05
48	0.98	0.327	3.34E+02	1.11E+05
50	0.978	0.286	2.92E+02	8.55E+04
55	0.974	0.21	2.16E+02	4.65E+04
60	0.969	0.157	1.62E+02	2.63E+04
65	0.965	0.107	1.11E+02	1.23E+04
70	0.959	0.093	9.70E+01	9.40E+03

A plot of G^2 against f is shown in Figure II.14.1.5.

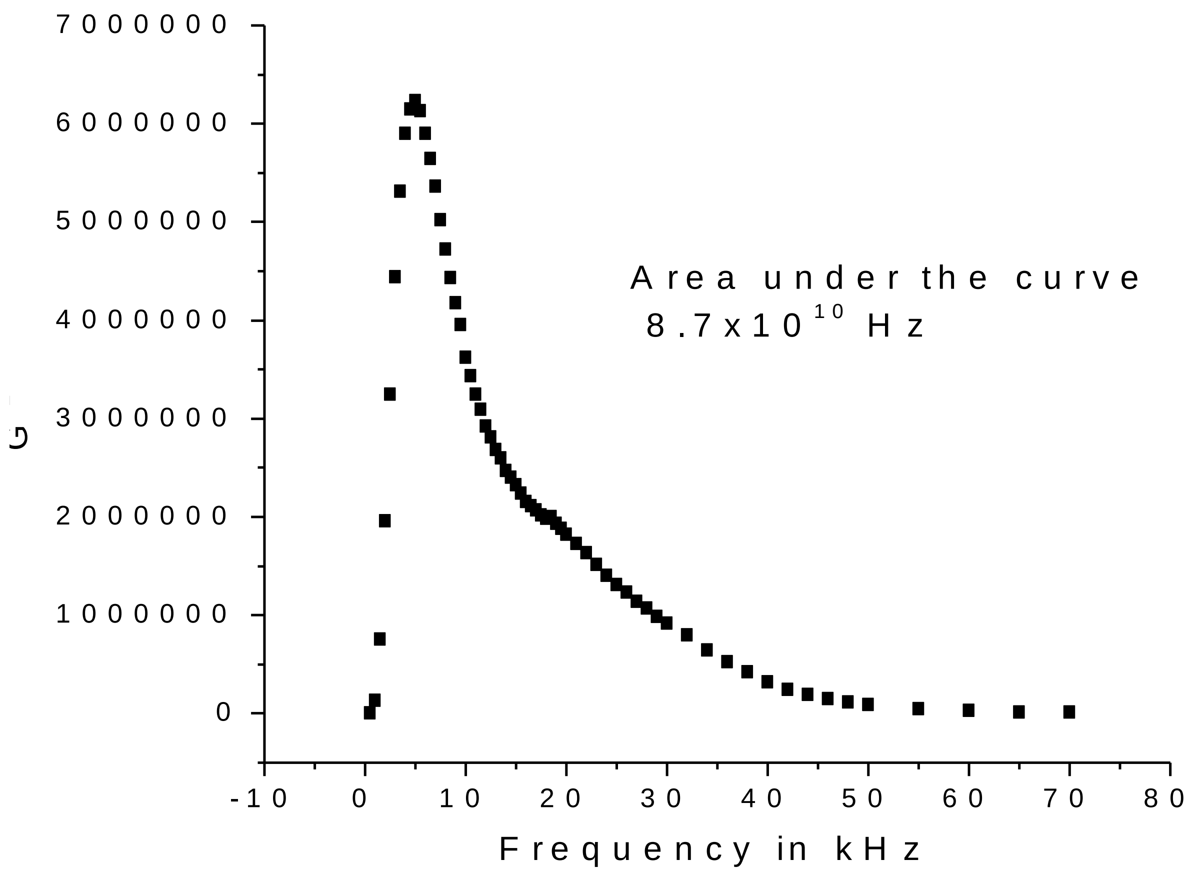


Figure II.14.1. 5: A plot of G^2 against frequency

The area under the curve gives G^2 BW. I have to redesign the Butterworth filter to get a less sharp peak in G^2 as a function of frequency. But since the noise is white the shape of this curve does not matter,

5. Shot Noise Source:

A LED is operated with a 9V battery and a potential divider. The voltage applied to the LED can be ramped up slowly by turning a POT. The brightness of the LED increases as the voltage across it increases. The light from the LED falls on a Motorola photo transistor. This is also driven by a 9V battery. The transistor is in series with a 10 k resistor and a 100k pot. The resistance connected to the transistor is a constant at 110 k. But the noise voltage is measured across a part of the resistor whose value can be changed by turning the pot. Thus we can vary the resistance (or equivalently <V>) across which noise is measured keeping the current through the circuit constant. The Dc voltage <V> will be blocked by a 0.47 microfarad capacitor. But the noise voltage will go through and will be the input to the noise detector.

The front panel diagram of the noise source is shown in Figure II.14.1.6

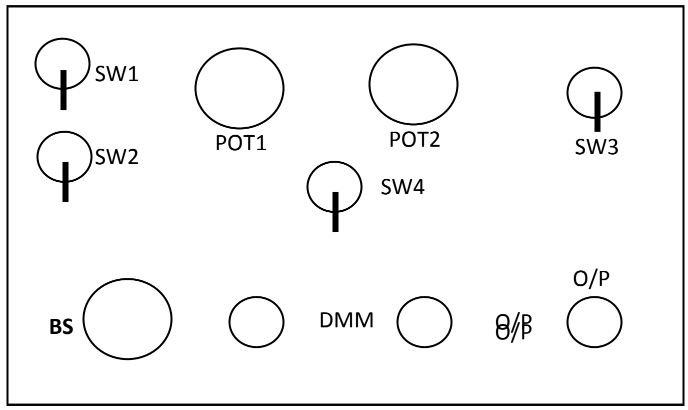


Figure II.14.1.6: Front Panel of Noise Source

There are four SPDT switches SW1 to SW4. SW1 switches on the power to the LED and SW2 the power to the Motorola Photo transistor when the switches are down. SW3 should be up for Shot noise measurements. The LED voltage is adjusted by POT1. When it is turned right, the brightness of the LED increases as seen by an increase in the phototransistor current. The Phototransistor is connected to a 10 k resistor and the two ends of a 100 k pot (POT2). A DMM connected to the banana terminals marked DMM measures the DC voltage in the 2 volt range. When SW4 is put down it measures the DC voltage across 10 k. So the average current <I> = this DC voltage divided by 10 k. The voltage across the movable contact of pot 2 relative to ground is measured on DMM when SW4 is up. If <I> is the current and the voltage across the moveable contact is <V>, we are measuring the Shot noise across a resistance R = <V>/<I>. This variable voltage for a given current <I> appears at the banana connector marked DMM when the switch SW3 is up. This voltage with noise appears at the B&C connector marked O/P. O/P is connected to the B&C connector marked Input on the front panel of the noise detector (see Figure II.14.1.3).

The current <I> can be varied from from 10 to 70 microamps. We work in the range 15 to 50 microamps. The current can be measured by measuring the voltage across 10 k. The variable resistance can be set at values of 20 k to 80 k in steps of 10 k, by measuring <V> for a given <I>. Thus we set <I>, change <V> in steps and measure v_{rms} at the output of the noise detector. The set of readings are shown in Table II.14.1.2 below.

Table II.14.1.2

Shot noise readings

	I in μamp	15		20		25		30
R	V mV	vrms mV	V mV	vrms mV	V mV	vrms mV	V mV	vrms mV
20000	300	27	400	29	500	30	600	31
30000	450	36	600	37	750	39	900	43
40000	600	43	800	45	1000	47	1200	48
50000	750	49	1000	51	1250	54	1500	56
60000	900	54	1200	57	1500	61	1800	63
70000	1050	60	1400	62	1750	67	2100	71
80000	1200	65	1600	68	2000	76	2400	80

	I in μamp	35		40		45		50
R	V mV	vrms mV	V mV	vrms mV	V mV	vrms mV	V mV	vrms mV
20000	700	31	800	31	900	32	1000	31
30000	1050	40	1200	41	1350	41	1500	41
40000	1400	49	1600	50	1800	50	2000	50
50000	1750	57	2000	58	2250	58	2500	58
60000	2100	64	2400	65	2700	66	3000	65
70000	2450	72	2800	74	3150	74	3500	74
80000	2800	83	3200	83	3600	84	4000	85

A plot of v_{rms} against $\langle V \rangle$ for a current of 35 μamp is shown in Figure II.14.1.7.

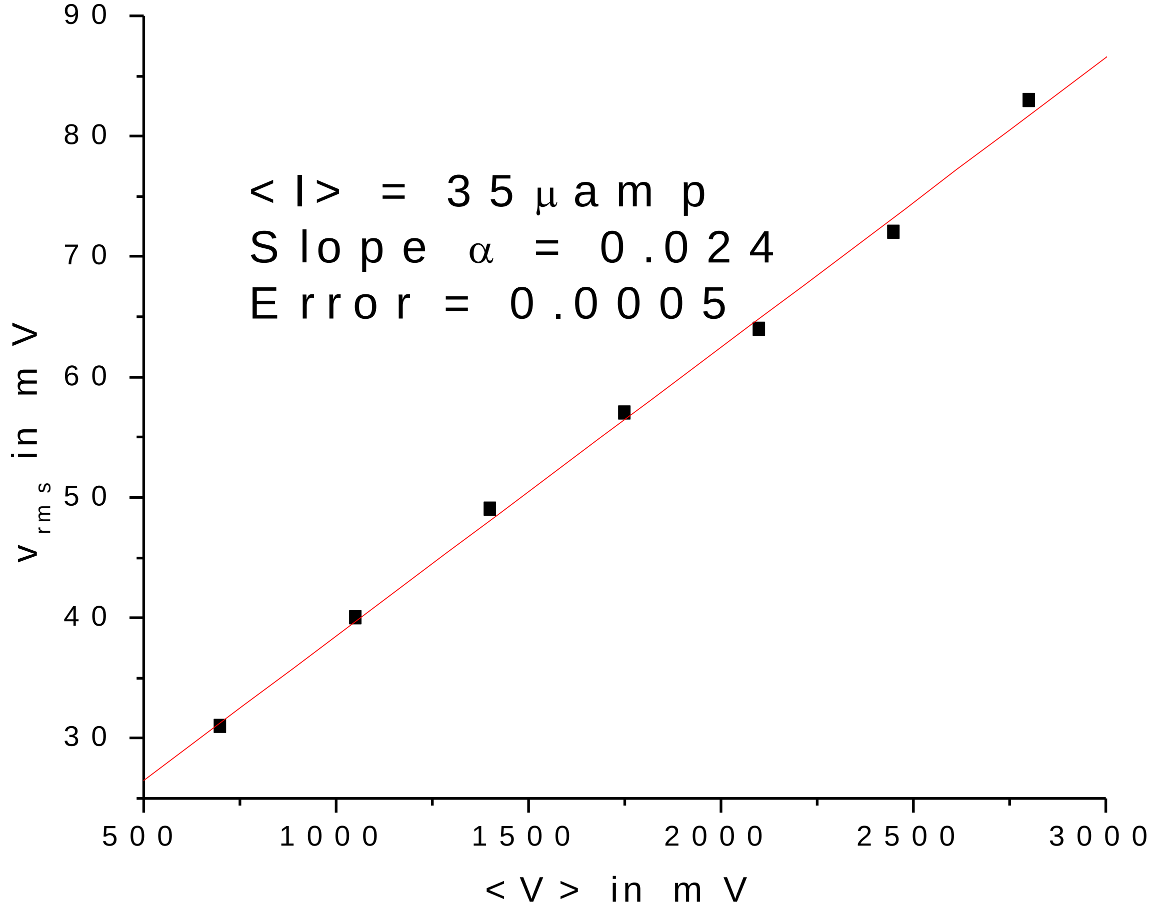


Figure II.14.1.7 Variation of v rms against <V> for <I> = 35 □ amp

We get a good straight line with an error in slope of about 2%. For every current from 15 to 50 μamp we get similar straight lines with different slopes with an error in slope within 3%.

We now collect the slope α for different currents <I> in Table II.14.1.3.

Table II.14.1.3

Slope α for different currents

<I> □ amps	1/<I> in 10^6	Slope α	Error	α^2
15	0.066666667	0.041	0.001	0.001681
20	0.05	0.032	0.001	0.001024
25	0.04	0.03	0.0007	0.0009
30	0.033333333	0.026	0.0009	0.000676
35	0.028571429	0.024	0.0005	0.000576
40	0.025	0.0211	0.0004	0.000445
45	0.022222222	0.0189	0.0003	0.000357
50	0.02	0.0174	0.0004	0.000303

Column 1 gives average current in micro amperes. Column two gives 1/<I>. Column 3 is slope α, and column 4 is the error in slope α. Column 5 gives α^2. We plot α^2 against 1/<I>. This plot is shown in FigureII.14.1. 8. The linear variation of α^2 vs 1/<I> is a **clear signature that what we are measuring is Shot Noise.** The slope β of this graph is 2.83×10^{-8} amp. The error in slope is about 5%.

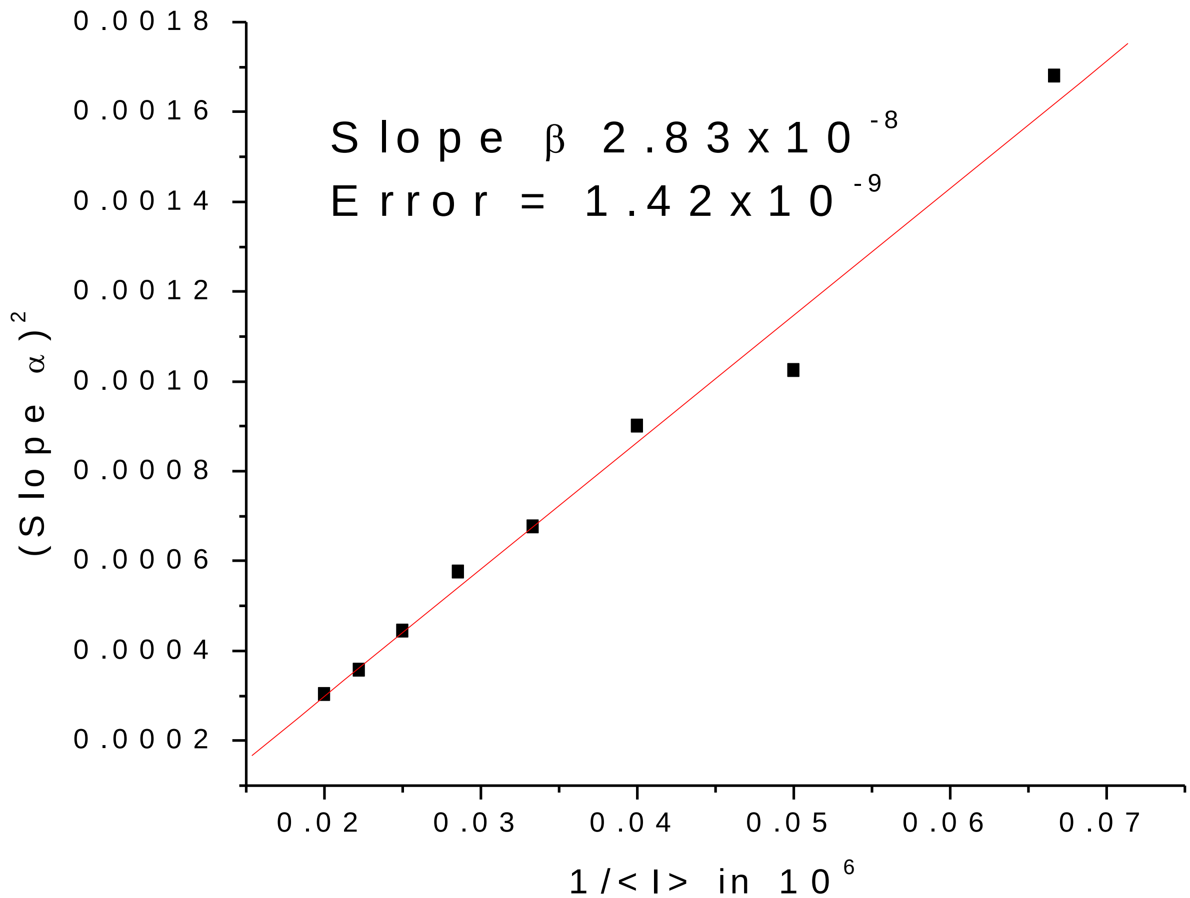


Figure II.14.1.8 Plot of slope α against 1/<I>

Using this value of β and the value of G^2BW, we get

$$e = \beta/(2*G^2BW) = 2.83x10^{-8}/(2*8.7x10^{10}) = 1.63x10^{-19} \text{ Coulombs}$$

with an error of 5%.

A second measurement on 12 Sep gave G^2B as $8.72x10^{10}$ Hz and slope □ as $2.82x10^8$ amp . This gives $e = 1.62x10^{-19}$ Coulombs with an error of 5%.

APPENDIX

NBS TABLE FOR THERMO-EMF

OF

CHROMEL-ALUMEL THERMOCOUPLE

THERMOELECTRIC EMF IN mV
CHROMEL-ALUMEL THERMOCOUPLE
REFERENCE JUNCTION AT 0^0 C

	0	1	2	3	4	5	6	7	8	9
0	0.000	0.039	0.079	0.119	0.158	0.198	0.238	0.277	0.317	0.357
10	0.397	0.437	0.477	0.517	0.557	0.597	0.637	0.677	0.718	0.758
20	0.798	0.838	0.879	0.919	0.960	1.000	1.041	1.081	1.122	1.163
30	1.203	1.244	1.285	1.326	1.366	1.407	1.448	1.489	1.530	1.571
40	1.612	1.653	1.694	1.735	1.776	1.817	1.858	1.899	1.941	1.982
50	2.023	2.064	2.106	2.147	2.188	2.230	2.271	2.312	2.354	2.395
60	2.436	2.478	2.519	2.561	2.602	2.644	2.685	2.727	2.768	2.810
70	2.851	2.893	2.934	2.976	3.017	3.059	3.100	3.142	3.184	3.225
80	3.267	3.308	3.350	3.391	3.433	3.474	3.516	3.557	3.599	3.640
90	3.682	3.723	3.765	3.806	3.848	3.889	3.931	3.972	4.013	4.055
100	4.096	4.138	4.179	4.220	4.262	4.303	4.344	4.385	4.427	4.468
110	4.509	4.550	4.591	4.633	4.674	4.715	4.756	4.797	4.838	4.879
120	4.920	4.961	5.002	5.043	5.084	5.124	5.165	5.206	5.247	5.288
130	5.328	5.369	5.410	5.450	5.491	5.532	5.572	5.613	5.653	5.694
140	5.735	5.775	5.815	5.856	5.896	5.937	5.977	6.017	6.058	6.098
150	6.138	6.179	6.219	6.259	6.299	6.339	6.380	6.420	6.460	6.500
160	6.540	6.580	6.620	6.660	6.701	6.741	6.781	6.821	6.861	6.901
170	6.941	6.981	7.021	7.060	7.100	7.140	7.180	7.220	7.260	7.300
180	7.340	7.380	7.420	7.460	7.500	7.540	7.579	7.619	7.659	7.699
190	7.739	7.779	7.819	7.859	7.899	7.939	7.979	8.019	8.059	8.099
200	8.138	8.178	8.218	8.258	8.298	8.338	8.378	8.418	8.458	8.499
210	8.539	8.579	8.619	8.659	8.699	8.739	8.779	8.819	8.860	8.900
220	8.940	8.980	9.020	9.061	9.101	9.141	9.181	9.222	9.262	9.302
230	9.343	9.383	9.423	9.464	9.504	9.545	9.585	9.626	9.666	9.707

	0	1	2	3	4	5	6	7	8	9
240	9.747	9.788	9.828	9.869	9.909	9.950	9.991	10.031	10.072	10.113
250	10.153	10.194	10.235	10.276	10.316	10.357	10.398	10.439	10.480	10.520
260	10.561	10.602	10.643	10.684	10.725	10.766	10.807	10.848	10.889	10.930
270	10.971	11.012	11.053	11.094	11.135	11.176	11.217	11.259	11.300	11.341
280	11.382	11.423	11.465	11.506	11.547	11.588	11.630	11.671	11.712	11.753
290	11.795	11.836	11.877	11.919	11.960	12.001	12.043	12.084	12.126	12.167
300	12.209	12.250	12.291	12.333	12.374	12.416	12.457	12.499	12.540	12.582
310	12.624	12.665	12.707	12.748	12.790	12.831	12.873	12.915	12.956	12.998
320	13.040	13.081	13.123	13.165	13.206	13.248	13.290	13.331	13.373	13.415
330	13.457	13.498	13.540	13.582	13.624	13.665	13.707	13.749	13.791	13.833
340	13.874	13.916	13.958	14.000	14.042	14.084	14.126	14.167	14.209	14.251
350	14.293	14.335	14.377	14.419	14.461	14.503	14.545	14.587	14.629	14.671
360	14.713	14.755	14.797	14.839	14.881	14.923	14.965	15.007	15.049	15.091
370	15.133	15.175	15.217	15.259	15.301	15.343	15.385	15.427	15.469	15.511
380	15.554	15.596	15.638	15.680	15.722	15.764	15.806	15.849	15.891	15.933
390	15.975	16.017	16.059	16.102	16.144	16.186	16.228	16.270	16.313	16.355
400	16.397	16.439	16.482	16.524	16.566	16.608	16.651	16.693	16.735	16.778
410	16.820	16.862	16.904	16.947	16.989	17.031	17.074	17.116	17.158	17.201
420	17.243	17.285	17.328	17.370	17.413	17.455	17.497	17.540	17.582	17.624
430	17.667	17.709	17.752	17.794	17.837	17.879	17.921	17.964	18.006	18.049
440	18.091	18.134	18.176	18.218	18.261	18.303	18.346	18.388	18.431	18.473

ACKNOWLEDGMENTS

This manual describes a list of experiments that can be done at the B.Sc, M.Sc and Post-M.Sc degree levels in Physics using the experimental kit developed for the Indian Academy of Sciences.

As mentioned in the preface, forty nine Refresher courses have been conducted so far in which about 1200 participants have done all the experiments. Of these more than hundred were students in the undergraduate and master's programs in some universities. The feedback from all the participants has been uniformly appreciative. The participants have found the experiments simple and highly reproducible and the instructions clear. It is hoped that with the availability of the kit at a reasonable price from Ajay Sensors, Bangalore, these experiments will be accepted in the curricula of all universities in the not too distant future. **It is high time that the laboratory syllabus is revamped to include more experiments. A good experimental background is a requisite for a good Physics graduate.**

In conducting these Refresher courses we have received support and encouragement from many people. Our sincere thanks are due to

1. **Prof. A.K. Sood, past president of the Indian Academy of Sciences, for his support and constant encouragement;**

2. **Prof. N. Mukunda, Chairman of the Science Education Panel of the Indian Academy of Sciences, Bangalore, all the members of the current and previous Science Education panels, and Sri G. Madhavan, Co-ordinator of the Science Education Programs of the Indian Academy of Sciences, Bangalore;**

3. **Prof. Sadique of Goa University, Prof. Efrem DeSa of Carmel College, Margoa, and Sri Manohar Nayak of Goa for designing some of the electronic circuits and for their sustained help and committed involvement in the Refresher courses ;**

4. **Dr. T.G. Ramesh of the Material Sciences Division, NAL, Bangalore for setting up experiments with the Thermo-emf analyzer, Resistograph and Differential Thermal Analysis and acting as Resource person in the recent courses;**

5. **Prof. M.S.R. Rao, Department of Physics, IIT Madras for making samples for percolation studies, Prof. V. Sankaranarayanan, Department of Physics, IIT Madras, for designing the dipstick cryostat, for Dr. V. Ganesan, UGC-DAE Consortium for Scientific Research, Indore for providing LSMO and YBCO samples and Prof. Kumar of Crystal Growth Center, Anna University, Chennai, for providing Silicon and Germanium samples.**

6. **All the resource persons, too numerous to list out, who have helped me to conduct these forty-nine courses.**

7. **All the local organizers of outstation courses for their untiring efforts in making local arrangements for the courses.**

8. **All staff of the Indian Academy of Sciences, Bangalore, for their unstinted support for courses organized in the IAS Residency in Jalahalli.**

9. **The Director, National Aerospace Laboratory, Bangalore for agreeing to gift the Themo-emf Analyser, Resistograph and the DTA set up developed at NAL to the Indian Academy and for Dr. Subha of the Materials Science Division of NAL for arranging for the gift.**

   and

10. **To Sri Ajay Kumar of Ajay Sensors and Instruments for all timely help.**

The late Dr. K.R. Rao of BARC was a great source of support to us which can never be forgotten.

We would not have been able to progress so far but for the interest and deep involvement of the participants who attended the Forty nine Refresher courses. To all of them we convey our sincere thanks. Their support will encourage us to develop additional user-friendly experiments.

Mysore R.Srinivasan
1 August 2013

Goa K.R. Priolkar

Prof. R.SRINIVASAN

A PROFILE

Srinivasan was born on 15-12-1931 in Vellore, Tamilnadu. After obtaining his B.Sc (Honours) degree in Physics from Madras University in 1951, he joined the Department of Physics, Indian Institute of Science, Bangalore, for doing research. He worked on Thermal Expansion of Crystals down to 90 K. For this work he was awarded the Ph.D degree of Madras University in 1957.

From 1957-1962 he continued as a post-doctoral fellow in the Indian Institute of Science, Bangalore. During this period he worked on the Dispersion of Photoelastic constants of Alkali Halides into the UV region and also started theoretical research work in Lattice Dynamics and Thermal Expansion of Crystals.

He joined the Physics Department of I.I.T., Madras, in October 1962 and continued his theoretical research. From 1965 to 1967 he was a Visiting Scientist at the Materials Research Center, Pennsylvania State University, U.S.A. During this time he developed the theory of Third Order Elastic Constants of Non-piezoelectric crystals and the Theory of the Elastic Dielectric for ionic crystals. He was promoted as Professor in I.I.T. Madras in 1969 and continued with his work on Third order Elastic Constants, and Surface Modes of Vibrations.

In 1971, a small Low Temperature Laboratory was started in the Department of Physics, I.I.T. Madras, with aid from the German Government and Srinivasan was put in charge. He learnt the operation and maintenance of the liquid nitrogen and helium plants, built a series of experiments to train students in low temperature physics and started research on the thermo-electric power and electrical transport properties of Chevrel Phase Superconductors, magnetic properties of rare earth oxide materials, cryopumping and dynamics of flux penetration in Type I superconductors. He also did some work on electrical conductivity and dielectric properties of poly-electrolyte complexes of biological polymers.

From 1979 to 1984, he was the Dean of Academic Research (2 Years), the Dean of Student Affairs(2 years) and the Deputy Director (2½ years) holding the last two posts concurrently for about a year. In 1984 he went on sabbatical to the Institut fur Technische Physik in Kernforsschungszentrum, Karlsruhe, Germany. Here he worked on heat transfer to flowing Liquid HeII, the flow being generated by a fountain effect pump. These studies showed that a fountain effect pump can be used to cool superconducting magnets with Liquid HeII and they have the advantage that there were no moving parts.

On his return to I.I.T., Madras, in 1985 he worked on thermal properties, electrical transport properties and point contact tunneling in high temperature superconductors.

In 1990 the UGC was setting up the Inter-University Consortium for DAE Facilities and invited Prof. Srinivasan to be the Director. From 1990 to 1995 he worked in Indore and set up three centers, one in Mumbai, one in Indore and one in Kolkotta. The Mumbai Center facilitated the use of neutron spectrometers on the Dhruva Reactor by University researchers. At Indore materials preparation and characterization facilities were set up. Measurements down to 1.8 K became possible through the installation of 25 year old LHe and LN2 machines obtained as gifts. He also oversaw the construction of a beamline for photoelectron spectroscopy in the VUV and Soft X ray region. This beamline was installed on the Indus 1 Synchrotron built at CAT, Indore. In Kolkatta the center facilitated the use of the Variable Energy Cyclotron by university researchers. IUC-DAEF is now renamed UGC-DAE Consortium for Scientiific Research. A National Center of Low Temperature- High magnetic Field Research has been established in UGC-DAE CSR in Indore by DST based on a report for the need of such a center prepared by Srinivasan.

From 1996 to 2007 he was a Visiting Scientist at the Raman Research Institute, Bangalore. Here he collaborated with Prof. Hema Ramachandran in setting up a laboratory for Optics and Atomic Physics Research and they initiated work on cold atoms generated by Laser cooling. During this time, he was also a consultant to the Cryogenics Division of the Institute for Plasma Research in Ahmedabad, for four years.

In 2001, the Indian Academy of Sciences asked Srinivasan to develop a Refresher Course in Experimental Physics. From 2001-2007 the course made slow, but steady, progress. In 2007 all three National Science Academies started supporting all Refresher Courses. Realizing that the activity of running the Experimental Physics Course was poised for an accelerated growth, Srinivasan relinquished his position at RRI, Bangalore, and devoted his full time for finding a company to make the kit commercially, and running the Refresher courses in Experimental Physics. To date 49 Refresher Courses have been conducted and more are planned before end of August 2014.

Srinivasan has produced more than twenty Ph.Ds, of which seven are in Theory, two in Bio-science and the rest in Low Temperature Physics. He has more than two hundred publications in refereed international journals of repute. He has taught B.Tech students, and handled theoretical and experimental physics courses at the M.Sc and post-M.Sc levels.

He has served on a number of committees. He was the Chairman of the Program Advisory Committee on Condensed Matter Physics of DST for six years, Chairman of the Program Advisory Committee in Cryogenics of DST for ten years, and a member of the Science and Engineering Council of the DST for three years. He was the first member from India on the International Cryogenic Engineering Committee for five years. He is a member of the ITER-India Review Committee.

He is a Fellow of the Indian Academy of Sciences, Fellow of the Indian Cryogenics Council, a Materials Research Society of India Medallist and an Awardee of the Instrumentation Society of India.

CONTACT ADDRESS

PROF. R. SRINIVASAN

143, 5 Cross, 3 Main,

Vijayanagar I Stage,

MYSORE 570017

Tel: 0821-2517504

e-mqil; rsvmys@gmail.com