Is the size of θ_{13} related to the smallness of the solar mass splitting?

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Abstract

θ_{13} is small compared to the other neutrino mixing angles. The solar mass splitting is about two orders smaller than the atmospheric splitting. We indicate how both could arise from a perturbation of a more symmetric structure. The perturbation also affects the solar mixing angle and can tweak alternate mixing patterns such as tribimaximal, bimaximal, or other variants to viability. For real perturbations only normal mass ordering with the lightest neutrino mass less than 10^{-2} eV can accomplish this goal. Both mass orderings can be accommodated by going over to complex perturbations if the lightest neutrino is heavier. The CP-phase in the lepton sector, fixed by θ_{13} and the lightest neutrino mass, distinguishes different options.

I Introduction

The recent experimental observation [1] of non-zero θ_{13}

$$\sin^2 2\theta_{13} = 0.090^{+0.008}_{-0.009} \quad (\text{Daya Bay}: 217\text{days}, \quad \text{Rate and Spectrum}) \tag{1}$$

$$\sin^2 2\theta_{13} = 0.100 \pm 0.010\,(\text{stat}) \pm 0.015\,(\text{syst}) \quad (\text{RENO}: 403 \ \text{live days}, \quad \text{Rate only}) \tag{2}$$

has triggered much model building.

The flavour basis neutrino mass matrix is diagonalized by a unitary matrix U such that $U^T M_\nu U = \text{diag}(m_1, m_2, m_3)$. In the standard parametrization this Pontecorvo, Maki, Nakagawa, Sakata (PMNS) mixing matrix U is expressed as:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{3}$$

As is evident, the entire PMNS matrix is completely determined by the oscillation observables which in turn dictates the lepton mixings. The association of the CP violating phase δ with

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s_{13} readily suggests the possibility of occurrence CP violation now as θ_{13} is observed to be non-vanishing. From global fits the currently favoured values are [2]:

$$\begin{aligned}
\Delta m^2_{21} &= (7.50^{+0.18}_{-0.19}) \times 10^{-5}\,\mathrm{eV}^2, \;\; \theta_{12} = (33.36^{+0.81}_{-0.78})^\circ, \\
|\Delta m^2_{31}| &= (2.473^{+0.070}_{-0.067}) \times 10^{-3}\,\mathrm{eV}^2, \;\; \theta_{23} = (40.0^{+2.1}_{-1.5} \oplus 50.4 \pm 0.13)^\circ \\
\theta_{13} &= (8.66^{+0.44}_{-0.46})^\circ, \;\; \delta = (300^{+66}_{-138})^\circ \; .
\end{aligned} \tag{4}$$

The atmospheric mixing angle, θ_{23}, displays an intriguing deviation from exact maximal mixing ($\theta_{23} = \pi/4$). θ_{12} is also large but not maximal while θ_{13} is the smallest of the three although it is close to its upper bound from earlier data. In this sense the latter is sometimes said to be large.

The mass spectrum harbours some fascinating unsettled issues. Although the magnitude of the solar and atmospheric neutrino mass splittings are now well measured, the absolute mass remains undetermined. Moreover, the sign of Δm^2_{31} remains unknown keeping the options open for both the *normal* ($m_1 < m_2 < m_3$) and the *inverted* ($m_3 < m_1 < m_2$) ordering depending upon whether this sign is positive or negative. Note that the solar splitting is about two orders of magnitude smaller than the atmospheric one. It is useful to define the ratio $R_{\rm mass} \equiv |\Delta m^2_{21}/\Delta m^2_{31}| = (3.03 \pm 0.16) \times 10^{-2}$.

Therefore, the oscillation data evince two small quantities namely θ_{13} and $R_{\rm mass}$. This observation kindled the motivation of our present analysis of starting with both these small quantities vanishing and generating both by deploying a single perturbation, thereby relating them [3].

II Perturbation Theory

II.A The Unperturbed Picture

To begin with we assume no solar splitting as well as $\theta_{13} = 0$ and produce both by a *single* perturbation. For three flavours[1] of neutrinos in the absence of solar splitting the unperturbed mass matrix in the mass basis will appear as[2] $M^0_{mass} = diag(m_1^{(0)}, m_1^{(0)}, m_3^{(0)})$. We define[3] $m^\pm = (m_3^{(0)} \pm m_1^{(0)})$. The unperturbed mass matrix[4] in the flavour basis is $M^0_{flavour} = U^0 diag(m_1^{(0)}, m_1^{(0)}, m_3^{(0)}) U^{0T}$, where U^0 is the lowest order leptonic mixing matrix. The columns of U^0 are the unperturbed flavour eigenstates. Popular lepton mixings like tribimaximal (TBM), bimaximal (BM), and the 'golden ratio' (GR) exhibit[5] $\theta_{13} = 0$ and $\theta_{23} = \pi/4$.

[1]The charged lepton mass matrix is diagonal in the flavour basis under consideration.

[2]The masses $m_i^{(0)}$ ($i = 1, 2, 3$) are chosen real and positive by appropriately adjusting the Majorana phases.

[3]m^- is positive (negative) for normal (inverted) mass ordering.

[4]Note that all the mass matrices both the unperturbed as well as the perturbation itself are of Majorana nature and therefore symmetric. M^0 is hermitian by construction.

[5]After the discovery of non-zero θ_{13}, none of them are consistent with the data and thus have to be corrected.

They differ only in θ_{12}^0. We opt for a general parametrization:

$$U^0 = \begin{pmatrix} \cos\theta_{12}^0 & \sin\theta_{12}^0 & 0 \\ -\frac{\sin\theta_{12}^0}{\sqrt{2}} & \frac{\cos\theta_{12}^0}{\sqrt{2}} & \sqrt{\frac{1}{2}} \\ \frac{\sin\theta_{12}^0}{\sqrt{2}} & -\frac{\cos\theta_{12}^0}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \tag{5}$$

For $\sin\theta_{12}^0 = 0.577$ one gets the Tribimaximal mixing, wheras $\sin\theta_{12}^0 = 0.707$ and $\sin\theta_{12}^0 = 0.526$ yields the BM and GR mixings respectively. At 1σ, $0.539 < \sin\theta_{12} < 0.561$. Therefore, none of the mixings above satisfy the 1σ limits of θ_{12}.

II.B The Perturbation

The symmetric perturbation mass matrix in the mass basis after removal of an irrelevant constant part has the most general form:

$$M' = m^+ \begin{pmatrix} 0 & \gamma & \xi \\ \gamma & \alpha & \eta \\ \xi & \eta & \beta \end{pmatrix} \quad . \tag{6}$$

The dimensionless entities $\alpha, \beta, \gamma, \xi, \eta$ should be small compared to unity for a valid perturbation theory. The perturbation – i.e., $\alpha, \beta, \gamma, \xi, \eta$ – can be real or complex.

A complex M' is not hermitian, therefore the combination $(M^0 + M')^\dagger(M^0 + M')$ has to be considered. $M^{0\dagger}M^0$ is the unperturbed term and $(M^{0\dagger}M' + M'^\dagger M^0) = M_{pert}$ serves as the perturbation to the lowest order. The unperturbed eigenvalues are $(m_i^{(0)})^2$ and the perturbation matrix is

$$M_{pert} = m^+ \begin{pmatrix} 0 & 2m_1^{(0)}\mathrm{Re}(\gamma) & m^+\mathrm{Re}(\xi) - i\; m^-\mathrm{Im}(\xi) \\ 2m_1^{(0)}\mathrm{Re}(\gamma) & 2m_1^{(0)}\mathrm{Re}(\alpha) & m^+\mathrm{Re}(\eta) - i\; m^-\mathrm{Im}(\eta) \\ m^+\mathrm{Re}(\xi) + i\; m^-\mathrm{Im}(\xi) & m^+\mathrm{Re}(\eta) + i\; m^-\mathrm{Im}(\eta) & 2m_3^{(0)}\mathrm{Re}(\beta) \end{pmatrix} \quad . \tag{7}$$

II.B.1 The solar sector

The perturbation splits the degeneracy and determines the eigenstates which are rotated by an angle ζ with respect to the first two columns of U^0. The resultant solar mixing angle is now $\theta_{12} = \theta_{12}^0 + \zeta$. The 2×2 perturbation submatrix responsible for the entire solar story is:

$$M'_{(2\times2)} = m^+\alpha \begin{pmatrix} 0 & r \\ r & 1 \end{pmatrix} \text{ for Real } M' \; . \tag{8}$$

with $r = \gamma/\alpha$. For complex M', $r \equiv \mathrm{Re}(\gamma)/\mathrm{Re}(\alpha)$ and

$$(M_{pert})_{(2\times2)} = 2m^+m_1^{(0)}\mathrm{Re}(\alpha) \begin{pmatrix} 0 & r \\ r & 1 \end{pmatrix} \text{ for Complex } M' \quad . \tag{9}$$

From simple calculation, one can easily show for Real Perturbation

$$m_{2,1} = m_1^{(0)} + m^+ \, \frac{\alpha}{2}\left[1 \pm \sqrt{1 + 4r^2}\right] \; , \tag{10}$$

Parameter	TBM		BM		GR	
	r_{min}	r_{max}	r_{min}	r_{max}	r_{min}	r_{max}
r ($\times 10^2$)	-4.59	-1.95	-23.1	-19.9	1.54	4.18

Table 1: The range of r in the perturbation (see Eqs. (8, 9)) for the TBM, BM, and GR alternatives that produces a θ_{12} consistent with the global fits at 1σ.

and for complex perturbation one has:

$$m^2_{2,1} = (m_1^{(0)})^2 + 2m_1^{(0)} m^+ \, \frac{\text{Re}(\alpha)}{2} \left[1 \pm \sqrt{1+4r^2}\right] \; . \tag{11}$$

Up to small perturbative corrections m^+m^- gives the atmospheric mass splitting. Hence:

$$R_{\text{mass}} = |(m_2^2 - m_1^2)/(m_3^2 - m_1^2)| = 2\frac{m_1^{(0)}}{|m^-|} \, \text{Re}(\alpha)\sqrt{1+4r^2} \; . \tag{12}$$

The angle ζ obtained from the above 2×2 submatrices – Eqs. (8, 9) – is $\zeta = \frac{1}{2}\tan^{-1}(2r)$. $r \neq 0$ is chosen so that the mass degeneracy is removed as well as the mixing angle is tuned within the allowed range. In Table 1 we show the ranges of r for each of the three models.

II.B.2 Generating $\theta_{13} \neq 0$

With first order corrections, the third wave-function $|\psi_3\rangle$ is given by:

$$|\psi_3\rangle = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} + \bar{\xi}^* \begin{pmatrix} \cos\theta_{12}^0 \\ -\sin\theta_{12}^0/\sqrt{2} \\ \sin\theta_{12}^0/\sqrt{2} \end{pmatrix} + \bar{\eta}^* \begin{pmatrix} \sin\theta_{12}^0 \\ \cos\theta_{12}^0/\sqrt{2} \\ -\cos\theta_{12}^0/\sqrt{2} \end{pmatrix} \; . \tag{13}$$

with

$$\bar{\xi} = \left(\frac{m^+}{m^-}\right) \text{Re}(\xi) + i \; \text{Im}(\xi), \;\; \bar{\eta} = \left(\frac{m^+}{m^-}\right) \text{Re}(\eta) + i \; \text{Im}(\eta) \text{ for Complex } M' \; . \tag{14}$$

The expressions for the real limit can be read off from the above.

For simplicity we assume $\theta_{23} = \pi/4$. Hence we get, $\left(\frac{\bar{\xi}}{\bar{\eta}}\right)^* = \tan\theta_{12}^0$. Now $\tan\theta_{12}^0$, being a real quantity, forces the phases of $\bar{\xi}$ and $\bar{\eta}$ to be exactly equal. Comparing $|\psi_3\rangle$ with Eq. (3) one has:

$$\sin\theta_{13} \; e^{-i\delta} = \left[\cos\theta_{12}^0 \; \bar{\xi}^* + \sin\theta_{12}^0 \; \bar{\eta}^*\right] = \frac{\bar{\xi}^*}{\cos\theta_{12}^0} \; , \tag{15}$$

θ_{13} and δ are now determined. It is evident that in the real case CP is conserved.

II.C Results

Eq. (12) gives an estimate of α while ξ is obtained from Eq. (15). This is utilized below to set bounds on the lightest neutrino mass, m_0.

II.C.1 Real Perturbation

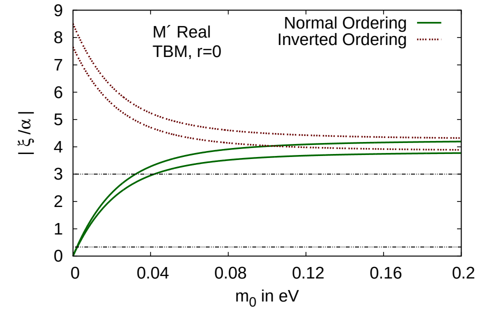

Figure 1: $|\xi/\alpha|$ is shown as a function of the lightest neutrino mass m_0 for both mass orderings when M' is real. The area between two curves of the same type is allowed when θ_{13} is varied over its 1σ range. Also indicated are the values $\frac{1}{3}$ and 3 for $|\xi/\alpha|$ – black dot-dashed lines.

It is not unreasonable to demand that the elements of the perturbation matrix should be of the same order. In Fig. 1 is shown $|\xi/\alpha|$ for both mass orderings as a function of m_0 for the TBM case. We have also indicated where this ratio corresponds to the values 3 and $\frac{1}{3}$ (dot-dashed black lines), two limits separated by an order of magnitude. For normal ordering the ratio is within the above range only if 2.3 meV $\leq m_0 \leq$ 3.7 meV. If other experiments establish a larger value of m_0 then that could be an indication that M' must be complex. In case of inverted ordering, α is more than an order of magnitude less than $|\xi|$ for almost the entire range of m_0. Thus, inverted ordering is a less favoured alternative if the perturbation is real but can be accommodated if it is complex.

II.C.2 Complex Perturbation

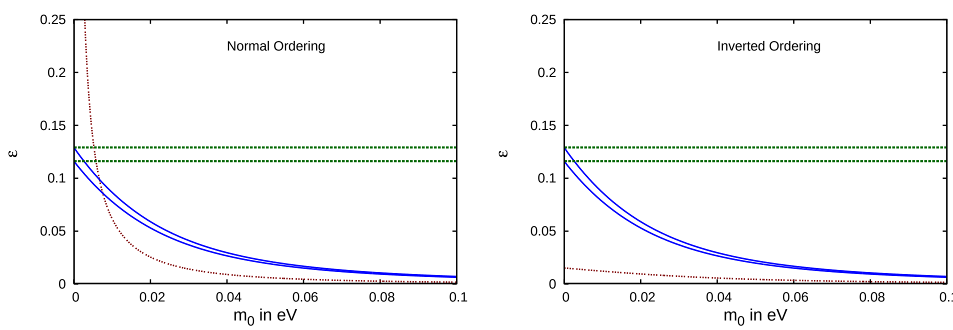

Figure 2: The limits on ϵ for normal (left) and inverted (right) mass orderings as a function of m_0. The upper (lower) limits from Eq. (16) for TBM are the green dashed (blue solid) curves. The region between the curves of the same type correspond to θ_{13} values in the 1σ range. The dotted maroon curves are the lower limits from solar splitting. Here $r = 0$ has been taken.

We take a conservative standpoint such that the complex perturbation matrix elements differ only in the phase while each of them have the same magnitude (ϵ), i.e., $\alpha = \epsilon \exp(i\phi_\alpha)$, $\gamma = \epsilon \exp(i\phi_\gamma)$, $\xi = \epsilon \exp(i\phi_\xi)$. ϵ sets the scale of perturbation. ϵ is not entirely arbitrary; Eq. (14)

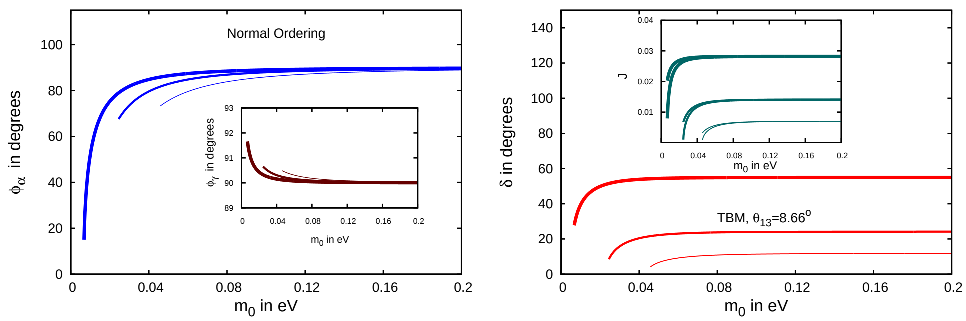

Figure 3: ϕ_α (ϕ_γ) for complex M' as a function of m_0 for normal ordering in the left panel (inset) for ϵ in decreasing order of line-thickness 0.1, 0.05 and 0.025. The right panel is for the TBM model. Same values of ϵ are chosen – in decreasing order of thickness and θ_{13} is taken at the best-fit value. In the inset is shown the Jarlskog parameter J for the chosen ϵ and the 1σ limits of θ_{13}. Both panels are for normal ordering. For inverted ordering $\delta \to (\pi - \delta)$ and J is unchanged.

implies:

$$\left|\frac{m^+}{m^-}\right| \epsilon \;\geq\; |\bar{\xi}| \;\geq\; \epsilon \;\; . \tag{16}$$

These limits are presented in the left (right) panel of Fig. 2 for the normal (inverted) mass ordering. The upper and lower limits on ϵ are shown as the green dashed and blue solid curves. The two curves of each type show how the limit changes as θ_{13} is allowed to vary over its 1σ range. Tribimaximal mixing has been assumed for these plots.

In addition, Eq. (12) also puts a bound indicated by the dotted maroon curves in the two panels of Fig. 2. It is seen that $\epsilon < 0.13$ is a safe choice.

The phases ϕ_α, ϕ_γ and ϕ_ξ can be found from the solar splitting, θ_{12}, and θ_{13}, respectively and δ can be predicted as shown in Fig. 3.

It is worthwhile to point out that the procedure for extracting δ using $|\bar{\xi}|$ leaves a two-fold uncertainty $\delta \leftrightarrow \pi + \delta$. Keeping this in mind we have shown δ in the first quadrant in Fig. 3 even though the 1σ range of the global fit – Eq. (4) – would prefer the partner $\pi + \delta$ solution.

III A Mass Model

In this segment we focus on a definite model. As already shown, the two small quantities θ_{13} and Δm^2_{solar} can originate from a single perturbation. We now intend to generate some other oscillation parameters perturbatively, causing them to get connected. Specifically, our present goal is to get non-zero θ_{12}, θ_{13}, Δm^2_{solar} and also allow θ_{23} to deviate from $\pi/4$ starting from a scenario where both the solar mixing and splitting together with θ_{13} are zero and the atmospheric mixing is also maximal. Here a sub-dominant Type I seesaw contribution perturbs the dominant unperturbed mass matrix arising from a Type II seesaw.

III.A Origin of the Unperturbed Piece

The unperturbed mass matrix remains the same as earlier. The mixing matrix now reduces to a simpler form obtained from eq. (5) setting $\theta_{12}^0 = 0$.

We assume that the Type II seesaw together with a $\mu \leftrightarrow \tau$ symmetry produces the unperturbed piece. The $SU(2)_L \times U(1)_Y$ conserving Lagrangian is of the form:

$$\mathcal{L}_{TypeII} = \sum_{i,j} \frac{1}{2} h_{ij} (\nu_L^i)^T C^{-1} \nu_L^j \langle \Delta_L \rangle + h.c. \tag{17}$$

Here Δ_L is the usual scalar triplet whose *vev* gives the Majorana mass, and $\nu_L \equiv (\nu_e, \nu_\mu, \nu_\tau)_L^T$. $h_{ij} = h_{ji}$ owing to the symmetric nature of the Majorana mass matrix. The additional $\mu \leftrightarrow \tau$ symmetry causes $h_{22} = h_{33}$. We demand ν_e is unmixed and set $h_{12} = h_{13} = 0$. This can be achieved using a $\mathbb{Z}_2$ symmetry satisfying:

$$\mathbb{Z}_2 : \nu_{eL} \to \nu_{eL}; \quad (\nu_{\mu,\tau})_L \to -(\nu_{\mu,\tau})_L; \quad \Delta_L \to \Delta_L \quad (i,j = 1,2,3). \tag{18}$$

All the above properties are obeyed by a general mass matrix in the flavour basis:

$$M^0_{flavour} = \begin{pmatrix} x & 0 & 0 \\ 0 & y & z \\ 0 & z & y \end{pmatrix} \tag{19}$$

Its mass basis counterpart obtained with the help of U^0 that leads to $M^0_{mass} = diag(x, y-z, y+z)$. Absence of solar splitting is guaranteed if $x = y - z$, i.e., $h_{22} - h_{23} = h_{11}$.

III.B Model for the Perturbation

The perturbation originates from a Type - I seesaw. We choose the Dirac mass matrix to be proportional to the identity. The $SU(2)_L \times U(1)_Y$ preserving Lagrangian in this case is,

$$\mathcal{L}_{TypeI} = \sum_{i,j} \lambda_{ij} \bar{\nu}_L^i N_R^j \langle \Phi \rangle + \frac{1}{2} H_{ij} (N_R^i)^T C^{-1} (N_R^j) + h.c. \quad (i,j = 1,2,3) \tag{20}$$

The *vev* of the doublet scalar $\langle \Phi \rangle$ sets the Dirac mass scale (m_D). Choosing $\lambda_{ij} = \lambda_0 \delta_{ij}$, gives $M_D = m_D \mathbb{I}$.

One of the singlet right-handed neutrinos is taken decoupled from the rest to yield the mass matrix of the desired form via the Majorana mass term given by the second term in the above Lagrangian. The Majorana nature requires $H_{ij} = H_{ji}$. The perturbation can be both real and complex depending upon H_{ij}. We discuss the real case first.

III.B.1 Real Perturbation

Using Type - I seesaw Mechanism we get the perturbation matrix in flavour basis as:

$$M'^{flavour} = M_D^T M_R^{-1} M_D = \frac{m_D^2}{m_R} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ choosing } M_R^{flavour} = m_R \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{21}$$

In mass basis:

$$M'^{mass} = U^{0T} M'^{flavour} U^0 = \frac{m_D^2}{\sqrt{2}m_R}\begin{pmatrix} 0 & 1 & 1 \\ 1 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 1 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \tag{22}$$

Using this perturbation matrix the corrected wave-function $|\psi_3\rangle$ is:

$$|\psi_3\rangle = \begin{pmatrix} \sigma \\ \frac{1}{\sqrt{2}}(1-\frac{\sigma}{\sqrt{2}}) \\ \frac{1}{\sqrt{2}}(1+\frac{\sigma}{\sqrt{2}}) \end{pmatrix} \quad \text{where} \quad \sigma \equiv \frac{m_D^2}{\sqrt{2}m_R m^-} = s_{13}\cos\delta. \tag{23}$$

Since M' is real, CP-violation is absent. As m^- is positive (negative) for normal (inverted) ordering, δ is 0 (π) for normal (inverted) ordering. For the atmospheric mixing we have,

$$\tan\theta_{23} = \frac{1-\frac{\sigma}{\sqrt{2}}}{1+\frac{\sigma}{\sqrt{2}}} = \tan(45^\circ - \varphi) \quad \text{where} \quad \varphi = \tan^{-1}(\frac{\sigma}{\sqrt{2}}) = \tan^{-1}(\frac{s_{13}\cos\delta}{\sqrt{2}}). \tag{24}$$

Since $s_{13}\cos\delta$ is positive (negative) for normal (inverted) ordering θ_{23} lies in the first and second octant for normal and inverted ordering respectively.

The 2×2 submatrix of M'^{mass} relevant for the solar sector is:

$$M'^{mass}_{2\times 2} = \frac{m_D^2}{\sqrt{2}m_R}\begin{pmatrix} 0 & 1 \\ 1 & \frac{1}{\sqrt{2}} \end{pmatrix} = \sqrt{2}m^- s_{13}\cos\delta \begin{pmatrix} 0 & 1 \\ 1 & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{25}$$

This leads to:

$$\Delta m^2_{solar} = 3\sqrt{2}s_{13}\cos\delta\ m^- m_1^{(0)}. \tag{26}$$

The solar mixing angle as already introduced in terms of ζ is given by

$$\theta_{12} = \zeta = \frac{1}{2}\tan^{-1}(2\sqrt{2}) = 35.26^\circ. \tag{27}$$

This is actually the θ_{12} of Tribimaximal mixing. It can be inferred from Eqs. (4) and (27) that real M' is incapable of providing the observed value of θ_{12} within 1σ. But it is allowed at 3σ for which the region of lightest neutrino mass ranges from 2 meV to 3.25 meV for normal ordering. For inverted ordering, the solar splitting is too high to fit the data.

To accommodate θ_{12} within 1σ and produce CP violation, complex M' is imperative.

III.B.2 Complex Perturbation

The perturbation is made complex by allowing phases in M_R, keeping $M_D \propto \mathbb{I}$ fixed.

$$M_R^{flavour} = m_R \begin{pmatrix} 0 & e^{-i\phi_1} & 0 \\ e^{-i\phi_1} & 0 & 0 \\ 0 & 0 & e^{-i\phi_3} \end{pmatrix} \tag{28}$$

We will restrict ourselves to the choice of $\phi_3 = 0$. Employing Type - I seesaw:

$$M'^{mass} = U^{0^T} M'^{flavour} U^0 = \sigma m^- \begin{pmatrix} 0 & e^{i\phi_1} & e^{i\phi_1} \\ e^{i\phi_1} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ e^{i\phi_1} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{29}$$

In mass basis,

$$M_{pert} = \frac{m_D^2}{\sqrt{2} m_R} \begin{pmatrix} 0 & 2m_1^{(0)} \cos\phi_1 & m^+ \cos\phi_1 - i\ m^- \sin\phi_1 \\ 2m_1^{(0)} \cos\phi_1 & \frac{2}{\sqrt{2}} m_1^{(0)} & -\frac{m^+}{\sqrt{2}} \\ m^+ \cos\phi_1 + i\ m^- \sin\phi_1 & -\frac{m^+}{\sqrt{2}} & \frac{2}{\sqrt{2}} m_3^{(0)} \end{pmatrix}. \tag{30}$$

In analogy to the real case we write for the solar mixing angle

$$\theta_{12} = \zeta = \frac{1}{2} \tan^{-1}(2\sqrt{2} \cos\phi_1). \tag{31}$$

This is similar to eq. (27) apart from the factor of $\cos\phi_1$. One can utilize the 1σ values of θ_{12} as in eq. (4) to obtain a range for ϕ_1: $0.764 < \cos\phi_1 < 0.890$. Restricting ourselves to the lowest order in perturbation

$$\Delta m^2_{solar} = \sqrt{2} \sigma m_1^{(0)} m^- \sqrt{1 + 8\cos^2\phi_1} \tag{32}$$

where σ is as defined in eq. (23). σ is positive and negative for normal and inverted ordering respectively. With the solar sector now resolved, we turn towards the other two mixing angles. As computed before,

$$|\psi_3\rangle = \begin{pmatrix} \sigma m^- z_1 \\ \frac{1}{\sqrt{2}}(1 - \frac{\sigma}{\sqrt{2}}) \\ \frac{1}{\sqrt{2}}(1 + \frac{\sigma}{\sqrt{2}}) \end{pmatrix} \quad \text{where} \quad z_1 \equiv \frac{\cos\phi_1}{m^-} - i\, \frac{\sin\phi_1}{m^+}. \tag{33}$$

Thus $s_{13} e^{-i\delta} = \sigma m^- z_1$. Hence the identification of

$$s_{13} = \sigma |m^-| \sqrt{\frac{\cos^2\phi_1}{m^{-2}} + \frac{\sin^2\phi_1}{m^{+2}}} \text{ and } \delta = \tan^{-1}(\tan\phi_1 \frac{m^-}{m^+}) \tag{34}$$

is obvious. The atmospheric mixing angle is given by $\tan\theta_{23} = \tan(45^\circ - \varphi')$ where $\varphi' = \sqrt{2} s_{13} \frac{\cos\delta}{\cos\phi_1}$. As the sign of $s_{13}\cos\delta$ is completely determined by sign of σ, one can clearly exclude the second (first) octant of θ_{23} for normal (inverted) ordering. One can always check the correspondence of this section with the real case by substituting $\phi_1 = 0$.

Our results for Normal Ordering are graphically represented in Fig. 4 for 1σ. Here solar splitting is expressed as a function of m_0. We have shown the results for central value of θ_{13} for θ_{23} in first octant. The horizontal lines are the observed bounds of the solar splitting. The blue and maroon lines are for lower and upper bounds of θ_{12} respectively, that basically marks the range of m_0 in which our model holds. The CP violating phase δ lies between 24° to 36°. The Jarlskog parameter ranges between 1.4×10^{-2} and 2×10^{-2}. In this case also our target can not be achieved for Inverted Ordering.

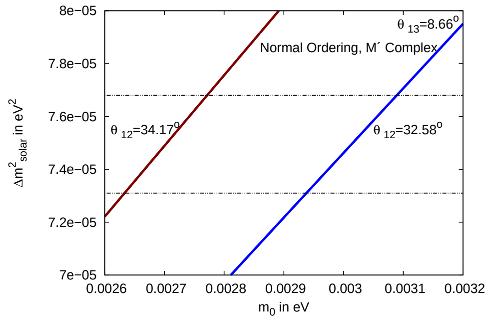

Figure 4: Solar splitting-vs-m_0 for the central value of θ_{13} with θ_{23} in first octant. The horizontal lines are the observed bounds. The blue (maroon) curve is for the lower (upper) bound of θ_{12}.

IV Conclusions

We have proposed that the neutrino mass matrix has a structure in which θ_{13} and Δm^2_{solar} are zero, $\theta_{23} = \pi/4$, and the atmospheric mass splitting, Δm^2_{atm}, is what is observed. The solar mixing angle θ_{12} can be chosen zero or as dictated by popular mixing patterns such as tribimaximal mixing. This is a reasonably good reflection of the observed data though a few finer details are missing here. We speculate the presence of a smaller contribution in addition, amenable to a perturbative treatment, that generates the small parameters in the neutrino mixing sector, namely, θ_{13} and Δm^2_{solar} and applies minor tweaks to θ_{12} and θ_{23}. CP-violation can also be incorporated. This leads to testable relationships between oscillation parameters. We also sketch a mass model based on the seesaw mechanism which embodies these features.

Acknowledgements: AR takes this opportunity to wish Professor C.S. Aulakh a happy birthday. He also thanks the organizers for the invitation to present this talk. The authors thank Biswajoy Brahmachari for discussions. SP acknowledges support from CSIR, India. AR is partially funded by the Department of Science and Technology Grant No. SR/S2/JCB-14/2009.

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