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Generalized Parton Distributions for the Nucleon in the Light-front Quark model

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Abstract. We calculate the generalized parton distributions (GPDs) for the up and down quark in nucleon using the effective light- front wavefunction (LFWF). Our results for the GPDs in momentum and impact parameter space are comparable with the other phenomenological models for the quark distribution functions.

Keywords. Form factors electromagnetic, properties of baryons, Compton scattering by hadrons

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1. Introduction

Generalized parton distributions (GPDs) are the important set of parameters that give us essential information about the nonperturbative structure of hadrons. GPDs have gained a lot of theoretical and experimental interest in the recent past. At the leading order, there are two GPDs: helicity dependent $H(x, \zeta, t)$ and helicity independent $E(x, \zeta, t)$. These are functions of three variables, namely, longitudinal momentum fraction (x), square of the total momentum transferred (t), and skewness (ζ), which represents the longitudinal momentum transferred in the process. GPDs are experimentally extracted from hard exclusive processes like deeply virtual compton scattering [1] and vector meson production [2]. Recent measurements at DESY [3] and Jefferson Lab [4] with the upcoming 12 GeV energy upgrade at Jefferson Lab will significantly advance the determination of GPDs in the valence quark region, whereas measurements at COMPASS [5] will explore the region of sea quarks and gluons with small momentum fractions.

The GPDs are expressed as off-forward matrix elements of bilocal light-front current in the overlap representation. Their first moments are related to form factors and they do not have probabilistic interpretation. For the zero skewness, the Fourier transform (FT) of the GPDs with respect to the momentum transfer in the transverse direction, gives the impact parameter dependent GPDs. The impact parameter GPDs have probabilistic interpretation [6] and provide us information about partonic distributions in the impact parameter or transverse position space [7].

The AdS/CFT correspondence between the string theory on a higher-dimensional anti-de Sitter (AdS) space and conformal field theory (CFT) in physical space-time is an important approach to study the hadron spectroscopy [8]. The AdS/CFT conjecture has led to a semiclassical approximation for strongly-coupled quantum field theories which

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provides physical insights into its non perturbative dynamics of meson and baryons [9]. These models incorporate confinement and chiral symmetry breaking, and successfully explain the important properties of hadron spectra, e.g., the mass spectra [10], form factors [11], etc. [12]. The idea of matching the matrix elements of AdS modes to the light-front QCD is referred to as light-front holography (LFH). This approach has been successfully extended to explain the GPDs for valence quarks indirectly via the sum rules that connect GPDs with electromagnetic form factors [13].

Recently a phenomenological light-front wave function (LFWF) has been proposed by matching the soft-wall model of AdS/QCD and Light-Front QCD for electromagnetic form factors of hadrons with arbitrary twist dimensions [14]. The light-front quark model (LFQM) successfully explain the results for hadronic form factors consistent with quark counting rules as well as the Drell Yan-West Duality. Using LFH approach we intend to investigate the LFQM to study the GPDs for the up and down quark in nucleon. We also investigate the GPDs in the impact parameter space. The qualitative behavior of our model predictions is compared with the recent parameterization method of parton distributions [15].

2. Generalized parton distributions in the Light-front quark model

In the light-front formalism, the Dirac and Pauli form factors are identified by the helicity conserving and the helicity non-conserving matrix element of the plus component of the electromagnetic current J^+ [16]. The quark electromagnetic form factors $F_1^q(q^2)$ and $F_2^q(q^2)$ in the overlap representation are defined as

$$\begin{aligned} F_1^q(t) &= \int \frac{\mathrm{d}x\,\mathrm{d}^2k_\perp}{16\pi^3}\left[\psi^{*\uparrow}_{1/2}(x,k'_\perp)\,\psi^{\uparrow}_{1/2}(x,k_\perp)+\psi^{*\uparrow}_{-1/2}(x,k'_\perp)\,\psi^{\uparrow}_{-1/2}(x,k_\perp)\right], \\ F_2^q(t) &= \frac{-2M_N}{q_1-\iota q_2}\int \frac{\mathrm{d}x\,\mathrm{d}^2k_\perp}{16\pi^3}\left[\psi^{*\uparrow}_{1/2}(x,k'_\perp)\,\psi^{\downarrow}_{-1/2}(x,k_\perp)+\right. \\ &\qquad \left.\psi^{*\uparrow}_{1/2}(x,k'_\perp)\,\psi^{\downarrow}_{-1/2}(x,k_\perp)\right], \end{aligned} \tag{1}$$

where $\psi^{\lambda N}_{\lambda q}(x,\,k_\perp)$ is LFWF describing the interaction of quark with scalar-diquark to form a nucleon, $\lambda_N = \uparrow\downarrow$ correspond to the specific helicities of the nucleon and $\lambda_q = \pm 1/2$ is helicity of struck quark, $k'_\perp = k_\perp + (1-x)q_\perp$, and $t = -q^2 = -q_\perp^2$ is the momentum transferred.

The quark-scalar diquark helicity component of the LFWFs are expressed as

$$\begin{aligned} \psi^{\uparrow}_{\frac{1}{2}}(x,k_\perp) &= \varphi_q^1(x,\,k_\perp)\,, \quad \psi^{\uparrow}_{-\frac{1}{2}}(x,k_\perp) = -\left(\frac{k^1+\iota k^2}{xM_n}\right)\varphi_q^2(x,\,k_\perp)\,, \\ \psi^{\downarrow}_{\frac{1}{2}}(x,k_\perp) &= \left(\frac{k^1-\iota k^2}{xM_n}\right)\varphi_q^2(x,\,k_\perp)\,, \quad \psi^{\downarrow}_{-\frac{1}{2}}(x,k_\perp) = \varphi_q^1(x,\,k_\perp)\,. \end{aligned} \tag{2}$$

where

$$\varphi_q^i(x,\,k_\perp) = \frac{4\pi}{\kappa}\,N_q^i\sqrt{\frac{\log(1/x)}{1-x}}\,x^{a_q^i}(1-x)^{b_q^i}\,e^{-\frac{k_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}}\,. \tag{3}$$

Here N_q^i is the normalization constant, a_q^i and b_q^i are the free parameters to be fitted to the experimental data on form factors for proton and neutron.

Sum rules connect the GPDs for unpolarized quarks with the electromagnetic form factors [6]

$$F_1^q(t) = \int_0^1 \mathrm{d}x\; H^q(x,t)\,, \qquad F_2^q(t) = \int_0^1 \mathrm{d}x\; E^q(x,t)\,. \tag{4}$$

We have used the standard convention to define the GPDs for valence quarks (minus antiquark) $H^q(x,t) = H^q(x,0,t)+H^q(-x,0,t)$; $E^q(x,t) = E^q(x,0,t)+E^q(-x,0,t)$. The GPDs at $-x$ for quarks is equal to the GPDs at x for antiquarks with a minus sign. After performing the matching of respective expressions for the nucleon form factors results the GPDs for the up and down quark are given as

$$\begin{aligned} H^q(x,t) \,&=\, n_q \frac{(N_q^1)^2}{I_1^q} x^{2a_q^1}(1-x)^{2b_q^1+1}\left[1+\frac{\sigma(x)^2\kappa^2}{\log(1/x)}\left(1-\frac{q_\perp^2\log(1/x)}{4\kappa^2}\right)\right] \\ &\quad \times \exp[-\frac{\log(1/x)}{4\kappa^2}q_\perp^2]\,, \\ E^q(x,t) \,&=\, \kappa_q \frac{2N_q^1N_q^2}{I_2^q}\, x^{a_{1q}+a_{2q}-1}(1-x)^{b_{1q}+b_{2q}+2}\, \exp[-\frac{\log(1/x)}{4\kappa^2}q_\perp^2]\,. \end{aligned} \tag{5}$$

where n_q is the number of valence quarks in nucleon, κ_q is the quark anomalous magnetic moment, and $\sigma(x) = \frac{N_q^2}{N_q^1}\, x^{(a_q^2-a_q^1)}(1-x)^{(b_q^2-b_q^1+1)}$. The integral in the above Eq. are defined as

$$\begin{aligned} I_1^q \,&=\, \int_0^1 \mathrm{d}x\,(N_q^1)^2\, x^{2a_q^1}(1-x)^{2b_q^1+1}\left[1+\frac{\sigma^2\kappa^2}{\log(1/x)}\right], \\ I_2^q \,&=\, 2\int_0^1 \mathrm{d}x\, N_q^1N_q^2\, x^{a_{1q}+a_{2q}-1}(1-x)^{b_{1q}+b_{2q}+2}\,. \end{aligned} \tag{6}$$

For the numerical computation of GPDs, we use the following set of parameters: $a_u^1 = 0.285$, $a_d^1 = 0.7$, $b_u^1 = 0.05$, $b_d^1 = 1$, $a_u^2 = 0.244$, $a_d^2 = 0.445$, $b_u^2 = 0.109$, $b_d^2 = 0.336$. In Fig. 1(a)-(b), we have presented the spin conserving GPD $H^{u/d}(x,t)$ as a function of x for different $t = -0.5, -1, -2$ GeV^2 for up and down quark. The overall behavior of the GPDs is same for up and down quarks, however, the fall off behavior is faster with the increasing values of x for the down quark. In Figs. 1(c)-(d), we present the GPD $E^{u/d}(x,t)$ as a function of x for different t values for the up and down quark. In this case the profile function increases to a maximum value and then decreases, however, the fall off behavior is same for both up and down quarks.

3. GPDS in impact parameter space

GPDs in the momentum space are related to the impact parameter dependent parton distribution by the Fourier transform [7]. The transverse impact parameter $b = |b_\perp|$ is a measure of the transverse distance between the struck parton and the center of momentum of the hadron. The GPDs in the impact parameter or transverse position space are

$$H^q(x,b) = \frac{1}{(2\pi)^2}\int \mathrm{d}^2q_\perp e^{-q_\perp.b_\perp} H^q(x,t)\,, \tag{7}$$

$$E^q(x,b) = \frac{1}{(2\pi)^2}\int \mathrm{d}^2q_\perp e^{-q_\perp.b_\perp} E^q(x,t)\,. \tag{8}$$

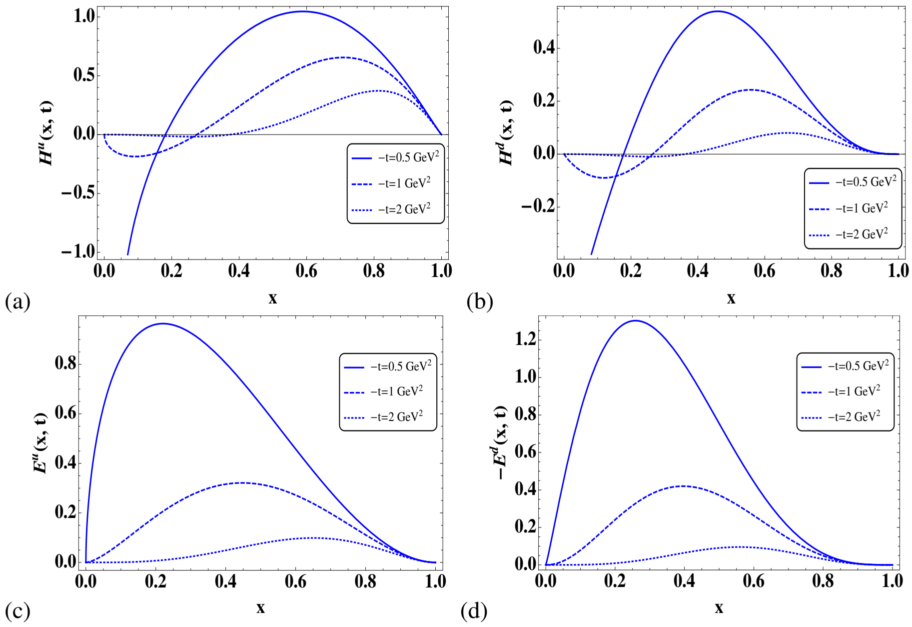

Figure 1. (Color online) Plots of (a) $H^u(x, b)$ vs x for fixed values of impact parameter $b = |b_\perp|$ (b) $H^u(x, b)$ vs b for fixed values of x, (c) same as in (a) but for d quark and (d) same as in (b) but for d-quark.

In order to have a comprehensive analysis of GPDs in impact parameter space, we compare our model predictions with other approaches. There are various phenomenological approaches based upon the global parton analysis [17], Gaussian ansatz [18], Regge parameterization [19], etc. [20]. In a recent parameterization method(PM), the GPDs for Dirac form factor [15]

$$\mathcal{H}^q(x, t) = q(x)\ \exp[1.1\frac{(1-x)^2 t}{x^{0.4}}]\,, \tag{9}$$

where the quark distribution function $q(x)$

$$\begin{aligned} u(x) &= 0.262x^{-0.69}(1-x)^{3.50}(1 + 3.83x^{0.5} + 37.65x)\,, \\ d(x) &= 0.061x^{-0.65}(1-x)^{4.03}(1 + 49.05x^{0.5} + 8.65x)\,. \end{aligned} \tag{10}$$

Similarly, for the GPD $\mathcal{E}^q(x, t)$, we have used the representation [15]

$$\mathcal{E}^q(x,\, t) = \mathcal{E}^q(x)\, \exp[1.1\frac{(1-x)^2 t}{x^{0.4}}]\,. \tag{11}$$

Here the function $\mathcal{E}^q(x)$ is expressed as

$$\mathcal{E}^u(x) = \frac{\kappa_u}{N_u}(1-x)^{\kappa_1}u(x)\,, \quad \mathcal{E}^d(x) = \frac{\kappa_d}{N_d}(1-x)^{\kappa_2}d(x)\,, \tag{12}$$

with $k_1 = 1.53$, $k_2 = 0.31$, normalization constants $N_u = 1.53$, and $N_d = 0.946$ for quark anomalous magnetic moment $\kappa_u = 1.673$, $\kappa_d = -2.033$.

We compare the behavior of impact parameter GPDs $H^{u/d}(x, b)$ and $E^{u/d}(x, b)$ in LFQM with the PM for both up and down quarks. In Fig. 2(a), we have plotted the

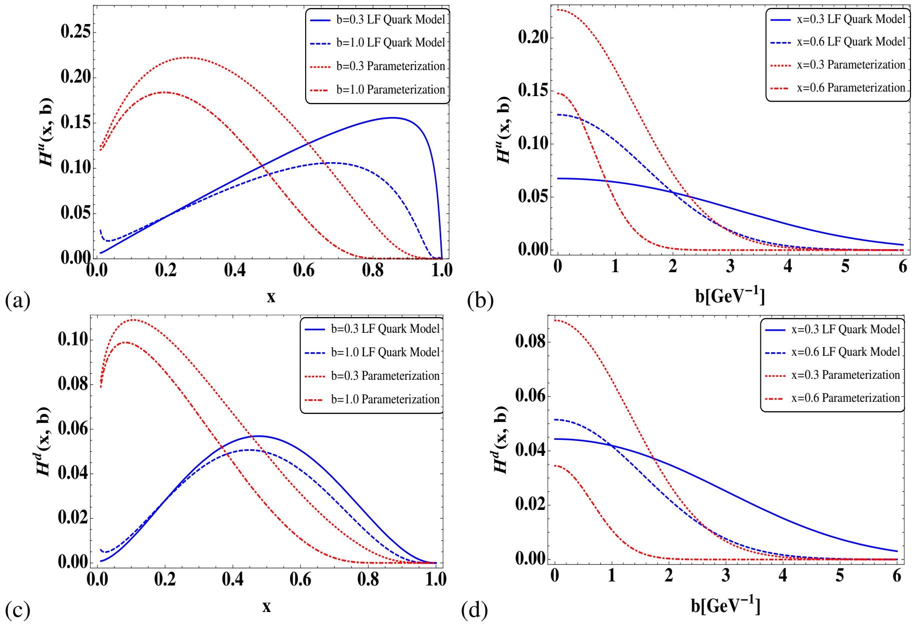

Figure 2. (Color online) Plots of (a) $H^u(x,b)$ vs x for fixed values of impact parameter $b = |b_\perp|$ (b) $H^u(x,b)$ vs b for fixed values of x, (c) same as in (a) but for d quark and (d) same as in (b) but for d quark.

behavior of $H^u(x,b)$ with x for fixed values $b = 0.3, 1$ GeV^{-1}, and in Fig. 2(b), we have shown the behavior of same GPD with impact parameter b for the fixed values $x = 0.3, 0.6$. In Figs. 2(c) and 2(d), we plot the GPDs $H^d(x,b)$ while considering its variation with x and b for the same set of parameters for the down quark. Similar plots showing the behavior of GPDs $E^{u/d}(x,b)$ are shown in Fig. 3(a)-(d). In both cases, the qualitative behavior of GPDs is almost same for both the up and down quarks in the impact parameter space. The profile of GPDs in the LFQM shifted towards a lower value of x as b increases, therefore the transverse profile is peaked at $b = 0$ and falls off further.

The GPD $H^{u/d}(x,b)$ is peaked at a higher value of x and falls off sharply in LFQM, whereas in parameterization method the same GPD is peaked at a much lower x. The GPD $E^{u/d}(x,b)$ show almost similar behavior with x in two approaches, however for the small values of x, GPD has a large magnitude for the up quark in LFQM. The small difference in the behavior of GPDs is due reason that we restricted to contribution of up and down quarks, while the contribution of the heavier quarks, such as, strange and charm quarks has been ignored. For the variation of both the GPDs $H^{u/d}(x,b)$ and $E^{u/d}(x,b)$ with impact parameter b, the overall behavior is same in different models and the transverse profile is peaked at $b = 0$. It is also interesting to observe that for the small values of b, the magnitude of GPD $H(x,b)$ is larger for up quark than down quark, whereas the magnitude of the GPD $E(x,b)$ is marginally larger for down quark than up quark.

4. Summary and Conclusions

In this work we have used a phenomenological Light-front quark model to calculate the GPDs for the up and down quark in nucleon. The effective light-front wavefunction is obtained by matching the matrix elements in the AdS/QCD and light-front QCD at an

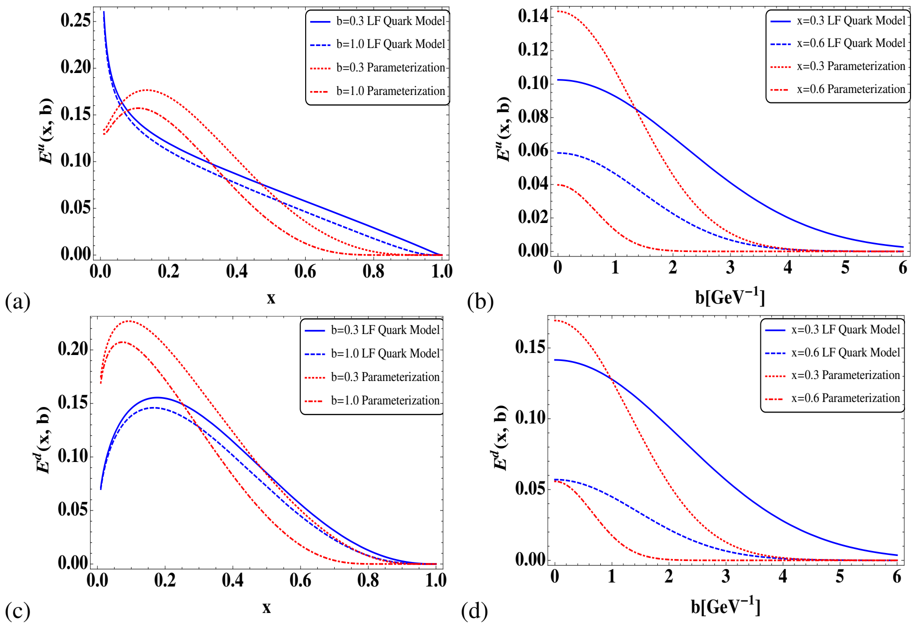

Figure 3. (Color online) Plots of (a) $E^u(x,b)$ vs x for fixed values of impact parameter $b = |b_\perp|$ (b) $E^u(x,b)$ vs b for fixed values of x, (c) same as in (a) but for d quark and (d) same as in (b) but for d quark.

initial scale. A detailed analysis of nucleon GPDs has been performed in the momentum space and transverse impact parameter space. The qualitative behavior of GPDs in transverse impact parameter space is same as the phenomenological parameterization method, though we have considered only the valence quarks contribution. In future, we plan to generalize the LFWF to sea quarks, antiquarks, and gluons, which could then be used in the evaluation of different hadronic processes.

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