Constraining the lightest neutrino mass and m_{ee} from general lepton mass matrices

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Abstract

Despite spectacular advances in fixing the neutrino mass and mixing parameters through various neutrino oscillation experiments, we still have little knowledge about the magnitudes of some vital parameters in the neutrino sector such as the absolute neutrino mass scale, effective Majorana mass m_{ee} measured in neutrinoless double beta decay. In this context, the present work aims to make an attempt to obtain some bounds for m_{ee} and the lightest neutrino mass using the most general lepton mass matrices in the Standard Model.

1 Introduction

In the last few years, significant developments have taken place in the context of phenomenology of neutrino oscillations, both from theoretical as well as experimental points of view. Owing to various solar, atmospheric, reactor and accelerator neutrino experiments, at present, the measurements of leptonic mixing angles and the neutrino mass squared differences have reached almost a precision level. However, despite intense experimental efforts, magnitudes of some of the key parameters still lack precise measurements. For example, the neutrino oscillation experiments provide no clue regarding the absolute neutrino mass scale and the related issue of the neutrino mass hierarchy. Another important issue which needs to be taken note of is regarding the rather small neutrino masses as compared to their charged counterparts. From the theoretical point of view, the most popular explanation for this observation is the 'See-Saw Mechanism' [1] which requires the neutrinos to be Majorana fermions. In this context, precise measurement of the effective Majorana mass m_{ee} in the neutrinoless double beta decay experiments can be pivotal in establishing or ruling out the Majorana neutrinos. To this end, till date, the experimental data has been able to provide only an upper bound viz.,

$$m_{ee} < 0.1 - 0.25\ eV \quad (90\%\ C.L.), \quad EXO\ and\ KamLAND - Zen, \tag{1}$$

In near future, several new NDBD experiments [3], such as NEXT, SuperNEMO, Majorana, GENIUS, CUORE etc., are in line which aim to achieve a sensitivity upto 0.01 eV for m_{ee}. Likewise, for the absolute neutrino mass scale, the most significant upper bounds on the $m_{\nu e}$ have been obtained by Mainz and Triotsk neutrino mass experiments viz. [2],

$$Mainz : m_{\nu e} < 2.30\ eV \ \ (95\% C.L.), \quad Triotsk : m_{\nu e} < 2.05\ eV \ \ (95\% C.L.). \tag{2}$$

Currently, the most important experiment in this context is the Karlsruhe Tritium Neutrino Experiment (KATRIN) [2] which is expected to push the sensitivity for the mass of the electron antineutrino down to a value of 200 meV (90% $C.L.$).

On the theoretical front, intense amount of activity has taken place to develop models for explaining neutrino masses and mixings. Despite large number of attempts [4] in various models, we still have not been able to obtain rigorous bounds on m_{ee} and the lightest neutrino mass from the most general considerations. In this context, it would be interesting to explore the possibility of obtaining some constraints on the above mentioned parameters from the general mass matrices within the framework of Standard Model (SM). The purpose of the present work, therefore, is to make an attempt to obtain bounds for m_{ee} and the lightest neutrino mass using the most general lepton mass matrices in the non-flavor basis using the facility of Weak Basis (WB) transformations.

2 Methodology

The lepton mass matrices in the SM can, in general, be given as

$$M_l = \frac{\upsilon}{\sqrt{2}} Y_{ij}^l, \quad M_{\nu D} = \frac{\upsilon}{\sqrt{2}} Y_{ij}^{\nu D}, \tag{3}$$

where M_l and $M_{\nu D}$ respectively correspond to the charged lepton and Dirac neutrino mass matrices while Y_{ij}'s and υ correspond to the Yukawa couplings and the vacuum expectation value of the Higgs field respectively. To this end, using the facility of weak basis (WB) transformations [5], it can be shown that the most general lepton mass matrices within the framework of SM can be expressed as

$$M_l = \begin{pmatrix} C_l & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} C_\nu & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}. \tag{4}$$

Details of the diagonalizing transformations for the above mass matrices and the methodology connecting the lepton mass matrices to the mixing matrix can be looked up in [6] and [7], however we mention the essentials here. A general mass matrix M_k can be expressed as

$$M_k = Q_k M_k^r P_k, \tag{5}$$

where Q_k, P_k are diagonal phase matrices given as $\text{Diag}(e^{i\alpha_k}, 1, e^{-i\beta_k})$ and Diag $(e^{-i\alpha_k}, 1, e^{i\beta_k})$ respectively and M_k^r is a real symmetric matrix. M_k^r can be diagonalized by an orthogonal transformation O_k, e.g.,

$$M_k^{diag} = O_k^T M_k^r O_k \tag{6}$$

which can be rewritten as

$$M_k^{diag} = O_k^T Q_k^\dagger M_k P_k^\dagger O_k. \tag{7}$$

Assuming fine tuning, the phase matrices $Q_{\nu D}^T$ and $Q_{\nu D}$ along with $-M_R$ can be taken as m_R diag$(1,1,1)$. Making this assumption as well as using the orthogonality of $O_{\nu D}$, it can be shown that using the See-Saw mechanism, the effective neutrino mass matrix can be expressed as

$$M_\nu = P_{\nu D} O_{\nu D} \frac{(M_{\nu D}^{diag})^2}{(m_R)} O_{\nu D}^T P_{\nu D}, \tag{8}$$

m_R being the right handed neutrino mass scale. Further, the lepton mixing matrix can be expressed as

$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D}, \tag{9}$$

where $Q_l P_{\nu D}$, without loss of generality, can be taken as $(e^{i\phi_1},\ 1,\ e^{i\phi_2})$, ϕ_1 and ϕ_2 being related to the phases of mass matrices and can be treated as free parameters.

3 Inputs used for the analysis

For the purpose of calculations, we have made use of the results of a latest global three neutrino oscillation analysis [8]. Further, the phases ϕ_1, ϕ_2 and the elements $D_{l,\nu}$, $C_{l,\nu}$ are considered to be free parameters. For all the three possible mass hierarchies of neutrinos, the explored range of the lightest neutrino mass is taken to be $10^{-8}\,\mathrm{eV} - 10^{-1}\,\mathrm{eV}$, our conclusions remain unaffected even if the range is extended further. In the absence of any constraint on the phases, ϕ_1 and ϕ_2 have been given full variation from 0 to 2π. Although $D_{l,\nu}$ and $C_{l,\nu}$ are free parameters, however, they have been constrained such that diagonalizing transformations O_l and O_ν always remain real. For the numerical analysis, we generate 10^7 random points (10^9 when the number of allowed points is small).

Parameter	1σ range	3σ range
Δm^2_{sol} $[10^{-5}eV^2]$	(7.32-7.80)	(6.99-8.18)
Δm^2_{atm} $[10^{-3}eV^2]$	(2.33-2.49)(NH); (2.31-2.49) (IH)	(2.19-2.62)(NH); (2.17-2.61)(IH)
$sin^2\theta_{13}$ $[10^{-2}]$	(2.16-2.66)(NH); (2.19-2.67)(IH)	(1.69-3.13)(NH); (1.71-3.15) (IH)
$sin^2\theta_{12}$ $[10^{-1}]$	(2.91-3.25)	(2.59-3.59)
$sin^2\theta_{23}$ $[10^{-1}]$	(3.65-4.10)(NH);(3.70-4.31)(IH)	(3.31-6.37)(NH);(3.35-6.63)(IH)

Table 1: Ranges of neutrino oscillation parameters [8].

4 Results and discussions

The effective Majorana mass in the neutrinoless double beta decay can be defined as

$$\langle m_{ee} \rangle = m_{\nu_1} U_{e1}^2 + m_{\nu_2} U_{e2}^2 + m_{\nu_3} U_{e3}^2. \tag{10}$$

Using the methodology outlined in Section (2), we calculate m_{ee} for the most general lepton mass matrices in SM given in equation (4) pertaining to normal as well as inverted neutrino mass orderings. To this end, in figure (1), we have plotted the parameter m_{ee} with respect to the phases ϕ_1 and ϕ_2 pertaining to normal hierarchy (NH) of neutrino masses. While plotting these figures, all the three mixing angles have been constrained by their 3σ experimental bounds. A careful look at these plots clearly establishes a lower bound $\sim$ 0.1 meV for m_{ee} in the NH scenario of neutrino masses. As a next step, we study the dependence of the parameter m_{ee} on the lightest neutrino mass. For this purpose, in figure (2) we have presented the parameter space of m_{ee} with respect to the lightest neutrino mass for mass matrices given in equation (4) pertaining to the normal and inverted hierarchy (IH) of neutrino masses respectively. These plots clearly indicate that for NH a lower bound $\sim$ 2 meV can be obtained for the lightest neutrino mass whereas for the IH case it remains largely unrestricted while a lower bound $\sim$ 1 meV can be obtained for m_{ee}.

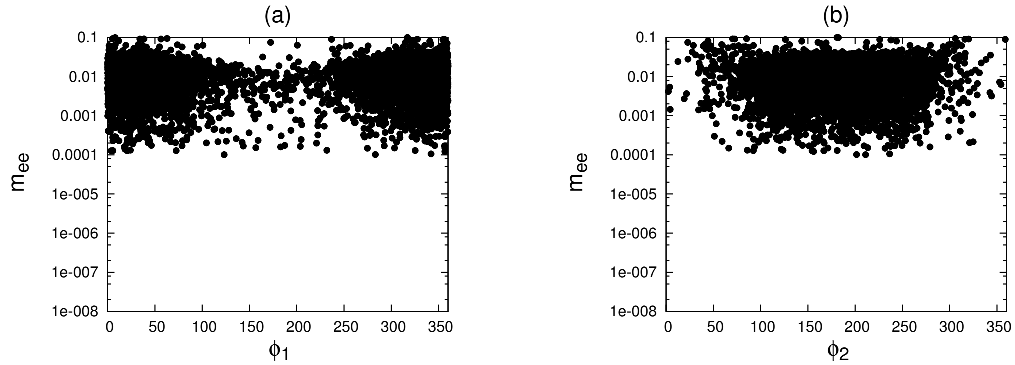

Figure 1: Plots showing the variation of the parameter m_{ee} with respect to the phase (a)ϕ_1 and (b) ϕ_2 pertaining to the normal hierarchy of neutrino masses.

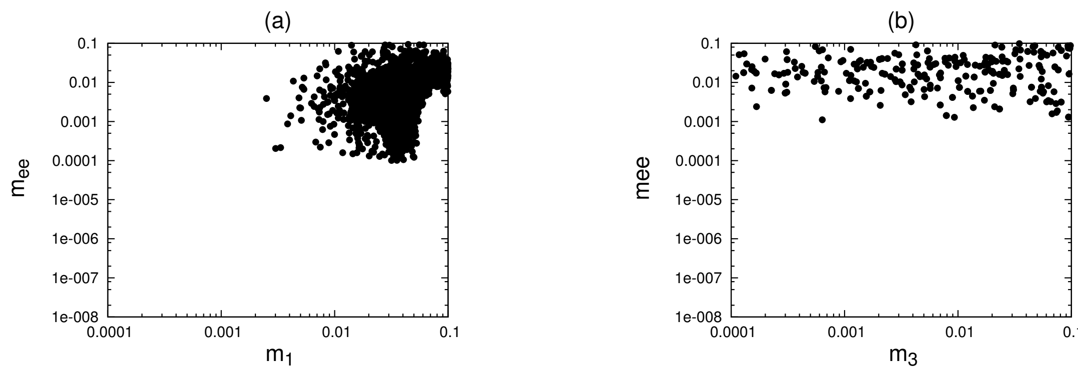

Figure 2: Plots showing the dependence of the parameter m_{ee} with the lightest neutrino mass pertaining to the (a) normal hierarchy and (b) inverted hierarchy of neutrino masses.

5 Summary and conclusions

This paper contains the preliminary results of our analyses wherein, starting with the most general lepton mass matrices within the framework of SM using the facility of WB transformations, we have attempted to obtain bounds on the parameter m_{ee} and the lightest neutrino mass for different neutrino mass hierarchies. In the light of the bounds so obtained, the future experiments in this direction are, thus, expected to have important implications for determining the texture structure of lepton mass matrices.

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