# Summary and Conclusions

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Understanding of fermion masses and mixings constitutes one of the most important problems in the present day high energy physics. The problem gets further complicated when one notices that the pattern of masses and mixings are quite different in case of quarks and leptons. In fact, in the case of quarks we have clearly hierarchical structure of the masses and mixing angles. In contrast, the two of the mixing angles in case of neutrinos are quite large, whereas the third angle although small as compared to the other two angles yet it is of the order of the Cabbibo angle in the quark sector. Similarly, the pattern of masses in the case of charged leptons has a very well defined hierarchy, whereas in the case of neutrino we may have normal/inverted hierarchy or degenerate scenario of neutrino masses. Further, we have knowledge of only mass square differences, absolute masses of the neutrinos are not known.

At present, there is no approach within or beyond the SM which provides a viable description of fermion masses, mixing parameters and related phenomenology. In this context, at present the approaches to understand fermion masses and mixings can be put in two broad categories 'top-down' and 'bottom-up'. The top-down approach essentially starts with the formulation of mass matrices at the GUT scale, whereas, the bottom-up approach starts with the phenomenological mass matrices at the weak scale. In the present thesis, as an example of the bottom-up approach, we have investigated texture specific mass matrices in the light of ever increasing precision in the mixing data of quarks and leptons. In this context, the recent measurement of neutrino mixing angle $\theta_{13}$ provides the necessary motivation to revisit some of the texture specific mass matrices already considered viable in the leptonic case. Further, this observation of not so small neutrino mixing angle $\theta_{13}$ also motivates one to look for possibilities of CP violation in the leptonic sector, perhaps taking clues from the quark sector where CP is very well established

To begin with, in Chapter III, in the light of recent refinements in lepton mixing data, we have made an attempt to extend the analysis of FSTY wherein for Majorana neutrinos they have carried out an analysis of texture 6 zero Fritzsch lepton mass matrices for normal hierarchy of neutrino masses. Apart from reproducing their results for texture 6 zero mass matrices, for texture 5 zero Majorana neutrino mass matrices also detailed predictions for cases pertaining to normal/inverted hierarchy as well as degenerate scenario of neutrino masses have been carried out. Detailed dependence of mixing angles on the lightest neutrino mass has also been investigated.

Interestingly, we find that all cases pertaining to inverted hierarchy and degenerate scenario of neutrino masses seem to be ruled out for both texture 6 zero as well as texture 5 zero Majorana neutrinos mass matrices. This has been concluded from the plots of the three mixing angles against the lightest neutrino mass.

The present analysis indicates that the present $3\sigma$ C.L. range of $\theta_{13}$ does not put a reasonable constraint on the value of lightest neutrino mass $m_{\nu_1}$, therefore, refinements of its value will have important implications for $m_{\nu_1}$. Regarding the effective Majorana mass $\langle m_{ee} \rangle$, one finds that for texture 5 zero lepton mass matrices when $D_l = 0$ and $D_\nu \neq 0$ the $1\sigma$ C.L. range of the mixing angle $\theta_{13}$ constrains the value of $\langle m_{ee} \rangle$ to be 2.3-8.7 meV. This range looks to be somewhat expanded in comparison to the one obtained in the case of texture 6 zero mass matrices due to the additional parameter $D_\nu$. Therefore, it seems that measurements of $m_{\nu_1}$ and $\langle m_{ee} \rangle$ would have important implications on texture specific mass matrices considered here.

As a next step, in Chapter IV, keeping in mind that Dirac neutrinos have not been ruled out experimentally, we have carried out an analysis of texture 6 zero and texture 5 zero Fritzsch-like lepton mass matrices. In particular, analogous to the analysis carried out for Majorana neutrinos, we have carried out detailed calculations pertaining to three cases, i.e., texture 6 zero Fritzsch mass matrices and two possible cases of texture 5 zero Fritzsch-like hermitian mass matrices, $D_l = 0$ case and $D_\nu = 0$ case. Corresponding to each of these cases, we have considered three possibilities of neutrino masses having normal/inverted hierarchy and degenerate scenario. The detailed dependence of mixing angles on the lightest neutrino mass have been investigated for texture 6 zero as well as for texture 5 zero cases. The analysis leads to several interesting results. For Dirac neutrinos, all the cases pertaining to inverted hierarchy and degenerate scenario of the neutrino masses have been ruled out for texture 6 zero as well as two cases of texture 5 zero mass matrices. Parallel to the Majorana case these conclusions have been arrived at from the plots of the mixing angles versus the lightest neutrino mass. Further, interestingly, in the case of texture 6 zero mass matrices and the texture 5 zero $D_\nu = 0$ case, the normal hierarchy of neutrino masses is also ruled out at $1\sigma$ C.L.. Refinements in the data can make these conclusion more rigorous.

Corresponding to the texture 5 zero $D_l = 0$ case, the normal hierarchy of neutrino masses is viable and the plot of the mixing angle $s_{12}$ versus $m_{\nu_1}$ provides an upper bound on $m_{\nu_1} \sim 0.01$eV. The PMNS matrix for this case has also been constructed which shows good deal of compatibility with a recently constructed PMNS matrix by Garcia . Further, one finds that out of the two free parameters of the mass matrices, $D_l$ and $D_\nu$, the parameter $D_\nu$ plays a more important role in establishing the compatibility of texture 5 zero Dirac neutrino mass matrices. In this context, variation of $D_\nu$ with the three mixing angles has been examined and one finds that the angle $s_{23}$ constrains the range of $D_\nu$ to $\sim 0.01 - 0.03$eV.

After presenting the analysis of texture 6 zero and texture 5 zero lepton mass matrices for both Majorana and Dirac neutrinos, we discuss the case of quarks in Chapter V. In this context, it is known that texture 6 zero mass matrices are completely ruled out whereas texture 5 zero mass matrices are largely ruled out.

On the other hand, texture 4 zero mass matrices are known to be compatible with the data. In Chapter V, we have analyzed texture 5 and texture 4 zero quark mass matrices keeping in mind the recently refined data. Further, the relationship of textures with the weak basis transformations, both in the particular scenario of Giraldo as well as in the general scenario of Fritzsch and Xing and Branco *et al.* have been investigated.

Using the latest inputs regarding masses and mixing parameters, we find that the texture 5 zero $D_D$=0 and $D_U \neq 0$ case is completely ruled out whereas the other texture 5 zero $D_U$=0 and $D_D \neq 0$ case has limited viability depending upon the mass of strange quark $m_s$ used. A similar analysis of texture 4 zero Fritzsch-like mass matrices reveals that these continue to be compatible with the recently refined data. This brings into fore the issue of compatibility of non Fritzsch-like texture 4 zero quark mass matrices as well as the relationship of textures with the general principles like the WB transformations, naturalness, etc..

In this context, we have analyzed the recent approach of Giraldo, wherein he has constructed texture specific mass matrices using WB transformations. Interestingly, when we subject these matrices to condition of naturalness as well as CP violation phase sensitive parameters, we find that one has to be more careful while formulating texture specific mass matrices.

To this end, we have used the approach of Branco *et al.* to formulate all possible types of texture 4 zero mass matrices using permutation symmetry as part of WB transformations. To begin with, we find that WB transformations reduce a general mass matrix to texture 1 zero mass matrix. Further, using the condition of naturalness, we have taken note of the fact these can be reduced to texture 2 zero mass matrices without the loss of generality. Therefore, we have considered all possible texture 4 zero quark mass matrices for our analysis. In this regard, using strict parallelism between $M_U$ and $M_D$ mass matrices we find that we are left with only Fritzsch-like viable possibility of texture 4 zero quark mass matrices.

As a next step in Chapter VI, we have made an attempt to explore CP violation in the quark and leptonic sector. In the absence of any hints from the data regarding leptonic CP violation, keeping in mind the parallelism between neutrino mixing and quark mixing, we have first explored the case of quarks to test the techniques used to carry out the analysis as well as to obtain useful hints for the case of leptons. It may also be mentioned that we have made an attempt to examine CP violation using two approaches namely the unitarity triangle approach and through the commutator formalism involving texture specific mass matrices.

For the case of quarks, as a first step we have carried out the analysis using the unitarity triangle approach. For *db* triangle, we have plotted histograms for the Jarlskog's rephasing invariant parameter $J$ and the CP violating phase $\delta$ and from these we find $J = (3.36 \pm 0.38) \times 10^{-5}$ and $\delta = 62.60^{\circ} \pm 10.98^{\circ}$. Interestingly, these $J$ and $\delta$ values are found to be compatible with those given by PDG 2012. As a next step, we have evaluated the parameter $J$ through the commutator of the mass matrices $M_U$ and $M_D$. In this context, it may be noted that for hermitian mass matrices, one has two possible formulae to find $J$, referred to as corresponding to $J_{m^2}$ and $J_m$ in the text. Using the texture 4 zero quark mass matrices, we find that out of the two, the one corresponding to $J_{m^2}$ seems appropriate for studying CP violation.

Coming to the case of leptons, in view of the latest T2K and MINOS observations regarding the mixing angle $s_{13}$ along with the other two well measured mixing angles $s_{12}$ and $s_{23}$, we have explored the possibility of existence of CP violation in the leptonic sector. Taking clues from the construction of the $db$ unitarity triangle in the quark sector, we have made an attempt to construct the '$\nu_1.\nu_3$' leptonic unitarity triangle, suggesting a good possibility of having non zero CP violation. In particular, we find likely value of the CP violating phase in the leptonic sector to be $55^o \pm 14^o$. Regarding the possibility of CP violation being explored using the commutator formalism involving lepton mass matrices $M_l$ and $M_\nu$, one again finds that the formula corresponding to $J_{lm^2}$ seems appropriate.

In conclusion, one may state that texture specific mass matrices, not only provide clues for quark-lepton unification but also suggests viable phenomenological structure for describing fermion mixing data. A rigorous analysis of these may provide guiding stones for the formulation of viable theories of flavor physics. Similarly, a detailed comparison of quark mixing and lepton mixing may reveal hints for CP violation in the leptonic sector.