Thermodynamics of Atomic Quantum Gases with Synthetic Spin-Orbit Coupling

G. S. Singh

Department of Physics
Indian Institute of Technology Roorkee

Collaborators: Reena Gupta (IIT Roorkee)
K. N. Pathak (PU, Chandigarh)
Jürgen Bosse (FU Berlin, Germany)

QGBECS2014

PU Chandigarh, 3-6 March 2014
Talk Based on the Papers:

1. Gupta, Singh, and Bosse
   Weyl spin-orbit-coupling-induced interactions in uniform and trapped atomic quantum fluids

2. Bosse, Pathak, and Singh
   Bunching and Antibunching in Quantum Gases

3. Gupta and Singh
   *To be published*
   Low & High Dimensional Systems with Synthetic SOC
OUTLINE OF THE TALK

- Motivation for the Problem
- Statistical Interactions: An Overview
- 3D Spin-Orbit Coupling (Weyl coupling)
  A. Hamiltonian & Symmetries
  B. Dispersion Relations
  C. Density of States
- Grand Potential: Exact Analytical Expression
- Other Thermodynamic Quantities: Analytical Expressions
- Conclusions
Motivation for the Problem

**SO Coupled Fermions:**

   - Isothermal compressibility increases, and the Pauli pressure decreases, with increase in SOC strength.

   - Fermionic superfluidity destroyed by the Zeeman field above a critical value can be restored by a finite SOC.

   - Two-body bound states on the BCS side of a resonance and BCS-BEC crossover even for vanishingly small scattering length on increasing SOC
**SO Coupled Fermions:**


   **SOC enhances the short-distance correlation length**

**SO Coupled Bosons:**


   **Coupling creates condensate depletion**


   **Coupling creates shift in the BEC transition temp.**

**Features of SO coupled fermionic systems explained plausibly by others in terms of enhancement in the single-particle density of states**
Spin-orbit coupling in Solids

Dresselhaus SOC
\[ \propto (k_x \sigma_y - k_y \sigma_x) \]

Rashba SOC
\[ \propto (k_x \sigma_x + k_y \sigma_y) \]
Experimental Principle for Synthetic SOC in Ultracold Atomic Fermionic or Bosonic Gases

Synthetic coupling turns out to be of pure Rashba or pure Dresselhaus or a mixture of both or Weyl. Concept of psuedo-spin
Interesting Concepts

- Integral spin (Bosons)
- Half-odd-integral spin (Fermions)
- Pseudospin-half bosons or fermions.

No contradiction with the spin-statistics theorem.

Pseudospin degree of freedom assigned to the dressed hyperfine states.

- Study of ultracold atoms and matter waves at the forefront since 1995 to mimic many condensed-matter phenomena.

Now additional impetus.

Synthetic SOC makes the systems behave in ways that have no known analogues in the condensed matter systems.
Motivation for the Problem

**Synthetic SO Coupled Fermions:**

Evolution from BCS to BEC superfluidity in the presence of spin-orbit coupling.

Rashba Coupling:
Isothermal compressibility increases, and the Pauli pressure decreases, with increase in SOC strength.

**Synthetic SO Coupled Bosons:**

Rashba Coupling:
The critical temperature is not affected in the thermodynamic limit in which the number of particles becomes infinitely large.
SO Coupled Bosons:

   
   **Coupling creates condensate depletion.**
   For both 3D SOC and 2D SOC (Rashba) in 2D systems.

   
   **Coupling creates shift in the BEC transition temp.**
   For equal Rashba-Dresselhaus SOC in 3D systems.
**Question:** Is there any intrinsic many-body effect due to synthetic spin-orbit coupling?

**Answer:** Yes, we have found in terms of modifications in “statistical interactions”.

In fact, many interesting physical phenomena predicted theoretically, both in fermions and bosons, can be explained in terms of modifications in the statistical potential.

**What are statistical interactions?**
Ideal Quantum Gases without SO coupling

Equation of state

\[ PV \approx N k_B T \left[ 1 \pm \frac{\Lambda^3}{8\sqrt{2} V} \frac{N}{N} \right] \]

at high T

+ Fermi

- Bose

Thermal de Broglie wavelength:

\[ \Lambda = \left( \frac{2\pi \hbar^2}{mk_B T} \right)^{1/2} \]

Quantum correction has appreciable value only at low temperatures; pressure of an ideal Fermi gas increases as compared to a Boltmann gas whereas that of a Bose gas decreases.

This implies that an ideal Fermi gas has an “effective” repulsion whereas an ideal Bose gas has an “effective” attraction.

This is due to symmetry requirements of the many-body wave functions
Spin-orbit coupled bosons or fermions; pseudospin-\(\frac{1}{2}\) particles

Effective magnetic field:
\[
\vec{B} = \vec{V} \times \vec{A}
\]

Effective vector potential:
\[
\vec{A} = -\hbar \kappa \vec{\sigma}
\]

Non-abelian gauge pot. because:
\[
\vec{\sigma} \times \vec{\sigma} = 2i\vec{i} \vec{\sigma}
\]

Single-particle Hamiltonian:
\[
H = \frac{1}{2m} (\vec{p} I - \vec{A})^2
\]
\[
H = \frac{1}{2m} (p^2 + \hbar^2 \kappa^2) I + v \vec{\sigma} \cdot \vec{p}
\]

Rewritten as


For 3D coupling, SO coupling term is
Weyl-like Hamiltonian. It has been termed as the
Weyl Spin-Orbit Coupling (Anderson et al., PRL 108, 2012.)
Weyl coupling is a 3D analogue of the Rashba coupling which is 2D.

Symmetries of

\[ H = \frac{1}{2m} (p^2 + \hbar^2 \kappa^2) I + v \vec{\sigma} \cdot \vec{p} \]
[For a trapping potential, \( U(r) \) to be added]

1. Highest rotational symmetry:
\[ [\hat{J}, \hat{H}] = 0; \quad \hat{J} = \hat{L} + \frac{1}{2} \hbar \vec{\sigma} \]

2. Time-reversal symmetry

3. No Galilean invariance

4. No parity conservation
Dispersion relation has two helicity branches:

$$\varepsilon_{\pm}(k) = \frac{\hbar^2}{2m} (k \pm \kappa)^2$$

There are two constant-energy surfaces. Consider fermions at $T=0$. As coupling strength increases, inner surface shrinks and the outer one grows. Ultimately, the inner surface reduces to a point. Thus there is a Fermi-surface topological transition.
Density of states of $s = +$ or $-$ branch

Total density of states

DOS as a function of energy

(a) Total DOS for various coupling strengths:
Lowest – No SOC;
SOC increases upward

(b), (c)
Green: upper (+) branch
Blue: lower (-) branch
Red: total
Grand potential:
\[ \Omega(T, V, \mu) = \frac{V}{\eta \beta} \int_0^\infty \rho(\varepsilon) \ln(1 - \eta z e^{-\beta \varepsilon}) d\varepsilon \]

Finally, exact analytical form (unified expression)
\[ \Omega = -\frac{2V}{\eta \beta \Lambda^3} \left[ Li_{5/2}(\eta z) + \varphi(\kappa \Lambda) Li_{3/2}(\eta z) \right] \quad \varphi(\kappa \Lambda) = \frac{\kappa^2 \Lambda^2}{2\pi} \]

\( \eta = 0 \) (Boltzmann), -1 (Fermi), +1 (Bose):
Unifying parameter


Polylogarithmic function:
\[ Li_{v+2}(u) = -\frac{1}{\Gamma(v + 1)} \int_0^\infty x^v \ln(1 - ue^{-x}) dx; \quad v > -1 \]
Parameter attached
With curves:
values of coupling
in (a) & (c);
Values of temperature
in (b) & (d).

As coupling strength increases keeping
temperature fixed,
pressure decreases
For fermions (a) & (b);
pressure increases for
bosons (c) & (d).
Isothermal compressibility shows just reverse trend except at very low temperatures in fermions.

The anomaly has been attributed to the anomalous behaviour of chemical potential and hence some other thermodynamic quantities due to dimensional reduction for spin-orbit coupled fermions.
We examine the effect of spin-orbit coupling analytically also. For this we consider virial expansion in the presence of SO coupling:

\[ P \beta \cong n \left[ 1 - \frac{\eta \bar{\xi}}{2^2} \frac{1 + 2 \varphi(\kappa \Lambda)}{1 + \varphi(\kappa \Lambda)} + \frac{\eta^2 \bar{\xi}^2}{2^3} \left( \frac{1 + 2 \varphi(\kappa \Lambda)}{1 + \varphi(\kappa \Lambda)} \right)^2 - \frac{2^4}{3^{5/2}} \frac{1 + 3 \varphi(\kappa \Lambda)}{1 + \varphi(\kappa \Lambda)} \right] \]

\[ \varphi(\kappa \Lambda) = \frac{\kappa^2 \Lambda^2}{2\pi} ; \quad \bar{\xi} = \frac{n \Lambda^3}{2(1 + \varphi(\kappa \Lambda))} \]

Thus the result discussed earlier gets corroborated analytically.

Also, complete thermodynamics has been studied using exact analytical expression for the grand potential.
Experimental Observations:

- Spin–Orbit-Coupled Bose–Einstein Condensates
  *Nature* 471, 83 (03 March 2011)

- Collective Dipole Oscillations of a Spin-Orbit Coupled Bose-Einstein Condensate
  *Phys. Rev. Lett.* 109, 115301 (14 September 2012)

- Spin-Orbit Coupled Degenerate Fermi Gases
  *Phys. Rev. Lett.* 109, 095301 (31 August 2012)

- Spin-Injection Spectroscopy of a Spin-Orbit Coupled Fermi Gas
  *Phys. Rev. Lett.* 109, 095302 (31 August 2012)
Summary and Conclusions

- SO coupled fermions (bosons) have decreased (increased) pressure as compared to the gas without coupling;
- The result is corroborated analytically through virial expansion and also by plots of isothermal compressibility.

Thus effective statistical attraction in bosons has decreased.
Also, effective repulsion in fermions has decreased.

This implies that the statistical interaction is weakened due to SO coupling both in bosons and fermions.
Summary and Conclusions

The dimensional reduction due to SO coupling shows anomalous behaviour of thermodynamic quantities in fermions. Also, the fermi system has different behaviour when the chem. pot. is above or below the Dirac point.

Very weak coupling has precursor effect of BEC although the system does not really go to the BEC phase.

“Effective” attraction in SO coupled fermions and “effective” repulsion in SO coupled bosons as compared to the corresponding gas without coupling can be utilized to explain many features of the gases studied theoretically and explained earlier for fermions in terms of enhanced single-particle density of states.