Frequency/wavelength of Hawking radiations as characteristics of non-spinning black holes

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Abstract

Aims:To derive an expression for the wavelength/frequency of Hawking radiation emitted by non-spinning black holes in terms of the radius of event horizon($\lambda = 8\pi R_s$ & $\nu = c/8\pi R_s$), which may be regarded as the characteristics of non-spinning black holes.

Study Design: Data for the frequencies and wavelengths of Hawking radiation emitted from black holes have been calculated with the help of rest masses for stellar – mass black holes (M ~ 5 - 20 $M_\odot$) in X-ray binaries and for the super massive black holes (M ~ 10^6 – $10^{9.5}$ $M_\odot$) in active galactic nuclei using $\lambda = 8\pi R_s$ & $\nu = c/8\pi R_s$ which corresponds to the research work entitled: Frequency of Hawking radiation from black holes by Mahto et al. in International Journal of Astrophysics and Space Science (Dec.2013),

Place and Duration of Study: Department of Physics, Marwari College under University Department of Physics, T.M.B.U. Bhagalpur between January 2014 and May 2014.

Methodology: It is completely theoretical based work using Laptop done at Marwari College Bhagalpur and the residential research chamber of the first author.

Results: The astrophysical objects emitting the radiations of frequencies (8.092×10^2Hz to 2.023×10^2Hz) or wavelengths (3.707×10^5 m to 14.828×10^5m) in X-ray binaries and frequencies (4.046×10^{-3} Hz to 0.809×10^{-6} Hz) or wavelengths (7.414×10^{10} m to 37.070×10^{13} m) in active galactic nuclei may be classified as non-spinning black holes

Conclusion: The frequencies or wavelengths of Hawking radiation emitted from non-spinning black holes may be regarded as the characteristics of black holes in addition to the mass, spin and charge.

Keywords

Schwarzschild radius, Hawking radiation, XRBs and AGN.

Introduction: A stationary black hole is parameterized by just a few number: mass, electric charge and angular momentum (and magnetic monopole charge, except its actual existence in nature has not been demonstrated yet)(Ruffini and Wheeler 1970; Mahto et al. 2011). Kanak Kumari et al. used the Schwarzschild radius to characterize the non-spinning and spinning black holes in addition to the mass, spin and charge (Kanak et al. 2012). In classical theory black holes can only absorb and not emit particles. However, it is shown that quantum mechanical effects cause black holes to create and emit particles as if they were hot bodies with temperature $\frac{h\kappa}{2\pi k} \approx 10^{-6}\left(\frac{M_{\odot}}{M}\right)\kappa$, where κ is surface gravity of black holes and k is the Boltzmann constant(Hawking 1975), but according to the general theory of relativity: A black hole is a solution of Einstein's gravitational field equations in the absence of matter that describes the space time around a gravitationally collapsed star. Its gravitational pull is so strong that even light cannot escape from it (Dabholkar 2005; Mahto et al. 2012**)**

In this present research paper, we have derived an expression for the wavelength and frequency of Hawking radiation emitted by non-spinning black holes in terms of the radius of event horizon, which may be regarded as the characteristics of non-spinning black holes in addition to mass, spin, charge, Schwarzschild radius etc.

Expression for the frequency/wavelength of black holes in terms of Schwarzschild radius: According to the quantum mechanics, the empty space is not empty at all. In this empty space, there are always particle flashing in to existence and disappear again. They always come in pairs; one particle and one anti-particle, like an electron and a positron, or a photon and another photon with opposite spin and impulse. These particles are called virtual particles. If one virtual particle falls into the black hole and the other escapes as Hawking radiation from black hole. The energy of radiated photons is given by the following equation (Mahto et al. 2013)

$$E = h\nu \quad \text{...(1)}$$

The energy of a photon of Hawking radiation is given by the following equation (Hawking radiation, htt://library.thinkquest.org/c007571/English/printcore.htm, 2011)

$$E = \frac{hc^3}{16\pi GM} \quad \text{..(2)}$$

From equation(1) and (2), we have

$$\nu = \frac{c^3}{16\pi GM} \quad \text{..(3)}$$

All the terms like gravitational constant (G), Planck constant (h) and velocity of light(c) in right hand side of equation (5) are constant except mass (M) of the black hole. These constants have vital role discussed in the research paper (Dabholkar 2005)
The frequency of radiation is given by

$$\nu = \frac{c}{\lambda} \quad \text{..(4)}$$

From equations (3) and (4), we have

$$\lambda = \frac{16\pi GM}{c^2} \quad \text{..(5)}$$

$$\lambda = 8\pi \frac{2GM}{c^2} \quad \text{..(6)}$$

For non-spinning black holes, the radius of event horizon is given by (Narayan, 2005)

$$R_s = \frac{2GM}{c^2} \quad \text{..(7)}$$

Putting the value of equation (7) into equation(6), we have

$$\lambda = 8\pi R_s \quad \text{..(8)}$$

From equation (4), we have

$$\nu = \frac{c}{8\pi R_s} \quad \text{..(9)}$$

The relation (9) shows that the frequency of Hawking radiation emitted by the black holes is inversely proportional to the radius of event horizon of black holes where as from relation (8) it is clear that the wavelength of Hawking radiation emitted by the black holes is directly proportional to the radius of event horizon of the black holes. This means that heavier black holes will emit the Hawking radiation of lower frequency or longer wavelength and vice-versa.

Data in support of Schwarzschild radius/radius of the event horizon of black holes in XRBs and AGN:

There are two categories of Black holes classified on the basis of their masses clearly very distinct from each other, with very different masses M ~ 5 - 20 $M_\odot$ for stellar – mass Black holes in X-ray binaries and M ~ 10^6 – $10^{9.5}$ $M_\odot$ for super massive black holes in Galactic Nuclei (Narayan 2005,Mahto et al. 2012). The Schwarzschild radius/radius of the event horizon of non-spinning black holes corresponding to the masses M ~ 5 - 20 $M_\odot$ for stellar – mass Black holes in X-ray binaries are 14750 metre to 59000 metre and for the masses M ~ 10^6 – $10^{9.5}$ $M_\odot$ in super massive black holes in the Active Galactic Nuclei are 2.950x10^9 metre to 1475x10^{13}metre (Mahto et al., 2013)

Table 1:

Wavelength and frequency of non-spinning black holes in XRBs.				
S. No	Mass of BH_s (M)	$R_s = 2950\frac{M}{M_\odot}$ (in metre)	Wavelength $\lambda = 8\pi R_s$ metre.	.Frequency $\nu = \frac{c}{8\pi R_s}$ Hz
1	5$M_\odot$	14750	3.707x10^5	8.092x10^2
2	6 $M_\odot$	17700	4.448 x10^5	6.744 x10^2
3	7 $M_\odot$	20650	5.189 x10^5	5.781 x10^2
4	8 $M_\odot$	23600	5.913 x10^5	5.073 x10^2
5	9 $M_\odot$	26550	6.672 x10^5	4.496 x10^2
6	10 $M_\odot$	29500	7.414 x10^5	4.046 x10^2
7	11 $M_\odot$	32450	8.155 x10^5	3.678 x10^2
8	12 $M_\odot$	35400	8.996 x10^5	3.334 x10^2
9	13 $M_\odot$	38350	9.638 x10^5	3.112 x10^2
10	14 $M_\odot$	41300	10.379 x10^5	2.890 x10^2
11	15 $M_\odot$	44250	11.121 x10^5	2.697 x10^2
12	16 $M_\odot$	47200	11.862 x10^5	2.529 x10^2
13	17 $M_\odot$	50150	12.604 x10^5	2.380 x10^2
14	18 $M_\odot$	53100	13.345 x10^5	2.248 x10^2
15	19 $M_\odot$	56050	14.086 x10^5	2.129 x10^2
16	20 $M_\odot$	59000	14.828 x10^5	2.023 x10^2

Table 2:

Wavelength and frequency of non-spinning black holes in AGN.							
S. No.	Mass of BH_s (M)	$R_s = 2950 \frac{M}{M_\odot}$ (in metre)	$\log(R_s)$	$\lambda = 8\pi R_s$ (in metre)	$\log(\lambda)$	$\nu = \frac{c}{8\pi R_s}$ Hz	$Mod \log(\nu)$
1	1 x 10^6 $M_\odot$	2.950 x10^9	9.4698	7.414×10^{10}	10.8700	4.046×10^{-3}	3.6070
2	2 x 10^6 $M_\odot$	5.950 x10^9	9.7709	14.953×10^{10}	11.1747	2.006×10^{-3}	3.3023
3	3 x 10^6 $M_\odot$	8.850 x 10^9	9.9469	22.242×10^{10}	11.3471	1.348×10^{-3}	3.1296
4	4 x 10^6 $M_\odot$	1.180x10^{10}	10.0719	29.656×10^{10}	11.4721	1.011×10^{-3}	3.0047
5	5 x 10^6 $M_\odot$	1.475x10^{10}	10.1688	37.070×10^{10}	11.5690	0.809×10^{-3}	3.0920
6	6 x 10^6 $M_\odot$	1.770x10^{10}	10.2480	44.448×10^{10}	11.6478	0.674×10^{-3}	3.1713
7	7 x 10^6 $M_\odot$	2.065x10^{10}	10.3149	51.899×10^{10}	11.7151	0.578×10^{-3}	3.2380
8	8 x 10^6 $M_\odot$	2.360x10^{10}	10.3729	59.313×10^{10}	11.7731	0.505×10^{-3}	3.2967
9	9 x 10^6 $M_\odot$	2.655x10^{10}	10.4241	66.727×10^{10}	11.8243	0.449×10^{-3}	3.3477
10	1 x 10^7 $M_\odot$	2.950x10^{10}	10.4698	7.414×10^{11}	11.8700	4.046×10^{-4}	4.6070
11	2 x 10^7 $M_\odot$	5.950x10^{10}	10.7709	14.953×10^{11}	12.1747	2.006×10^{-4}	4.3023
12	3 x 10^7 $M_\odot$	8.850x10^{10}	10.9469	22.242×10^{11}	12.3471	1.348×10^{-4}	4.1296
13	4 x 10^7 $M_\odot$	1.180x10^{11}	11.0719	29.656×10^{11}	12.4721	1.011×10^{-4}	4.0047
14	5 x 10^7 $M_\odot$	1.475x10^{11}	11.1688	37.070×10^{11}	12.5690	0.809×10^{-4}	4.0920
15	6 x 10^7 $M_\odot$	1.770x10^{11}	11.2480	44.448×10^{11}	12.6478	0.674×10^{-4}	4.1713
16	7 x 10^7 $M_\odot$	2.065x10^{11}	11.3149	51.899×10^{11}	12.7151	0.578×10^{-4}	4.2380
17	8x 10^7 $M_\odot$	2.360x10^{11}	11.3729	59.313×10^{11}	12.7731	0.505×10^{-4}	4.2967
18	9 x 10^7 $M_\odot$	2.655x10^{11}	11.4241	66.727×10^{11}	12.8243	0.449×10^{-4}	4.3477
19	1 x 10^8 $M_\odot$	2.950x10^{11}	11.4698	7.414×10^{12}	12.8700	4.046×10^{-5}	5.6070
20	2 x 10^8 $M_\odot$	5.950x10^{11}	11.7709	14.953×10^{12}	13.1747	2.006×10^{-5}	5.3023
21	3 x 10^8 $M_\odot$	8.850x10^{11}	11.9469	22.242×10^{12}	13.3471	1.348×10^{-5}	5.1296

22	4 x 10^8 M_Θ	1.180x10^{12}	12.0719	29.656×10^{12}	13.4721	1.011×10^{-5}	5.0047
23	5 x 10^8 M_Θ	1.475x10^{12}	12.1688	37.070×10^{12}	13.5690	0.809×10^{-5}	5.0920
24	6 x 10^8 M_Θ	1.770x10^{12}	12.2480	44.448×10^{12}	13.6478	0.674×10^{-5}	5.1713
25	7 x 10^8 M_Θ	2.065x10^{12}	12.3149	51.899×10^{12}	13.7151	0.578×10^{-5}	5.2380
26	8 x 10^8 M_Θ	2.360x10^{12}	12.3729	59.313×10^{12}	13.7731	0.505×10^{-5}	5.2967
27	9 x 10^8 M_Θ	2.655x10^{12}	12.4241	66.727×10^{12}	13.8243	0.449×10^{-5}	5.3477
28	1 x 10^9 M_Θ	2.950x10^{12}	12.4698	7.414×10^{13}	13.8700	4.046×10^{-6}	6.6070
29	2 x 10^9 M_Θ	5.950x10^{12}	12.7709	14.953×10^{13}	14.1747	2.006×10^{-6}	6.3023
30	3 x 10^9 M_Θ	8.850x10^{12}	12.9469	22.242×10^{13}	14.3471	1.348×10^{-6}	6.1296
31	4 x 10^9 M_Θ	1.180x10^{13}	13.0719	29.656×10^{13}	14.4721	1.011×10^{-6}	6.0047
32	5 x 10^9 M_Θ	1.475x10^{13}	13.1688	37.070×10^{13}	14.5690	0.809×10^{-6}	6.0920

Result and discussion: In the present paper, we have derived an expression for the wavelength and frequency of Hawking radiation emitted by black holes in terms of the radius of event horizon using the energy of radiated photons of black holes ($E = h\nu$), the energy of a photon of Hawking radiation ($E = \frac{hc^3}{16\pi GM}$) and the radius of event horizon of the non-spinning black holes ($R_s = \frac{2GM}{c^2}$) with proper mathematical operation. From the table 1&2, it is clear that the radiations emitted by black holes are within the range of frequencies from 8.092x10^2Hz to 2.023x10^2Hz or wavelengths from 3.707x10^5 m to 14.828x10^5m in X-ray binaries and frequencies from 4.046×10^{-3}Hz to 0.809×10^{-6}Hz or wavelengths from 7.414×10^{10}m to 37.070×10^{13}m in active galactic nuclei. The observations from the table 1&2, it is also clear that the wavelength of radiations emitted by black holes increases with increase the radius of event horizon of the non-spinning black holes and vice-versa in the case of XRBs as well as AGN, but the frequency of radiations emitted by black holes decreases with increase the radius of event horizon of the non-spinning black holes. These two parameters like wavelength and frequency may be regarded as the characteristics of black holes in addition to the mass, spin and charge, because other characteristics the non-spinning black holes can be estimated with the help wavelength and frequency. Hence it may be concluded that the astrophysical objects emitting the radiations of frequencies (8.092x10^2Hz to 2.023x10^2Hz) or wavelengths (3.707x10^5 m to 14.828x10^5m) in X-ray binaries and frequencies (4.046×10^{-3}Hz to 0.809×10^{-6}Hz) or wavelengths (7.414×10^{10}m to 37.070×10^{13}m) in active galactic nuclei may be characterized as non-spinning black holes.

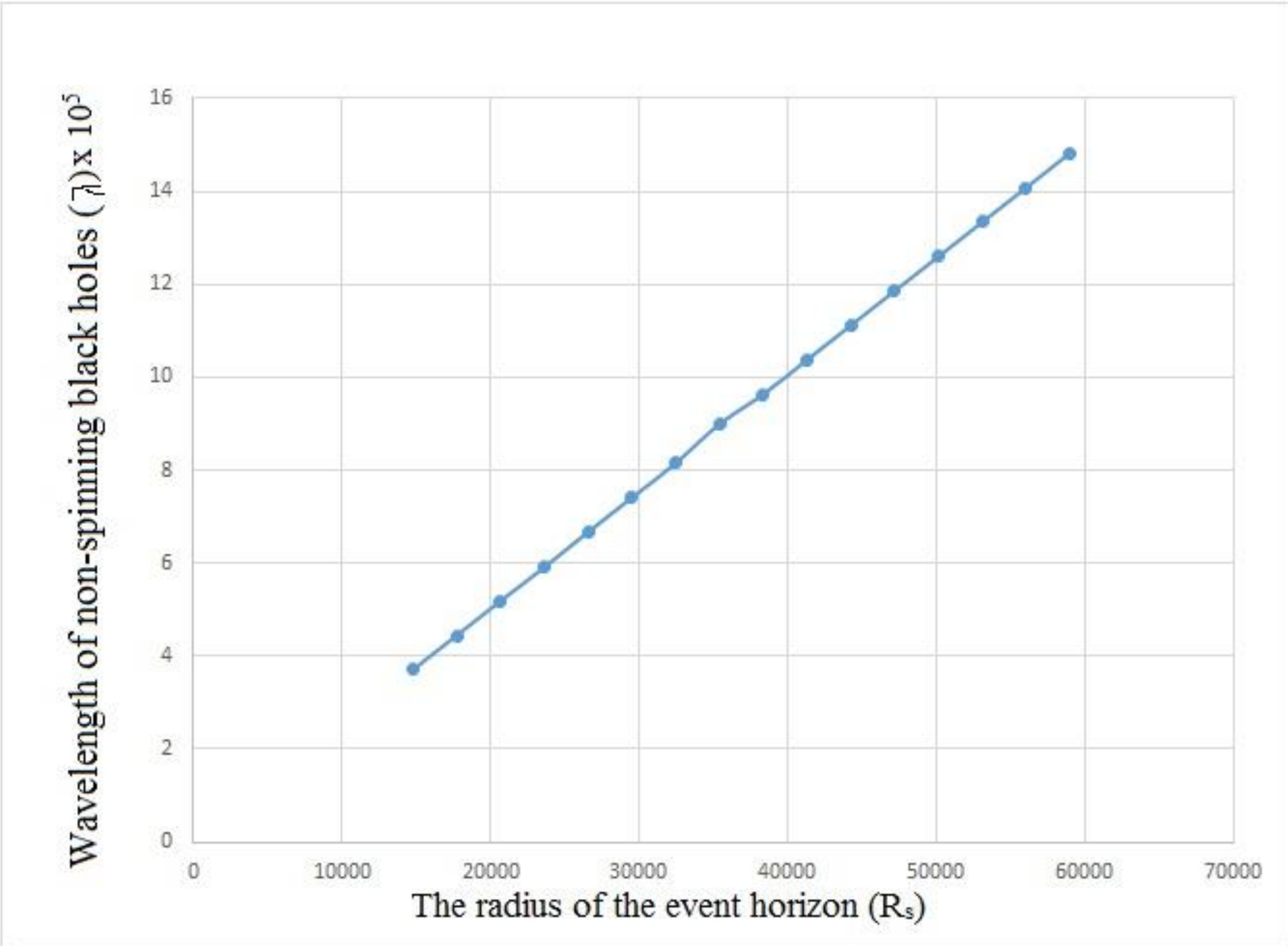
The graphs have been plotted between

(i) the radius of event horizon (R_s) of different black holes and their corresponding wavelength in XRBs (fig.1)

(ii) the radius of event horizon (R_s) of different black holes and their corresponding frequency in XRBs (fig.2)

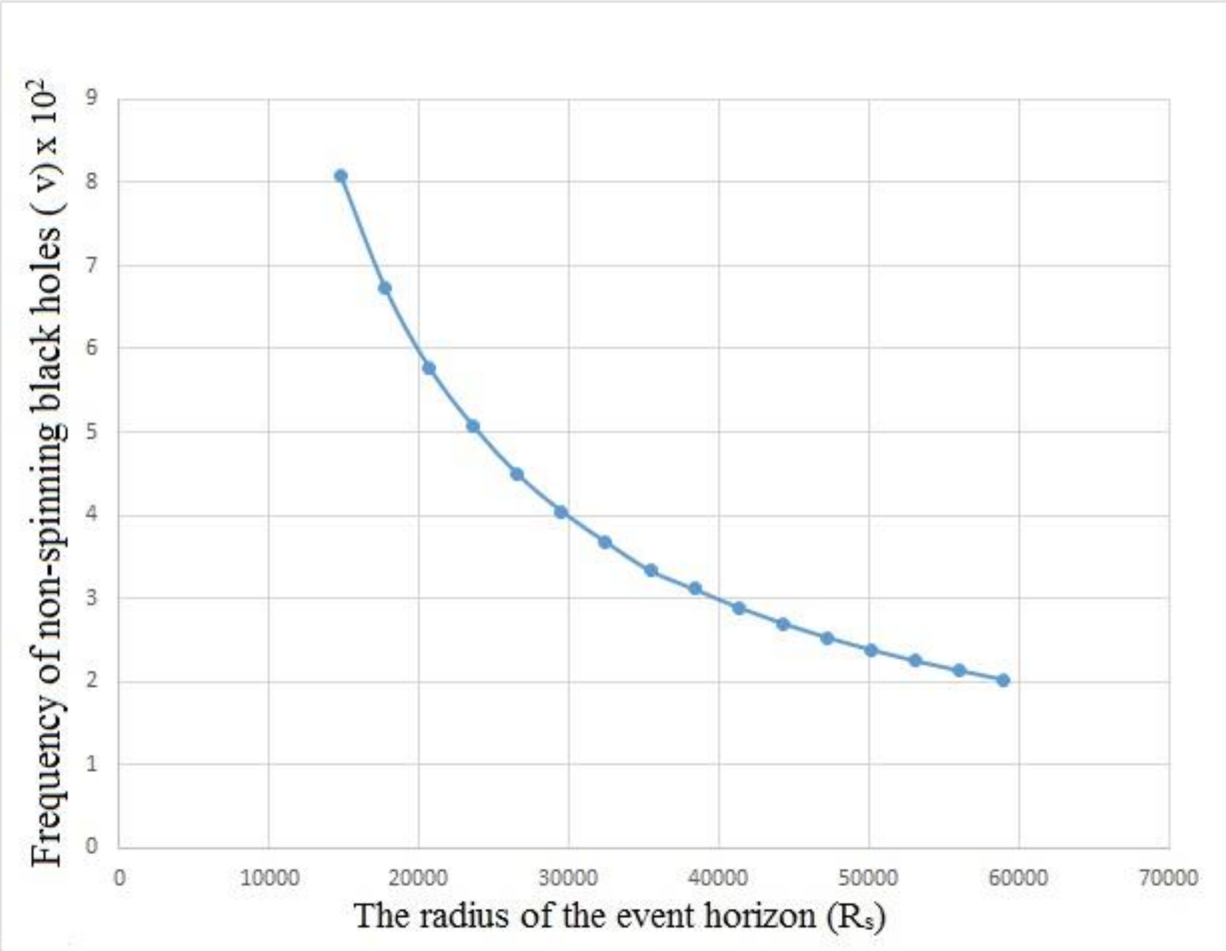
(iii) the radius of event horizon (R_s) of different black holes and their corresponding wavelength in AGN (fig.3).
(iv) the radius of event horizon (R_s) of different black holes and their corresponding frequency in AGN (fig.4).
Figures 1 &3 obtained for XRBs and AGN in the case of the radius of event horizon verses corresponding wavelength of non-spinning black holes are in a straight line showing that there is a uniform variation between the radius of event horizon and their corresponding wavelength of non-spinning black holes. The straight line also shows that there is a linear relationship between the radius of event horizon and wavelength of non-spinning black holes and justifies the validity of model ($\lambda = 8\pi R_s$) while in the case of XRBs, the graph 2 plotted between radius of event horizon and their corresponding frequency of non-spinning black holes shows that the frequency decreases gradually with increase of the radius of event horizon. A peculiar nature of graph 4 is obtained for AGN between radius of event horizon and their corresponding frequency of non-spinning black holes. The variation of frequency emitted by non-spinning black holes with increase in the radius of event horizon can be understood from fig.4.

Graph1:



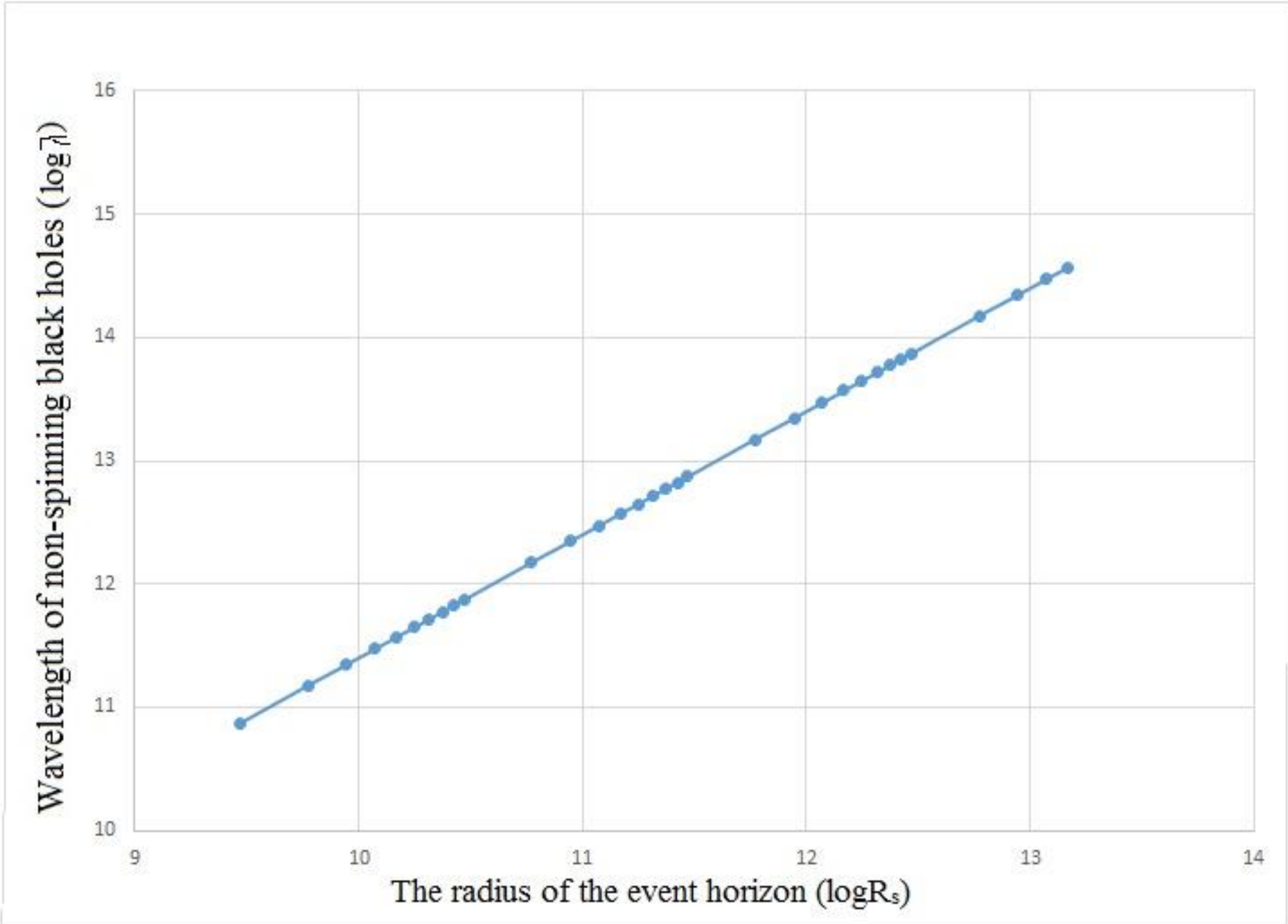
Wavelength of non-spinning black holes (λ) x 10^5
16
14
12
10
8
6
4
2
0
0
10000
20000
30000
40000
50000
60000
70000
The radius of the event horizon (Rs)

Graph2:

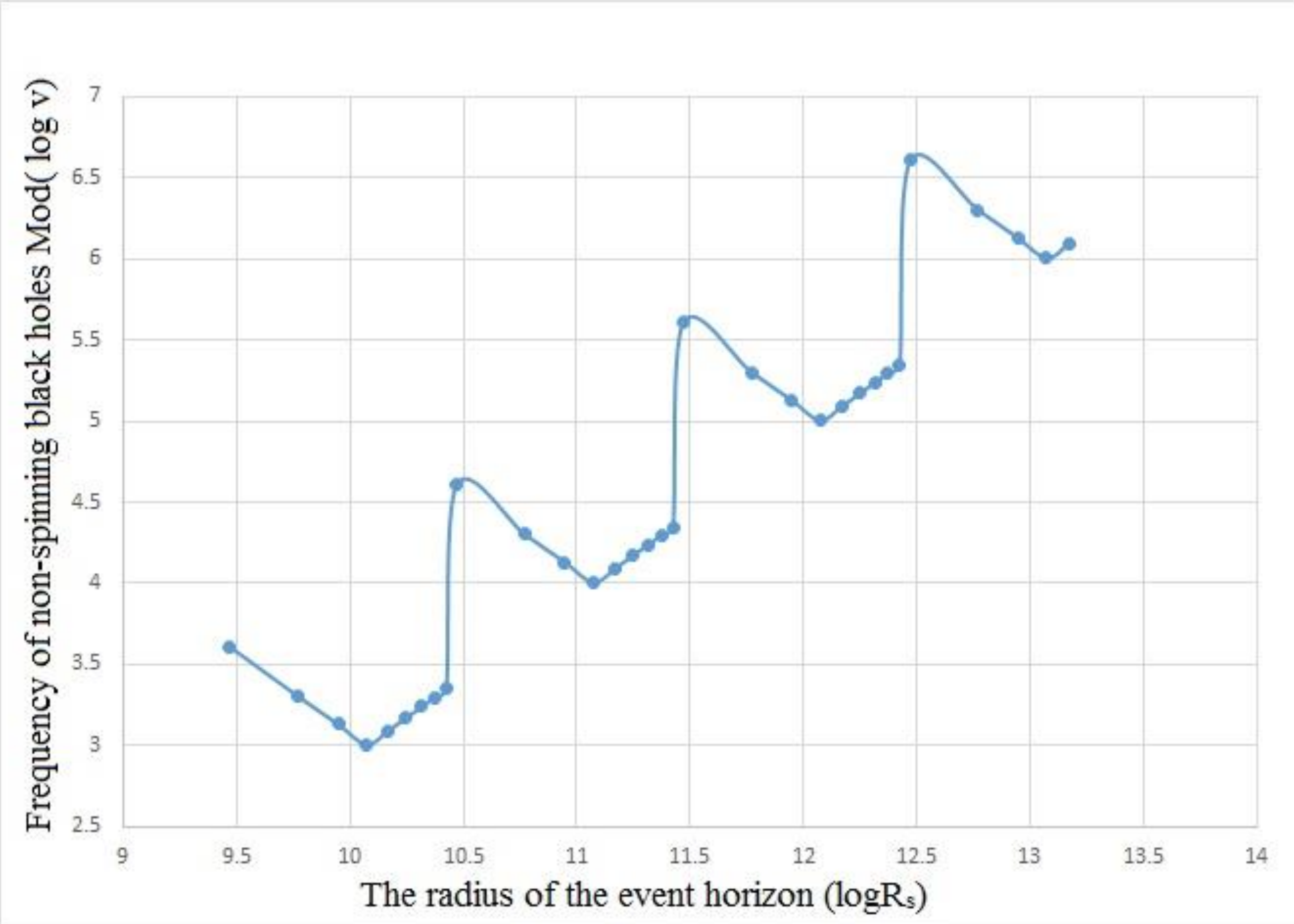


Frequency of non-spinning black holes (v) x 10²
9
8
7
6
5
4
3
2
1
0
0
10000
20000
30000
40000
50000
60000
70000
The radius of the event horizon (Rₛ)

Graph3:



Graph4:



Conclusion: In the present work, we can draw the following conclusions:

(1) The Hawking radiation emitted by the black holes may be regarded as the characteristics of non-spinning black holes in terms of wavelength or frequency.

(2) The frequencies of the Hawking radiation emitted by the non-spinning black hole decreases with the increase of the mass of different test non-spinning black holes.

(3) The wavelength of the Hawking radiation increases with the increase of the mass of different test non-spinning black holes.

(4) The graph plotted between the radius of event horizon verses corresponding wavelength of non-spinning black holes is in a straight line showing that there is a uniform variation between them in XRBs and AGN.

(5) The graph plotted between radius of event horizon and their corresponding frequency of non-spinning black holes in the case of XRBs shows that the frequency decreases gradually with increase of the radius of event horizon, while likely wave nature variation in AGN.

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