Constraining the lightest neutrino mass and m_{ee} from general lepton mass matrices

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Abstract

Despite spectacular advances in fixing the neutrino masses and mixing parameters through various neutrino oscillation experiments, we still do not have any clues for some very important parameters in the neutrino sector such as the lightest neutrino mass, effective Majorana mass m_{ee} measured in neutrinoless double beta decay etc. . In this context, it would be of paramount interest if one can obtain some bounds on these parameters from general considerations using the 'precisely' measured ranges for the other parameters. The purpose of the present work is to make an attempt to obtain bounds for m_{ee} and the lightest neutrino mass using the most general lepton mass matrices in the non-flavor basis using the facility of Weak Basis (WB) transformations, resulting into texture specific mass matrices.

Introduction

In the last few years, significant developments have taken place in the context of phenomenology of neutrino oscillations, both from the theoretical as well as experimental points of view. Owing to various solar, atmospheric, reactor and accelerator neutrino experiments, at present, the measurements of lepton mixing angles and the neutrino mass squared differences have reached almost a precision level. However, despite decades of rigorous experimental efforts, we still have little understanding about some vital parameters in the neutrino sector. For example, the neutrino oscillation experiments provide no clue regarding the absolute neutrino mass scale, leading to the absence of any consensus about the neutrino mass hierarchy. Another important issue which needs to be taken note of is the rather small neutrino masses as compared to their charged counterparts. From the theoretical point of view, the most popular explanation for this observation is the 'See-Saw Mechanism' which requires the neutrinos to be Majorana fermions. However, in the absence of any direct experimental evidence, Dirac neutrinos can not be ruled out. In this context, precise measurement of the effective Majorana mass m_{ee} in the neutrinoless double beta decay can be pivotal in establishing or ruling out the Majorana neutrinos. On the theoretical front, intense amount of activity has taken place to develop models for explaining neutrino masses and mixings. However, no focused attempt has been made in the literature which could have obtained unambiguous bounds on the crucial parameters such as the lightest neutrino mass and m_{ee}. In this context, it would be of paramount interest if one can obtain some bounds on these parameters from general considerations which can provide some clues to the experimentalists. The purpose of the present work is to make an attempt to obtain bounds for m_{ee} and the lightest neutrino mass using the most general lepton mass matrices in the non-flavor basis using the facility of Weak Basis (WB) transformations.

Current experimental bounds

- Although the neutrino oscillation experiments have proved that neutrinos have non zero masses, the absolute neutrino mass scale can not be determined by these. For this purpose, there are many direct neutrino mass experiments which investigate the kinematics of β decay of specific isotopes (^3H, ^{187}Re, ^{163}Ho) to derive model independent information on the averaged electron (anti)-neutrino mass $m_{\nu e}$. Till date, the most significant upper bounds on the $m_{\nu e}$ have been obtained by Mainz and Triotsk neutrino mass experiments, viz.,

$$m_{\nu e} < 2.30\, eV \;\; (95\% C.L.) \quad Mainz \tag{1}$$

$$m_{\nu e} < 2.05\, eV \;\; (95\% C.L.) \quad Triotsk \tag{2}$$

  Currently, the most important experiment in this context is the Karlsruhe Tritium Neutrino Experiment (KATRIN) which is expected to push the sensitivity for the mass of the electron antineutrino down to a value of 200 meV ($90\%\ C.L.$).

- For the effective Majorana mass measured in neutrinoless double beta decay (NDBD), defined as

$$\langle m_{ee} \rangle = m_{\nu_1} U_{e1}^2 + m_{\nu_2} U_{e2}^2 + m_{\nu_3} U_{e3}^2, \tag{3}$$

  till date, the experimental data has been able to provide only an upper bound viz.,

$$m_{ee} < 0.1 - 0.25\, eV \quad (90\%\ C.L.) \quad EXO\ and\ KamLAND-Zen, \tag{4}$$

  In near future, several new NDBD experiments, such as NEXT, SuperNEMO, Majorana, GENIUS, CUORE etc., are in line which aim to achieve a sensitivity upto 0.01 eV for m_{ee}.

Methodology

Using the facility of weak basis (WB) transformations [1], it can be shown that the most general lepton mass matrices within the framework of standard model (SM) can be expressed as

$$M_l = \begin{pmatrix} C_l & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} C_\nu & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}. \tag{5}$$

Details of the diagonalizing transformations for the above mass matrices and the methodology connecting the lepton mass matrices to the mixing matrix can be looked up in [2] and [3], however we mention the essentials here. A general mass matrix M_k by expressing as

$$M_k = Q_k M_k^r P_k, \tag{6}$$

where Q_k, P_k are diagonal phase matrices given as Diag$(e^{i\alpha_k}, 1, e^{-i\beta_k})$ and Diag $(e^{-i\alpha_k}, 1, e^{i\beta_k})$ respectively and M_k^r is a real symmetric matrix. M_k^r can be diagonalized by an orthogonal transformation O_k, e.g.,

$$M_k^{diag} = O_k^T M_k^r O_k \tag{7}$$

which can be rewritten as

$$M_k^{diag} = O_k^T Q_k^\dagger M_k P_k^\dagger O_k. \tag{8}$$

It can be shown that using the seesaw mechanism, the effective neutrino mass matrix can be expressed as,

$$M_\nu = P_{\nu D} O_{\nu D} \frac{(M_{\nu D}^{diag})^2}{(m_R)} O_{\nu D}^T P_{\nu D}, \tag{9}$$

m_R being the right handed neutrino mass scale. Further, the lepton mixing matrix can be expressed as

$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D}, \tag{10}$$

where $Q_l P_{\nu D}$, without loss of generality, can be taken as $(e^{i\phi_1},\ 1,\ e^{i\phi_2})$, ϕ_1 and ϕ_2 being related to the phases of mass matrices and can be treated as free parameters.

Inputs used for the analysis

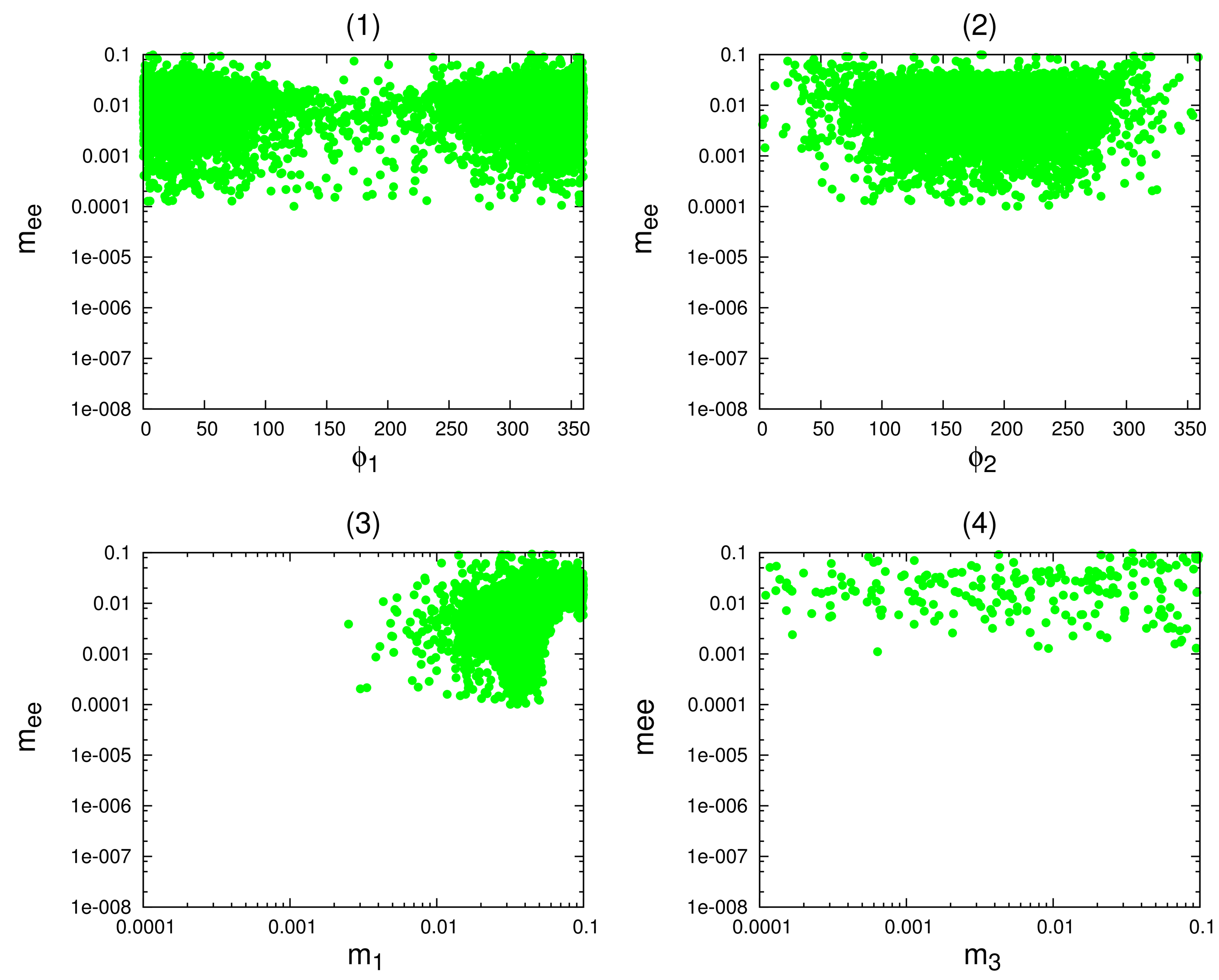
We have made use of the results of a latest global three neutrino oscillation analysis [4]

Parameter	1σ range	3σ range
Δm^2_{sol} $[10^{-5}eV^2]$	(7.32-7.80)	(6.99-8.18)
Δm^2_{atm} $[10^{-3}eV^2]$	(2.33-2.49)(NH); (2.31-2.49) (IH)	(2.19-2.62)(NH); (2.17-2.61)(IH)
$sin^2\theta_{13}$ $[10^{-2}]$	(2.16-2.66)(NH); (2.19-2.67)(IH)	(1.69-3.13)(NH); (1.71-3.15) (IH)
$sin^2\theta_{12}$ $[10^{-1}]$	(2.91-3.25)	(2.59-3.59)
$sin^2\theta_{23}$ $[10^{-1}]$	(3.65-4.10)(NH);(3.70-4.31)(IH)	(3.31-6.37)(NH);(3.35-6.63)(IH)

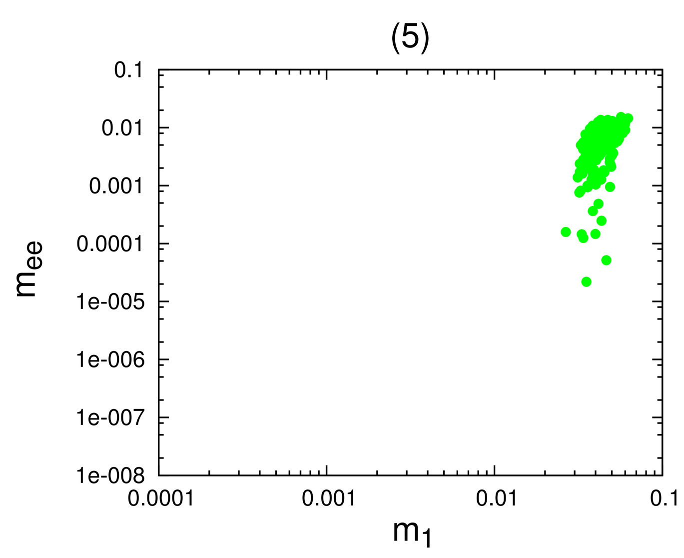
Further, the phases ϕ_1, ϕ_2 and the elements $D_{l,\nu}$, $C_{l,\nu}$ are considered to be free parameters. For all the three possible mass hierarchies of neutrinos, the explored range of the lightest neutrino mass is taken to be $10^{-8}\,\mathrm{eV} - 10^{-1}\,\mathrm{eV}$, our conclusions remain unaffected even if the range is extended further. In the absence of any constraint on the phases, ϕ_1 and ϕ_2 have been given full variation from 0 to 2π. Although $D_{l,\nu}$ and $C_{l,\nu}$ are free parameters, however, they have been constrained such that diagonalizing transformations O_l and O_ν always remain real. For the numerical analysis, we generate 10^7 random points (10^9 when the number of allowed points is small).

Results

In figures (1) and (2), we have plotted the parameter m_{ee} with respect to the phases ϕ_1 and ϕ_2 respectively for the texture two zero mass matrices given in equation (5) pertaining to normal hierarchy (NH) of neutrino masses, which clearly establishes a lower bound $\sim\ 0.1\ meV$ for m_{ee}. Figures (3) and (4) show the parameter space of m_{ee} with respect to the lightest neutrino mass for mass matrices given in eqn.(5) pertaining to the normal and inverted hierarchy (IH) of neutrino masses respectively which indicate that for NH a lower bound $\sim\ 2\ meV$ can be obtained for the lightest neutrino mass whereas for the IH case it remains largely unrestricted while a lower bound $\sim\ 1\ meV$ can be obtained for m_{ee}. While plotting these figures, all the three mixing angles have been constrained by their 3σ experimental bounds.



On carrying out a similar analysis for Fritzsch like texture four zero mass matrices, interestingly one finds an upper bound $\sim\ 0.01\ eV$ for m_{ee}, while the range of the lightest neutrino mass gets quite constrained as can be seen from figure (5). Future experiments are, thus, expected to have important implications for determining the texture structure of lepton mass matrices.



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