Hawking radiation from dynamical horizons

Ayan Chatterjee

Department of Physics and Astronomical Science,
Central University of Himachal Pradesh,
Dharamsala, India.

Talk presented at
UNICOS-14, Punjab University.

May 15, 2014

Black hole and the entropy puzzle.

- Classically, black holes cannot emit.

Black hole and the entropy puzzle.

- Classically, black holes cannot emit.
- Take a box of gas and throw into a black hole.

Black hole and the entropy puzzle.

- Classically, black holes cannot emit.
- Take a box of gas and throw into a black hole.Entopy of the gas vanishes into the singularity and thus decreases the entropy of the universe.

Black hole and the entropy puzzle.

- Classically, black holes cannot emit.
- Take a box of gas and throw into a black hole.Entopy of the gas vanishes into the singularity and thus decreases the entropy of the universe.Violation of the second law of entropy.
- Bekenstein (1973): Entropy of the hole goes up when it swallows the box of gas.

Black hole and the entropy puzzle.

- Classically, black holes cannot emit.
- Take a box of gas and throw into a black hole.Entopy of the gas vanishes into the singularity and thus decreases the entropy of the universe.Violation of the second law of entropy.
- Bekenstein (1973): Entropy of the hole goes up when it swallows the box of gas.Black holes have entropy proportional to their area.

$$S_{Bek} = \frac{A}{4G}.$$

- The change in the total entropy is

$$\frac{dS_{total}}{dt} = \frac{dS_{matter}}{dt} + \frac{dS_{Bek}}{dt} \geq 0.$$

- The change in the total entropy is

$$\frac{dS_{total}}{dt} = \frac{dS_{matter}}{dt} + \frac{dS_{Bek}}{dt} \geq 0.$$

- Hawking (1973- 1975): Black holes have surface gravity (which behaves like temperature) and obey a law similar to the second law of thermodynamics.

- The change in the total entropy is

$$\frac{dS_{total}}{dt} = \frac{dS_{matter}}{dt} + \frac{dS_{Bek}}{dt} \geq 0.$$

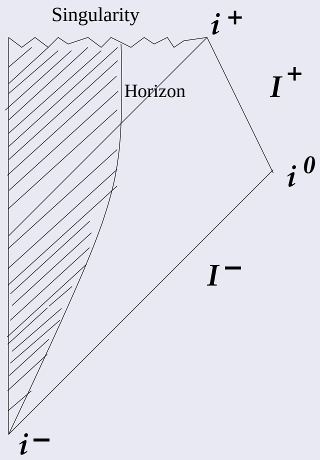
- Hawking (1973- 1975): Black holes have surface gravity (which behaves like temperature) and obey a law similar to the second law of thermodynamics.But to call them laws of thermodymanics, one must consider quantum fluctuations.

- The change in the total entropy is

$$\frac{dS_{total}}{dt} = \frac{dS_{matter}}{dt} + \frac{dS_{Bek}}{dt} \geq 0.$$

- Hawking (1973- 1975): Black holes have surface gravity (which behaves like temperature) and obey a law similar to the second law of thermodynamics.But to call them laws of thermodymanics, one must consider quantum fluctuations. Black holes radiate just like an ordinary thermal object.

Hawking's proof uses global geometry



In the collapsing geometry, data can only be specified on I^- and on $I^+ \cup$ Horizon.

- Give +ve freq modes f_i on I^- .

- Give +ve freq modes f_i on I^- .Give +ve freq modes g_i on I^+ but no data on Δ.

- Give +ve freq modes f_i on I^- .Give +ve freq modes g_i on I^+ but no data on Δ. Give +ve freq modes h_i on Δ but no data on I^+.

- Give +ve freq modes f_i on I^- .Give +ve freq modes g_i on I^+ but no data on Δ. Give +ve freq modes h_i on Δ but no data on I^+.
- Any field configuration is then expanded in these bases.

- Give +ve freq modes f_i on I^- .Give +ve freq modes g_i on I^+ but no data on Δ. Give +ve freq modes h_i on Δ but no data on I^+.
- Any field configuration is then expanded in these bases.
- $\phi = \sum_i \left(a_i f_i + a^{\dagger} f_i^*\right)$.

- Give +ve freq modes f_i on I^- .Give +ve freq modes g_i on I^+ but no data on Δ. Give +ve freq modes h_i on Δ but no data on I^+.
- Any field configuration is then expanded in these bases.
- $\phi = \sum_i \left(a_i f_i + a^\dagger f_i^*\right)$.
- $\phi = \sum_i \left(b_i\, g_i + c_i\, h_i + b_i^\dagger\, g_i^* + c_i^\dagger\, h_i^*\right)$.

- Vacuum can be defined in I^- with respect to f_i basis: $a_i|0>=0, \forall i$.

- Vacuum can be defined in I^- with respect to f_i basis: $a_i|0>=0, \forall i$.
  But this vacuum contains particles as seen by an observer in the future infinity I^+.

- Vacuum can be defined in I^- with respect to f_i basis: $a_i|0>= 0, \forall i$. But this vacuum contains particles as seen by an observer in the future infinity I^+.
- The number of particles are determined to be $|B|^2$, where B_{ij} is the Bogoliubov coefficient of the expansion $g_i = \sum_j \left(A_{ij} f_j + B_{ij} f_j^*\right)$.

- Vacuum can be defined in I^- with respect to f_i basis: $a_i|0>=0, \forall i$.
  But this vacuum contains particles as seen by an observer in the future infinity I^+.
- The number of particles are determined to be $|B|^2$, where B_{ij} is the Bogoliubov coefficient of the expansion $g_i = \sum_j \left(A_{ij}f_j + B_{ij}f_j^*\right)$.
- $|B|^2 \sim \frac{1}{\exp(2\pi\omega/\kappa)-1}$.

- Vacuum can be defined in I^- with respect to f_i basis:
  $a_i|0>= 0,\ \forall i$.
  But this vacuum contains particles as seen by an observer in the future infinity I^+.
- The number of particles are determined to be $|B|^2$, where B_{ij} is the Bogoliubov coefficient of the expansion
  $g_i = \sum_j \left(A_{ij} f_j + B_{ij} f_j^*\right)$.
- $|B|^2 \sim \frac{1}{\exp(2\pi\omega/\kappa) - 1}$.
- This is a Planckian spectrum at temperature $\kappa/2\pi$.

- Hawking's proof is independent of the gravitational field equations.

- Hawking's proof is independent of the gravitational field equations.Relies only on the beheviour of quantum fields in a spacetime formed out of collapse.

- Hawking's proof is independent of the gravitational field equations.Relies only on the beheviour of quantum fields in a spacetime formed out of collapse.
- Wald and Kay (1991): Any globally hyperbolic spacetime, with bifurcate Killing horizon has a vacuum thermal state at temperature $\kappa/2\pi$ and remains invariant under isometries generating the horizon.

- Hawking's proof is independent of the gravitational field equations.Relies only on the beheviour of quantum fields in a spacetime formed out of collapse.
- Wald and Kay (1991): Any globally hyperbolic spacetime, with bifurcate Killing horizon has a vacuum thermal state at temperature $\kappa/2\pi$ and remains invariant under isometries generating the horizon.
- All so well for global geometries.

- Hawking's proof is independent of the gravitational field equations.Relies only on the beheviour of quantum fields in a spacetime formed out of collapse.
- Wald and Kay (1991): Any globally hyperbolic spacetime, with bifurcate Killing horizon has a vacuum thermal state at temperature $\kappa/2\pi$ and remains invariant under isometries generating the horizon.
- All so well for global geometries.Can we extend Hawking's proof if we know the existence of horizon only but not the infinity ? (Chatterjee, Chatterjee and Ghosh, 2013.)

A local proof

- Hawking radiation as a tunelling phenomenon (Parikh and Wilczek,2000).

A local proof

- Hawking radiation as a tunelling phenomenon (Parikh and Wilczek,2000).
- Consider the classically forbidden s-wave emission from inside the horizon.

A local proof

- Hawking radiation as a tunelling phenomenon (Parikh and Wilczek,2000).
- Consider the classically forbidden *s*-wave emission from inside the horizon.Use WKB approximation to obtain the tunelling probability for a classically forbidden trajectory, $\Gamma = \exp\{-2\text{Im}S\}$, S being the classical action.

A local proof

- Hawking radiation as a tunelling phenomenon (Parikh and Wilczek,2000).
- Consider the classically forbidden *s*-wave emission from inside the horizon.Use WKB approximation to obtain the tunelling probability for a classically forbidden trajectory, $\Gamma = \exp\{-2\mathrm{Im}S\}$, S being the classical action.Compare with $\exp\{-\beta E\}$ and get the Hawking temperature.

- The formalism is local and has also been applied to horizons which are dynamical (evolving). (Hayward, 2007,2009)

- The formalism is local and has also been applied to horizons which are dynamical (evolving). (Hayward, 2007,2009)
- Two problems: The method is highly semiclassical.

- The formalism is local and has also been applied to horizons which are dynamical (evolving). (Hayward, 2007,2009)
- Two problems: The method is highly semiclassical.While evaluating the imaginary part of S, a singular integral appears with pole at the location of the horizon.

- The formalism is local and has also been applied to horizons which are dynamical (evolving). (Hayward, 2007,2009)
- Two problems: The method is highly semiclassical.While evaluating the imaginary part of S, a singular integral appears with pole at the location of the horizon.How to evaluate it for a evolving horizon ?

Future outward trapping horizon

- Consider spherical symmetrical geometry in $2+2$ formalism

$$ds^2 = -2e^{-f}dx^+dx^- + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right),$$

where $x_\pm = t \pm x$ and f and r are smooth functions of $x_\pm$.

Future outward trapping horizon

- Consider spherical symmetrical geometry in $2+2$ formalism

$$ds^2 = -2e^{-f} dx^+ dx^- + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right),$$

where $x_\pm = t \pm x$ and f and r are smooth functions of $x_\pm$.

- Definition: FOTH is defined to be a three dimensional surface such that on each foliation, one null normal has zero expansion ($\theta_+ = 0$) and another has negative expansion ($\theta_- < 0$).

Future outward trapping horizon

- Consider spherical symmetrical geometry in $2+2$ formalism

$$ds^2 = -2e^{-f}dx^+dx^- + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right),$$

where $x_\pm = t \pm x$ and f and r are smooth functions of $x_\pm$.

- Definition: FOTH is defined to be a three dimensional surface such that on each foliation, one null normal has zero expansion ($\theta_+ = 0$) and another has negative expansion ($\theta_- < 0$). Further, the directional derivative of θ_+ along the other null normal ($\partial_-\theta_+$) is negative. (Hayward 1994,1998)

- Can be shown that the FOTH is timelike iff the out-out component of the energy- momentum tensor $T_{++} < 0$.

- Can be shown that the FOTH is timelike iff the out-out component of the energy- momentum tensor $T_{++} < 0$.This implies that for emitting Hawking radiation the horizon must remain timelike for some affine time.

- Can be shown that the FOTH is timelike iff the out-out component of the energy- momentum tensor $T_{++} < 0$.This implies that for emitting Hawking radiation the horizon must remain timelike for some affine time.Further, it can be shown that the area and the (Misner-Sharp) energy of the FOTH decreases in the process.

- Can be shown that the FOTH is timelike iff the out-out component of the energy- momentum tensor $T_{++} < 0$.This implies that for emitting Hawking radiation the horizon must remain timelike for some affine time.Further, it can be shown that the area and the (Misner-Sharp) energy of the FOTH decreases in the process.
- Physically, this implies that the horizon receives an incoming negative energy flux.

- Can be shown that the FOTH is timelike iff the out-out component of the energy- momentum tensor $T_{++} < 0$.This implies that for emitting Hawking radiation the horizon must remain timelike for some affine time.Further, it can be shown that the area and the (Misner-Sharp) energy of the FOTH decreases in the process.
- Physically, this implies that the horizon receives an incoming negative energy flux.
- How to define positive frequency modes in a dynamical spacetime ? No time like Killing vector exists to give preffered time.

- Can be shown that the FOTH is timelike iff the out-out component of the energy- momentum tensor $T_{++} < 0$.This implies that for emitting Hawking radiation the horizon must remain timelike for some affine time.Further, it can be shown that the area and the (Misner-Sharp) energy of the FOTH decreases in the process.
- Physically, this implies that the horizon receives an incoming negative energy flux.
- How to define positive frequency modes in a dynamical spacetime ? No time like Killing vector exists to give preffered time. But can define Kodama vector which matches with the Killing vector at spatial infinity, becomes null on FOTH and gives a preferred timelike direction.

Defining the field modes on curved spacetime

- Define the modes of positive frequency using the Kodama vector K^a.

$$iKZ_\omega = \omega Z_\omega,$$

where Z_ω are the eigenfunctions

Defining the field modes on curved spacetime

- Define the modes of positive frequency using the Kodama vector K^a.

$$iKZ_\omega = \omega Z_\omega,$$

where Z_ω are the eigenfunctions and are given by

$$Z_\omega \simeq \exp\left(i\omega \int_r \frac{d\theta_+}{\kappa\theta_+}\right),$$

has singularities at the location of the horizon.

- Quantum fields are operator valued distributions. Using results from distribution theory, we can find modes which are well defined for all values of θ_+.

- Quantum fields are operator valued distributions. Using results from distribution theory, we can find modes which are well defined for all values of θ_+.We evaluate the probablity density in a single particle Hilbert space

$$\begin{aligned}\rho(\omega) &= \omega \text{ outside the horizon}\\ &= \omega\, \text{exp}\{2\pi\omega/\kappa\} \text{ inside the horizon}\end{aligned}$$

- Quantum fields are operator valued distributions. Using results from distribution theory, we can find modes which are well defined for all values of θ_+.We evaluate the probablity density in a single particle Hilbert space

$$\begin{aligned} \rho(\omega) &= \omega \text{ outside the horizon} \\ &= \omega \exp\{2\pi\omega/\kappa\} \text{ inside the horizon} \end{aligned}$$

- Thus the conditional probability that the particle emits when it is incident from inside is $\exp\{-2\pi\omega\kappa\}$.

- Quantum fields are operator valued distributions. Using results from distribution theory, we can find modes which are well defined for all values of θ_+.We evaluate the probablity density in a single particle Hilbert space

$$\begin{aligned} \rho(\omega) &= \omega \text{ outside the horizon} \\ &= \omega \exp\{2\pi\omega/\kappa\} \text{ inside the horizon} \end{aligned}$$

- Thus the conditional probability that the particle emits when it is incident from inside is $\exp\{-2\pi\omega\kappa\}$. Comparing with the Boltzmann factor, $T = \kappa/2\pi$.

Emission of matter flux

- We can also evaluate the outgoing flux of matter energy that crosses the dynamical horizon.

Emission of matter flux

- We can also evaluate the outgoing flux of matter energy that crosses the dynamical horizon.Let n^a be the normal to the 3- surface then the flux is

$$\mathcal{F} = \int d\mu T_{ab} n^a K^b .$$

Emission of matter flux

- We can also evaluate the outgoing flux of matter energy that crosses the dynamical horizon.Let n^a be the normal to the 3- surface then the flux is

$$\mathcal{F} = \int d\mu T_{ab} n^a K^b.$$

- $\mathcal{F} \sim (r_1 - r_2)$.

Emission of matter flux

- We can also evaluate the outgoing flux of matter energy that crosses the dynamical horizon.Let n^a be the normal to the 3- surface then the flux is

$$\mathcal{F} = \int d\mu T_{ab} n^a K^b .$$

- $\mathcal{F} \sim (r_1 - r_2)$.Since area is decreasing, the outgoing flux is positive definite.

What We Have Shown

- Dynamical horizons can be assigned a temperature. It is given by $\kappa/2\pi$, where κ is slowly varying.

What We Have Shown

- Dynamical horizons can be assigned a temperature. It is given by $\kappa/2\pi$, where κ is slowly varying.
- The outgoing flux is positive definite and is exactly equal to the difference of radius at the start and the end of the emission process.

What We Have Shown

- Dynamical horizons can be assigned a temperature. It is given by $\kappa/2\pi$, where κ is slowly varying.
- The outgoing flux is positive definite and is exactly equal to the difference of radius at the start and the end of the emission process.
- The results are valid for large black holes (κ is slowly varying) and spherical geometry.

References

- S. Hayward, PRD, **49**, 6467, 1994.
- R. Crisienzo, M. Nadalini, L. Vanzo and S. Zerbini, CQG, **26**, 062001, 2009.
- A. Chatterjee, B. Chatterjee and A. Ghosh, PRD, **87**, 084051, 2014.