Interplay between grand unification and supersymmetry in SU(5) and E_6

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- Higgs sector with $351' + \overline{351}' + 27 + \overline{27} + 78$
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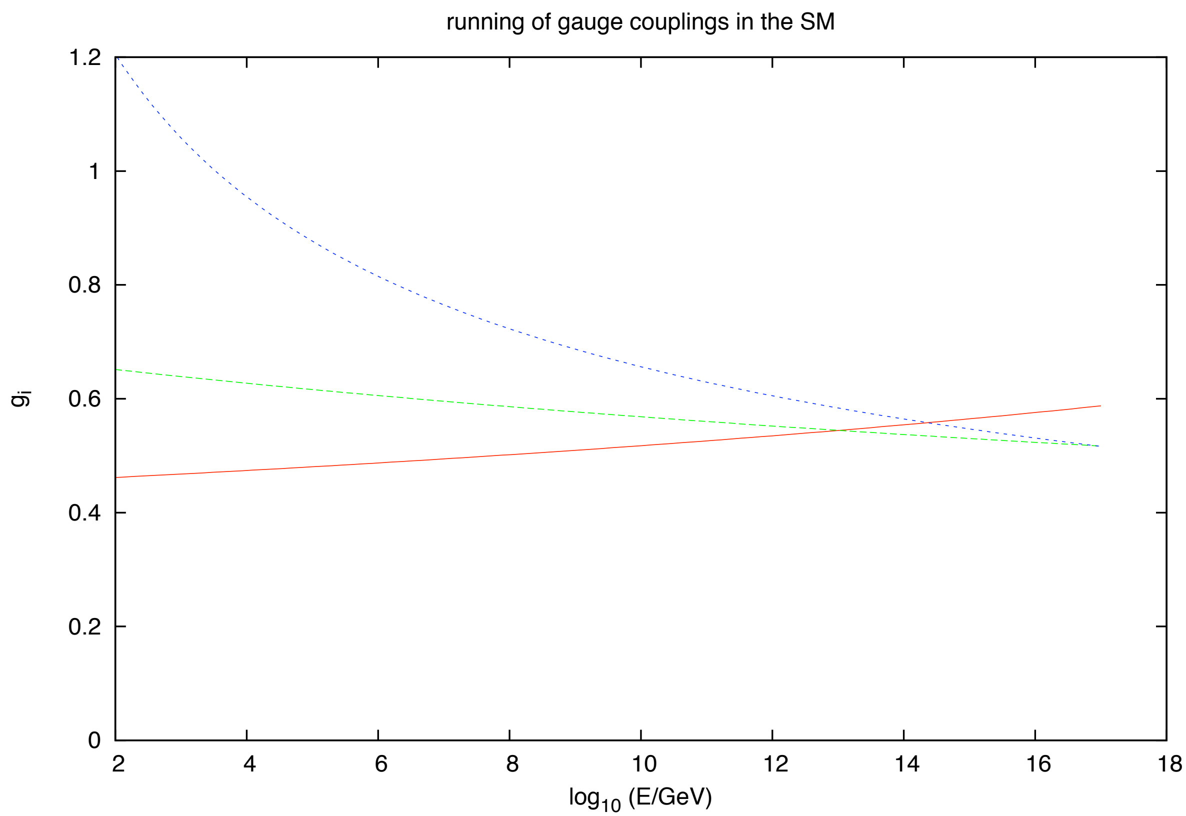
Introduction

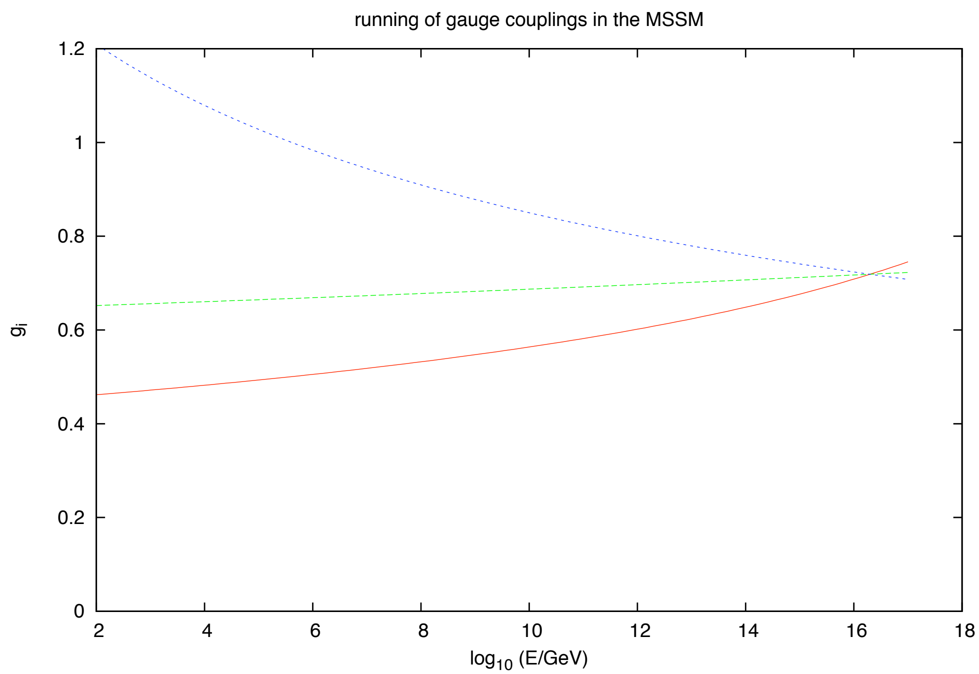
I had the pleasure to work with Charan (3 papers together) and it was a very fruitful experience (average citation per paper is 102), from which I learned a lot. Magic years spent together in ICTP. It was mainly on SO(10), one of Charan's strongholds.

He had given a very nice review on the subject, so I will try to cover the two other realistic groups, SU(5) and E_6.

The best known example of interplay between susy and gut is the gauge coupling unification. In SM:



New states needed. If we addMSSM at ≈ 1 TeV and run at 1-loop: unification at $M_{GUT} \approx 10^{16}$ GeV



Solution not unique, but enough to motivate supersymmetry

Another, not less important case is connected with Charan:

Usually GUTs do not give new ingredients in the search for dark matter candidates.

Susy has its own candidate, the light neutralino, *providing* we assume R-parity conservation

But, R-parity is just a subgroup of SO(10).

So, taking large representation (126) to break the rank, Charan with collaborators showed that R-parity is exact to low energy!

Grand Unification tells us something about supersymmetry and even dark matter!

In this talk the interplay between supersymmetry and grand unification will be the following:

- in minimal SU(5) the requirement of unification of couplings, Higgs mass, proton decay bounds, perturbativity and correct fermion masses, put constraints on susy parameters like sfermion spectrum
- in E_6 the relation is only tiny, the usual one: the renormalizable superpotential gives a restricted potential and the search of vacua is simplified

Minimal supersymmetric SU(5)

Usual reaction: hasn't this been ruled out long ago?

Unification constraint of the gauge couplings at 2-loop order needs light color triplet $m_T \lesssim 10^{15}$ GeV.

Proton decay constraint needs heavy color triplet $m_T \gtrsim 10^{17}$ GeV.

But, this is true only if

- only renormalizable couplings
- gaugini, higgsino and 3^{rd} generation superpartners $\mathcal{O}$ (TeV)

Renormalizability crucial for this conclusion. In fact in general

- triplet mass can get large threshold correction from the color octet (m_8) and weak triplet (m_3) in SU(5) adjoint:

$$m_T \approx \left(\frac{m_3}{m_8}\right)^{5/2} 10^{15}\ \mathrm{GeV}$$

In renormalizable case $m_3 = m_8$, in general arbitrary.

- higher order contributions to superpotential change relation between Higgs doublet Yukawa and color triplet Yukawa $\rightarrow$ proton decay estimates can change
- these terms can change also relations between fermion and sfermion mixings (without endanger fcnc constraints)

Is the second requirement - spartners $\mathcal{O}$ (TeV) - also crucial to rule out the model?

This is what I want to discuss now.

We will be talking about

- renormalizable minimal supersymmetric SU(5)

$$3 \times (10_F + \bar{5}_F) + (24_H + 5_H + \bar{5}_H) + 24_V$$

- soft terms SU(5) symmetric at M_{GUT} but otherwise arbitrary; to help that we will assume

$$\tilde{m}_1 \approx \tilde{m}_2$$

Several constraints:

- Higgs mass
- fermion masses
- perturbativity (couplings $\lesssim 1$)
- vacuum metastability (no tachyons, UFB, CCB)
- proton decay (small $\tan\beta \lesssim 5$)
- unification constraints ($g_1 = g_2 = g_3$, $y_b = y_\tau$)

Higgs mass

$$m_h^2 = 2\lambda(m_h)v^2$$

But the matching scale between SM and MSSM is $m_{\tilde{t}}$

$$\lambda(\tilde{m}_t) = \underbrace{\lambda_0(\tan\beta)}_{tree\ level} + \underbrace{\lambda_1\left(y_t, \frac{X_t}{\tilde{m}_t}\right)}_{>0} + \underbrace{\lambda_1\left(y_b, \frac{X_b}{\tilde{m}_b}\right)}_{<0} + \ldots$$

$$\rightarrow \quad m_h\left(\tan\beta, \tilde{m}_t, \frac{X_t}{\tilde{m}_t}, \frac{X_b}{\tilde{m}_b}\right)$$

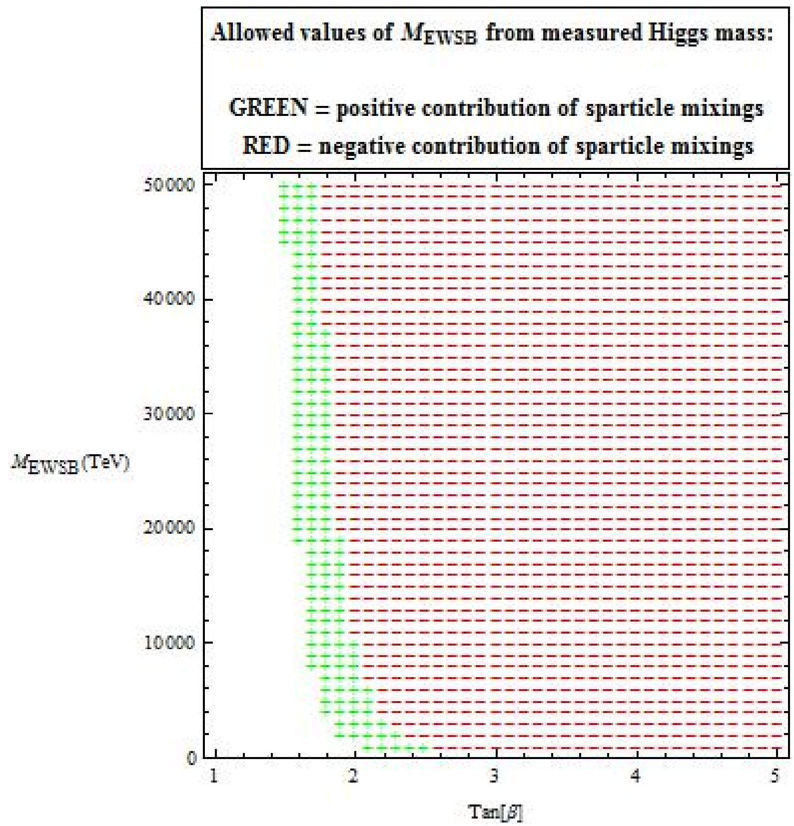
$$\begin{aligned} \tilde{m}_t &= M_{EWSB} \equiv \sqrt{\tilde{m}_{t_L}\tilde{m}_{t_R}} \\ X_t &= A_t/y_t - \mu/\tan\beta \\ X_b &= A_b/y_b - \mu\tan\beta \end{aligned}$$

$$\lambda(\tilde{m}_t) = \underbrace{\frac{m_Z^2}{2v^2}(\tilde{m}_t)\cos^2(2\beta)}_{small\ for\ \tan\beta=\mathcal{O}(1)}$$

$$+ \underbrace{\frac{6(y_t \sin\beta)^4}{(4\pi)^2}\left(\frac{X_t}{\tilde{m}_t}\right)^2\left[1-\frac{1}{12}\left(\frac{X_t}{\tilde{m}_t}\right)^2\right]}_{maximally\ positive\ for\ |X_t/\tilde{m}_t|=\sqrt{6}}$$

$$+ \underbrace{\frac{6(y_b \cos\beta)^4}{(4\pi)^2}\left(\frac{X_b}{\tilde{m}_b}\right)^2\left[1-\frac{1}{12}\left(\frac{X_b}{m_{\tilde{b}}}\right)^2\right]}_{maximally\ negative\ for\ |X_b/\tilde{m}_b|\approx 1/y_b} + \ldots$$

$|X_f/\tilde{m}_f| \lesssim 1/y_f$ because of vacuum metastability



$M_{EWSB} = \tilde{m}_t$

Fermion masses

SU(5) constraints at M_{GUT}: $y_b = y_\tau$, $y_s = y_\mu$, $y_d = y_e$

$\rightarrow$ at low energy we need corrections (assuming leptons correct):

$$\frac{\delta m_d}{m_d} \approx 2$$
$$\frac{\delta m_s}{m_s} \approx -3$$
$$\frac{\delta m_b}{m_b} \approx -0.3$$

1-loop finite susy threshold corrections:

$$\frac{\delta m_i}{m_i} = -\frac{\alpha_3}{3\pi}\,\frac{X_i}{\tilde{m}_i}\;I\left(\frac{m_{\tilde{g}}}{\tilde{m}_i}\right)$$

To survive the age of the universe:

$$\left|\frac{X_i}{\tilde{m}_i}\right| \lesssim \frac{1}{y_i}$$

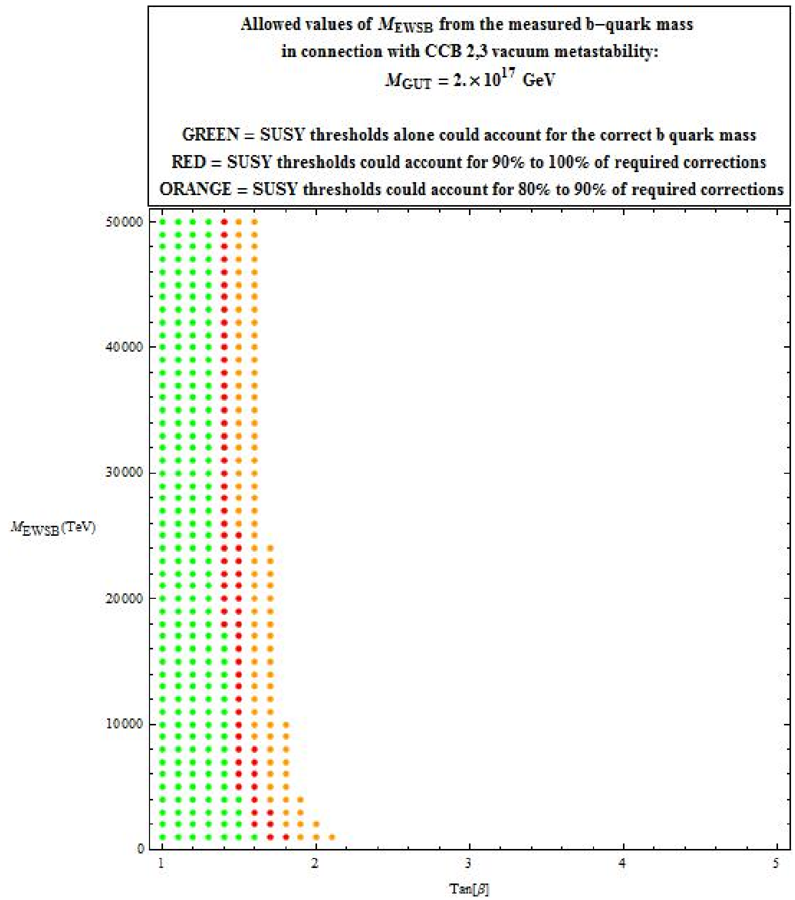
Harder to get corrections for b than for s or d !

Only bottom could be a problem!

$I(x)$ peaked around $x = 2$ ($I_1(2) \approx 1$)

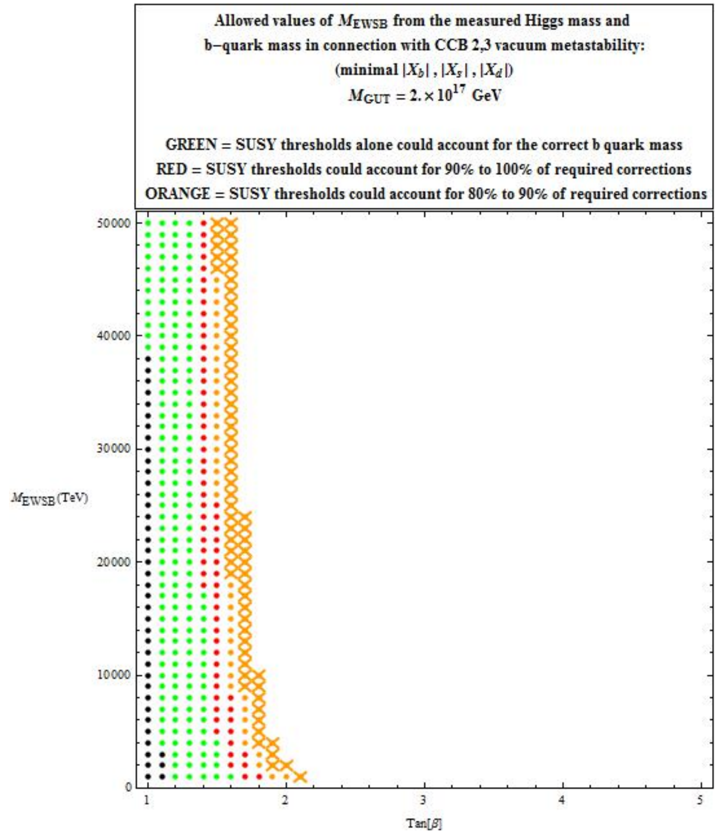
$\to m_{\tilde{g}} \approx \tilde{m}_b$ (the heaviest among $\tilde{b}_L$, $\tilde{b}_R$) to maximize corrections

$\to X_i \gg \tilde{m}_i \to$ vacuum is metastable



Allowed values of M_{EWSB} from the measured b–quark mass
in connection with CCB 2,3 vacuum metastability:
$M_{GUT} = 2. \times 10^{17}$ GeV
GREEN = SUSY thresholds alone could account for the correct b quark mass
RED = SUSY thresholds could account for 90% to 100% of required corrections
ORANGE = SUSY thresholds could account for 80% to 90% of required corrections
M_{EWSB}(TeV)
50000
40000
30000
20000
10000
0
1
2
3
4
5
Tan[β]

Put together Higgs mass and fermion masses constraints (crosses):



Black: forbidden region (y_t non-perturbative)

Very little region survives $\qquad m_{\tilde{t}} \leftrightarrow \tan\beta$

Summary on SU(5) results

- fermion masses $\rightarrow$ MSSM vacuum is metastable
- correction to b mass $\rightarrow \tilde{m}_b \approx m_{\tilde{g}}$
- SU(5) $\rightarrow m_{\tilde{g}} \approx m_{\tilde{w}}$
- Higgs mass and correction to b mass $\rightarrow \tan\beta(\tilde{m}_t)$
- corrections to s and d quarks much easier
  ($X/\tilde{m}$ allowed to be much larger)

Minimal supersymmetric E_6

In spite of being proposed almost soon after SU(5) very little is known, most works consider just Yukawa sector

Until recently little explicit examples of renormalizable realistic Higgs sectors except that with 78, 27, $\overline{27}$ only $E_6 \to SO(10)$

Here I will assume 1-step unification, i.e. $m_{susy} \approx 1$ TeV

Generic Yukawa sector in E_6

In all generality three types of Yukawas

$$W = 27_i \left(Y_{27}^{ij}\, 27 + Y_{\overline{351}'}^{ij}\, \overline{351}' + Y_{\overline{351}}^{ij}\, \overline{351} \right) 27_j$$

$$Y_{27,\overline{351}'} = Y_{27,\overline{351}'}^T \qquad \text{symmetric}$$

$$Y_{\overline{351}} = -Y_{\overline{351}}^T \qquad \text{antisymmetric}$$

Completely analogous to SO(10):

$$W = 16_i \left(Y_{10}^{ij}\, 10 + Y_{\overline{126}}^{ij}\, \overline{126} + Y_{120}^{ij}\, 120 \right) 16_j$$

$$Y_{10,\overline{126}} = Y_{10,\overline{126}}^T \qquad \text{symmetric}$$

$$Y_{120} = -Y_{120}^T \qquad \text{antisymmetric}$$

351, similar to **120** in SO(10), less promising, so we drop it out

$$
\begin{aligned}
W &= \begin{pmatrix} 16 & 10 & 1 \end{pmatrix} Y_{27} \begin{pmatrix} 10 & 16 & 0 \\ 16 & 1 & 10 \\ 0 & 10 & 0 \end{pmatrix} \begin{pmatrix} 16 \\ 10 \\ 1 \end{pmatrix} \\
&+ \begin{pmatrix} 16 & 10 & 1 \end{pmatrix} Y_{\overline{351}'} \begin{pmatrix} \overline{126} + 10 & 144 & \overline{16} \\ 144 & 54 & 10 \\ \overline{16} & 10 & 1 \end{pmatrix} \begin{pmatrix} 16 \\ 10 \\ 1 \end{pmatrix}
\end{aligned}
$$

- several new Higgs doublets (not only in 10 and $\overline{126}$)
- some fields have large $\mathcal{O}(M_{GUT})$ vevs $\rightarrow$
  - mixing between $\bar{5} \in 16$ and $\bar{5} \in 10$ (d^c, L)
  - mixing between $1 \in 1$ and $1 \in 16$ (ν^c)
- $M^U_{3\times 3}$, $M^D_{6\times 6}$, $M^E_{6\times 6}$, $M^N_{15\times 15} \rightarrow$ light $(M_{U,D,E,N})_{3\times 3}$

Higgs sector with $351' + \overline{351}' + 27 + \overline{27}$

- What are the large vevs that produce family mixings with vectorlike extra matter?
- Where are the MSSM Higgs doublets?

The full model needed.

The minimal Higgs sector with $E_6 \to$ SM composed of $351' + \overline{351}' + 27 + \overline{27}$.

$$
\begin{aligned}
W &= m_{351'}\, \overline{351}'\, 351' + \lambda_1\, 351'^3 + \lambda_2\, \overline{351'}^3 \\
&+ m_{27}\, \overline{27}\, 27 + \lambda_3\, 27\, 27\, \overline{351}' + \lambda_4\, \overline{27}\, \overline{27}\, 351' \\
&+ \lambda_5\, 27^3 + \lambda_6\, \overline{27}^3
\end{aligned}
$$

The SM singlets:

$$
\begin{aligned}
27 &: \quad c_1, c_2 \\
\overline{27} &: \quad d_1, d_2 \\
351' &: \quad e_1, e_2, e_3, e_4, e_5 \\
\overline{351'} &: \quad f_1, f_2, f_3, f_4, f_5
\end{aligned}
$$

More than one solution. For example:

$$c_2 = e_2 = e_4 = 0, \qquad d_2 = f_2 = f_4 = 0$$

$$d_1 = \frac{m_{351'} m_{27}}{2\lambda_3 \lambda_4 c_1}$$

$$e_1 = -\frac{m_{351'}}{6\lambda_1^{2/3}\lambda_2^{1/3}}, \qquad f_1 = -\frac{m_{351'}}{6\lambda_1^{1/3}\lambda_2^{2/3}}$$

$$e_3 = -\lambda_3 {c_1}^2 / m_{351'}, \qquad f_3 = -\frac{m_{351'} m_{27}^2}{4\lambda_3^2 \lambda_4 {c_1}^2}$$

$$e_5 = \frac{m_{351'}}{3\sqrt{2}\lambda_1^{2/3}\lambda_2^{1/3}}, \qquad f_5 = \frac{m_{351'}}{3\sqrt{2}\lambda_1^{1/3}\lambda_2^{2/3}}$$

with

$$0 = |m_{351'}|^4 |m_{27}|^4 + 2|m_{351'}|^4 |m_{27}|^2 |\lambda_3|^2 |c_1|^2 - 8|m_{351'}|^2 |\lambda_3|^4 |\lambda_4|^2 |c_1|^6 - 16|\lambda_3|^6 |\lambda_4|^2 |c_1|^8$$

This case seems really minimal: 27 and $\overline{351}'$ that participate to symmetry breaking could contribute to Yukawa terms!

Can the weak doublets with $Y = \pm 1$ in 27 and $\overline{351'}$ be the Higgses $H, \bar{H}$ of the MSSM?

Since E_6 is a GUT, this means:

Can we make the doublet-triplet splitting with the massless eigenvector living in both 27 and $\overline{351'}$?

The doublet-triplet splitting

Problem present in all minimal GUTs. The prototype example in SU(5):

$$5_H = \begin{pmatrix} T \\ H \end{pmatrix} \quad , \quad \bar{5}_H = \begin{pmatrix} \bar{T} \\ \bar{H} \end{pmatrix}$$

$$\begin{aligned} W_{Yukawa} &= Y_{\bar{5}}^{ij}\ \bar{5}_i\ 10_j\ \bar{5}_H + Y_{10}^{ij}\ 10_i\ 10_j\ 5_H \\ &\to Y_{\bar{5}}^{ij} \left(d_i^c\ Q_j + L_i\ e_j^c \right) \bar{H} + Y_{10}^{ij}\ u_i^c\ Q_j\ H \\ &+ Y_{\bar{5}}^{ij} \left(L_i\ Q_j + d_i^c\ u_j^c \right) \bar{T} + Y_{10}^{ij} \left(Q_i\ Q_j + u_i^c\ e_j^c \right) T \end{aligned}$$

$H, \bar{H} \ldots$ Higgses of MSSM $\to M_H \approx m_Z$

$T, \bar{T}$ mediate proton decay $\tau \propto M_T^2 \to M_T \approx M_{GUT} \gg m_Z$

How to get such a large splitting from components of same multiplet?

$$W = \mu\, \bar{5}_H\, 5_H + \eta\, \bar{5}_H\, 24_H\, 5_H$$

Since

$$\langle 24_H \rangle = M_{GUT} \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}$$

$$W = \bar{H}\,(\mu - 3\eta\; M_{GUT})\,H + \bar{T}\,(\mu + 2\eta\; M_{GUT})\,T$$

$$\begin{aligned} M_H &= \mu - 3\eta\; M_{GUT} \approx 0 \\ M_T &= \mu + 2\eta\; M_{GUT} \approx M_{GUT} \end{aligned}$$

$$\rightarrow \mu = 3\eta M_{GUT} \approx M_{GUT}$$

Fine-tuning unavoidable in minimal models

In our E_6 case doublets and triplets live in $351'$, $\overline{351'}$, 27, $\overline{27}$.

$351'$ has 8 doublets (9 triplets)

$\overline{351}'$ has 8 doublets (9 triplets)

27 has 3 doublets (3 triplets)

$\overline{27}$ has 3 doublets (3 triplets)

All together 22 doublets (11 with $Y = +1$ and 11 with $Y = -1$):
doublet matrix M_D is 11×11

All together 24 triplets (12 with $Y = +2/3$ and 12 with $Y = -2/3$):
triplet matrix M_T is 12×12

analysis complicated by presence of would-be-Goldstones in
$16 + \overline{16} \in 78$

$\rightarrow M_{T,D}$ have automatically one zero eignevalue

We need the determinant without the zero-modes:

$$Det(M) \equiv \prod_{i=2}^{n} m_i$$

We would like to get

$$Det(M_D) = 0 \ \ , \ \ Det(M_T) \neq 0$$

But after long calculation the result is:

$$Det(M_T) = \# Det(M_D)$$

i.e doublet-triplet splitting impossible !

Bizarre situation: all was ok, we seems to fail on doublet-triplet splitting. And not because we don't like fine-tuning, we cannot even fine-tune!

Simplest solutions:

- add another $27 + \overline{27}$ pair with coupling

$$W_{DT} = m_{27}\, 27\, \overline{27} + \kappa_1\, 27\, 27\, \overline{351'} + \kappa_2\, \overline{27}\, \overline{27}\, 351' + \kappa_3\, 27\, 27\, 27 + \kappa_4\, \overline{27}\, \overline{27}\, \overline{27}$$

  with $\langle 27 \rangle , \langle \overline{27} \rangle = \mathcal{O}(m_Z)$

  DT splitting now possible: MSSM Higgs live only in $27\, ,\, \overline{27}$

  In spite of this 3 Yukawa matrices involved.

- add another 78: although it does not contribute to Yukawas, it changes the symmetry breaking pattern (not being needed) thus relaxing constraints on DT.

  DT now possible in the old sector: MSSM Higgses live also in $\overline{351'}$ and 27!

  This possibility more minimal, only 2 Yukawas.

Higgs sector with $351' + \overline{351}' + 27 + \overline{27} + 78$

$$
\begin{aligned}
W &= m_{351'}\,\overline{351}'\,351' + \lambda_1\,351'^3 + \lambda_2\,\overline{351'}^3 \\
&+ m_{27}\,\overline{27}\,27 + \lambda_3\,27^2\,\overline{351}' + \lambda_4\,\overline{27}^2\,351' \\
&+ \lambda_5\,27^3 + \lambda_6\,\overline{27}^3 \\
&+ m_{78}78^2 + \lambda_7\,27\,78\,\overline{27} + \lambda_8\,351'\,78\,\overline{351}'
\end{aligned}
$$

Other SM singlets:

$$78 \quad : \quad a_1, a_2, a_3, a_4, a_5$$

Solution with $a_i \neq 0$ shown explicitly to be possible. Disconnected with the previous one (no limit gives the previous solution with $a_i \to 0$).

Yukawa sector in the minimal E_6 model

As an example of what happens let's see the down sector:

$$\begin{pmatrix} d^{cT} & d'^{cT} \end{pmatrix} \begin{pmatrix} \bar{v}_2 Y_{27} + \left(\frac{1}{2\sqrt{10}} \bar{v}_4 + \frac{1}{2\sqrt{6}} \bar{v}_8 \right) Y_{\overline{351}'} & c_2 Y_{27} \\ -\bar{v}_3 Y_{27} - \left(\frac{1}{2\sqrt{10}} \bar{v}_9 + \frac{1}{2\sqrt{6}} \bar{v}_{11} \right) Y_{\overline{351}'} & \frac{1}{\sqrt{15}} f_4 Y_{\overline{351}'} \end{pmatrix} \begin{pmatrix} d \\ d' \end{pmatrix}$$

$\bar{v}_{2,3,4,8,9,11} = \mathcal{O}(m_Z)$; $c_2, f_4 = \mathcal{O}(M_{GUT})$

$$\left. \begin{array}{l} d^c \in \bar{5}_{SU(5)} \in 16_{SO(10)} \\ d'^c \in \bar{5}_{SU(5)} \in 10_{SO(10)} \end{array} \right\} \text{mix}$$

$$d \in 10_{SU(5)} \in 16_{SO(10)}$$

$$d' \in 5_{SU(5)} \in 10_{SO(10)} \ldots \text{ heavy}$$

The matrix above has the form

$$\mathcal{M} = \begin{pmatrix} m_1 & M_1 \\ m_2 & M_2 \end{pmatrix}$$

with $m_{1,2} = \mathcal{O}(m_Z)$ and $M_{1,2} = \mathcal{O}(M_{GUT})$

All are 3×3 matrices.

the idea is to find a 6×6 unitary matrix $\mathcal{U}$ that

$$\mathcal{U} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \text{something} \end{pmatrix}$$

The solution is

$$\mathcal{U} = \begin{pmatrix} \left(1 + X X^\dagger\right)^{-1/2} & -\left(1 + X X^\dagger\right)^{-1/2} X \\ X^\dagger \left(1 + X X^\dagger\right)^{-1/2} & \left(1 + X^\dagger X\right)^{-1/2} \end{pmatrix}$$

with

$$X = M_1\, M_2^{-1}$$

so that

$$\mathcal{U}\mathcal{M} = \begin{pmatrix} \underbrace{\mathcal{O}(m_Z)}_{\text{light sector}} & 0 \\ \mathcal{O}(m_Z) & \mathcal{O}(M_{GUT}) \end{pmatrix}$$

For charged fermions they turn out to be

$$M_U = -v_1 Y_{27} + \left(\frac{1}{2\sqrt{10}} v_5 - \frac{1}{2\sqrt{6}} v_7\right) Y_{\overline{351}'},$$

$$M_D^T = \left(1 + XX^\dagger\right)^{-1/2} \left(\left(\bar{v}_2 - \bar{v}_3 X\right) Y_{27}\right.$$

$$\left. + \left(\frac{1}{2\sqrt{10}}\left(\bar{v}_4 - \bar{v}_9 X\right) + \frac{1}{2\sqrt{6}}\left(\bar{v}_8 - \bar{v}_{11} X\right)\right) Y_{\overline{351}'}\right)$$

$$M_E = \left(1 + \frac{4}{9} XX^\dagger\right)^{-1/2} \left(\left(-\bar{v}_2 - \frac{2}{3}\bar{v}_3 X\right) Y_{27}\right.$$

$$\left. + \left(-\frac{1}{2\sqrt{10}}\left(\bar{v}_4 + \frac{2}{3}\bar{v}_9 X\right) + \sqrt{\frac{3}{8}}\left(\bar{v}_8 + \frac{2}{3}\bar{v}_{11} X\right)\right) Y_{\overline{351}'}\right)$$

with

$$X = -3\sqrt{\frac{5}{3}}\, \frac{c_2}{f_4}\, Y_{27}\, Y_{\overline{351}'}^{-1},$$

$X \to 0$ gives minimal SO(10), but here not available ($c_2 \neq 0$) !

Y_{27} and $Y_{\overline{351}'}$ symmetric $\rightarrow M_U$ symmetric

Not true for X and so not for $M_{D,E}$

Choose system with $M_U = M_U^d$ (diagonal). Then we can always parametrize

$$X = M_U^d Y$$

with

$$Y = Y^T \quad \text{symmetric}$$

$$
\begin{aligned}
M_D^T &= \left(1 + M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(a + b\,(M_U^d Y) + c\,(M_U^d Y)^2\right)\left(d + (M_U^d Y)\right)^{-1} M_U^d \\
M_E &= \left(1 + (4/9) M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(a' + b'\,(M_U^d Y) + c'\,(M_U^d Y)^2\right)\left(d + (M_U^d Y)\right)^{-1} M_U^d \\
M_N &= \left(1 + (4/9)\, M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(a'' + b''\,(M_U^d Y) + c''\,(M_U^d Y)^2 + d''\,(M_U^d Y)^3\right. \\
&\qquad \left. + e''\,(M_U^d Y)^4\right)\left(d + (M_U^d Y)\right)^{-1} M_U^d \\
&\times \left(1 + (4/9)\, M_U^d Y^* Y M_U^d\right)^{-1/2}
\end{aligned}
$$

- Neutrino mass sum of type I and type II contributions
- a, b, c, d, a', b', c', a'', b'', c'', d'', e'' are $f(c_a, f_b, v_i, \bar{v}_j, m_i, \lambda_j)$
- Highly nonlinear, seems hopeless (unless numerically)

But remember that (let's simplify our life assuming $N_g = 2$)

- any function of a 2×2 matrix M can be always written as

$$f(M) = \alpha + \beta M$$

  with α, β written with invariants of M.

- Any 2×2 matrix A can be written as (with basis chosen)

$$A = a_1 + a_2 M_U^d + a_3 Y + a_4 M_U^d Y$$

  This simplifies the work and decreases number of unknowns (combinations)

$$
\begin{aligned}
M_D^T &= \left(1+M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(\alpha+\beta M_U^d Y\right) M_U^d \\
M_E &= \left(1+(4/9) M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(\alpha'+\beta' M_U^d Y\right) M_U^d \\
M_N &= \left(1+(4/9)\, M_U^d Y Y^* M_U^d\right)^{-1/2} \\
&\times \left(\alpha''+\beta'' M_U^d Y\right) M_U^d \\
&\times \left(1+(4/9)\, M_U^d Y^* Y M_U^d\right)^{-1/2}
\end{aligned}
$$

$N_g = 2$ case

Unknowns (9):

α, β, α', β', α'', β'',

$Y_1 \equiv Tr(Y)$, $Y_2 \equiv det(Y)$, $Z \equiv Tr(M_U^d Y)$

To fit (7):

m_s, m_b, m_μ, m_τ, V_{cb},

Δm_{23}^2, $\sin^2\theta_{23}$

Possible to fit, shown explicitly

$N_g = 3$ case

$$f(M) = \alpha + \beta M + \gamma M^2$$

Unknowns (15):

α, β, γ, α', β', γ', α'', β'', γ'',

$Y_{1,2,3}$, $Z_{1,2,3}$

To fit (14):

m_d, m_s, m_b, m_e, m_μ, m_τ, $\theta^q_{1,2,3}$,

$\theta^l_{1,2,3}$, Δm^2_{23}, Δm^2_{12}

Looks possible to fit, but harder than before, not checked yet

Summary of E_6

- E_6 a respectable (although complicated) theory
- shown examples of (so far) possibly realistic cases ($N_g = 2$)

Some open questions:

- Neutrino mass scale should be lower than M_{GUT}. To get it the full mass spectrum at that scale should be known and included in gauge couplings RGEs
- Landau pole very close just above M_{GUT}. Any possibility to treat it ?

Thank you, Charan !