Dynamical Generation of Flavour

Charanjit S. Aulakh and Charanjit Kaur

Department of Physics,
Panjab University, Chandigarh

Outline

- *NMSGUT*
  - Structure
  - Features
- $SO(10) \times O(N_g)$ (YUMGUTs)[1]
  - SM fermion and Neutrino Yukawa
  - SSB and Bajc-Melfo Superpotential[2]
  - Toy solution with flavour generation
- Summary

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[1] C. S. Aulakh and C. K. Khosa, arXiv:hep-ph/1308.5665

[2] C. S. Aulakh, arXiv:hep-ph/1402.3979

New Minimal Supersymmetric SO(10) Grand Unified Theory

- *Higgs irreps:* $\overbrace{10_H, 120_H, \overline{126}_H}^{FM}, \underbrace{\overline{126}_H, 126_H, 210_H}_{AM}$[3]
- *Matter fields:* $3 \times (16 = \{Q_L,\ L_L,\ u^c_L,\ d^c_L,\ l^c_L \oplus \nu^{\mathbf{c}}_{\mathbf{L}}\})$
- *Gauge fields:* $45 = 12$ SM $+ 33$(additional)
- *Fermion Masses:* $16 \times 16 = 10 \oplus 120 \oplus 126 \Rightarrow 10 + 120 + \overline{126}$

$$\overline{126} = (15,2,2) + \Delta_R(10,1,3) + \Delta_L(\overline{10},3,1) + (6,1,1)$$

- *Seesaw Mechanism $\Rightarrow$ Massive Neutrino*

$$M_{B-L} \sim < \vec{\Delta}_R > \ \Rightarrow M_{\nu^c} \ \Rightarrow M^I_\nu$$

$$\frac{v_W^2}{M_{B-L}} \sim < \vec{\Delta}_L > \ \Rightarrow M^{II}_\nu$$

[3] Aulakh, Garg(2012)

Superpotential & SSB

$$\begin{aligned} W_{NMSGUT} &= m\ 210^2 + M_H\ 10^2 + \lambda\ 210^3 + M\ 126 \cdot \overline{126} + \eta\ 210 \cdot 126 \cdot \overline{126} \\ &+ 10 \cdot 210(\gamma\ 126 + \bar{\gamma}\ \overline{126}) + M_O\ 120 \cdot 120 + k\ 10 \cdot 120 \cdot 210 \\ &+ \rho\ 120 \cdot 120 \cdot 210 + \zeta\ 120 \cdot 126 \cdot 210 + \bar{\zeta}\ 120 \cdot \overline{126} \cdot 210 \\ &+ h_{AB}\ 10 \cdot 16_A \cdot 16_B + f_{AB}\ \overline{126} \cdot 16_A \cdot 16_B + g_{AB}\ 120 \cdot 16_A \cdot 16_B \end{aligned}$$

- *GUT scale VEVS* : SUSY $SO(10) \rightarrow MSSM$[4]

$$\langle(15,1,1)\rangle_{210} : a \qquad \langle(15,1,3)\rangle_{210} : \omega$$
$$\langle(1,1,1)\rangle_{210} : p \qquad \langle(10,1,3)\rangle_{126,\overline{126}} : \sigma, \bar{\sigma}$$

- D Terms, preserve SUSY : $|\sigma| = |\overline{\sigma}|$
- F Terms : SSB completely analyzed by single **cubic**(in $x = -\frac{\lambda\omega}{m}$) eqn[5].

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1-x)^2 \quad ; \quad \xi = \frac{\lambda M}{\eta m}$$

[4] Aulakh, Mohapatra(1982), Clark, Kuo and Nakagawa (1983)

[5] Aulakh, Bajc, Melfo, Senjanovic and Vissani(2003)

MSSM Higgs

- Various color singlet bidoublets in $10_H, \overline{126}_H, 126_H, 210_H, 120_H$ mix to form the light doublets of MSSM.
- Fine tune $Det\mathcal{H} = 0$ to keep 1 pair of Higgs doublets $H, \bar{H}$ light.
- Bi-Unitary transformation $\Rightarrow \bar{U}^\dagger \mathcal{H} U$ is diagonal.

$$H = \sum_i \alpha_i^* h_i \quad \bar{H} = \sum_i \bar{\alpha}_i^* \bar{h}_i$$

- NMSGUT : 45+48+ 10+252+210+120=685 Fields $\Rightarrow$ 26 MSSM-irrep types : Chiral GUT scale spectra and Threshold effects.[6]

[6] Aulakh, Girdhar(2003,2004), Fukuyama, Ilakovac, Kikuchi, Mejanac, Okada (2004), Bajc, Melfo, Senjanovic and Vissani(2004), Aulakh, Garg(2006), Aulakh, Garg and Khosa(2014)

Features of NMSGUT

- Completely realistic fit of all fermion mass mixing data[7]
- Prediction of distinctive MSSM spectra
  - Light smuon (muon g-2)
  - Large A_0 and μ parameter
  - Normal shierarchy ($m_{\tilde{q}_3,\tilde{l}_3} >> m_{\tilde{q}_{1,2},\tilde{l}_{1,2}}$)
  - Heavy stop,sbottom, large A_0 *necessary* for $M_H^{Susy} >> M_Z$
  - NMSGUT requires these to survive !
- GUT scale threshold corrections $\Rightarrow \tau_p \geq 10^{34} yrs >> 10^{28} yrs$(typical without threshold corrections) [8].

[7]C.S. Aulakh and S. K. Garg, Nucl. Phys.**B**857 (2012)101

[8]C. S. Aulakh, I. Garg and C. K. Khosa,"Baryon Stability on the Higgs Dissolution Edge : Threshold corrections and suppression of Baryon violation in the NMSGUT", Nucl.Phys.**B**882 (2014) 397-449, arXiv:1311.6100 [hep-ph]

SO(10) extended with $O(N_g)$ gauged family symmetry

- Higgs channels of SO(10) carry family index, are called **Yukawons**.

  $(\overline{\mathbf{126}}(\bar{\mathbf{\Sigma}}), \mathbf{126}(\mathbf{\Sigma}), \mathbf{10}(\mathbf{H}), \mathbf{210}(\mathbf{\Phi}), \mathbf{120}(\mathbf{\Theta})) \Rightarrow$ **Sym**/**AS** rep. of $O(N_g)$

- YUMGUT superpotential:

$$W = \mathrm{Tr}(m\Phi^2 + \lambda\Phi^3 + M\overline{\Sigma}.\Sigma + \eta\Phi.\overline{\Sigma}.\Sigma + \Phi.H.(\gamma\Sigma + \bar{\gamma}.\overline{\Sigma}) + M_H H.H)$$

$$W_F = \Psi_A.(hH_{AB} + f\Sigma_{AB} + g\Theta_{AB})\Psi_B$$

$$\text{Fermion Yukawa couplings} \Rightarrow \begin{cases} 15 \rightarrow 3 & \text{MSGUT} \\ 21 \rightarrow 5 & \text{NMSGUT} \end{cases}$$

- GUT scale VEVS : $SO(10) \times O(N_g) \rightarrow MSSM$

$$(15,1,1)_{210} : a_{AB} \quad (15,1,3)_{210} : \omega_{AB}$$
$$(1,1,1)_{210} : p_{AB} \quad (10,1,3)_{126,\overline{126}} : \sigma_{AB}, \bar{\sigma}_{AB}$$

- Higgs mass matrix $\mathcal{H}$ becomes $2N_g(N_g+1)$ dimensional.

$$\mathcal{H} = \overbrace{\begin{pmatrix} -M_H & \bar{\gamma}\sqrt{3}\Omega(w-a) & -\gamma\sqrt{3}\Omega(w+a) & -\bar{\gamma}\Omega(\bar{\sigma}) \\ \gamma\sqrt{3}\Omega(w-a) & -(2M+4\eta\Omega(a-w)) & \varnothing_d & -2\eta\sqrt{3}\Omega(\bar{\sigma}) \\ -\bar{\gamma}\sqrt{3}\Omega(w+a) & \varnothing_d & -(2M+4\eta\Omega(w+a)) & \varnothing_d \\ -\gamma\Omega(\sigma) & -2\eta\sqrt{3}\Omega(\sigma) & \varnothing_d & (-2m+6\lambda\Omega(w-a)) \end{pmatrix}}^{N_g(N_g+1)/2\text{-tuples}(\{\phi_{11},\phi_{22},\ldots\phi_{N_g N_g},\sqrt{2}\phi_{12},\sqrt{2}\phi_{13},\ldots\ldots,\sqrt{2}\phi_{N_g-1,N_g}\})\text{ of } \mathbf{10},\mathbf{126},\overline{\mathbf{126}},\mathbf{210}}$$

- For $N_g = 2$ (symmetric invariant $Tr[< V > .\overline{H}.H]$)

$$\Omega[V] = \begin{pmatrix} V_{11} & 0 & V_{12}/\sqrt{2} \\ 0 & V_{22} & V_{12}/\sqrt{2} \\ V_{12}/\sqrt{2} & V_{12}/\sqrt{2} & (V_{11}+V_{22})/2 \end{pmatrix}$$

SM fermion and neutrino Yukawas

- MSSM Yukawas are generated dynamically via **VEV of Yukawon** field.

$$Y_u = \begin{pmatrix} \hat{h}\hat{V}_1 + \hat{f}\hat{V}_4 & (\hat{h}\hat{V}_3 + \hat{f}\hat{V}_6)/\sqrt{2} \\ (\hat{h}\hat{V}_3 + \hat{f}\hat{V}_6)/\sqrt{2} & \hat{h}\hat{V}_2 + \hat{f}\hat{V}_5 \end{pmatrix}$$

$$Y_d = \begin{pmatrix} \hat{h}\hat{W}_1 + \hat{f}\hat{W}_7 & (\hat{h}\hat{W}_3 + \hat{f}\hat{W}_9)/\sqrt{2} \\ (\hat{h}\hat{W}_3 + \hat{f}\hat{W}_9)/\sqrt{2} & \hat{h}\hat{W}_2 + \hat{f}\hat{W}_8 \end{pmatrix}$$

$$\hat{h} = 2\sqrt{2}h \qquad ; \qquad \hat{f} = -4i\sqrt{\frac{2}{3}}f$$

- $\hat{V}, \hat{W}$ are the normalized right and left null eigenvectors of the mass matrix $\mathcal{H}$ (fine tune $Det\mathcal{H} = 0$).
- Neutrinos and charged leptons Yukawas are obtained from Y_u, Y_d by the replacement $\hat{f} \to -3\hat{f}$.

F and D term equations

$$W = \mathrm{Tr}[m(p^2+3a^2+6\omega^2)+2\lambda(a^3+3p\omega^2+6a\omega^2)+M\bar{\sigma}\sigma+\eta(p+3a-6\omega)\frac{(\bar{\sigma}\sigma+\sigma\bar{\sigma})}{2}]$$

$$2m(p-a)-2\lambda a^2+2\lambda\omega^2=0$$

$$2m(p+\omega)+\lambda(p+2a+3\omega)\omega+\lambda\omega(p+2a+3\omega)=0$$

$$\bar{\sigma}\sigma+\sigma\bar{\sigma}=-\frac{4}{\eta}(mp+3\lambda\omega^2)\equiv F \qquad \chi\equiv p+3a-6\omega$$

$$M\sigma+\eta(\chi\sigma+\sigma\chi)/2=0 \qquad M\bar{\sigma}+\eta(\chi\bar{\sigma}+\bar{\sigma}\chi)/2=0$$

Sys of Homogeneous eqns

$$\mathbf{\Xi}\cdot\hat{\mathbf{\Sigma}}=\mathbf{\Xi}\cdot\hat{\bar{\mathbf{\Sigma}}}=\mathbf{0}$$

Nontrivial sol. $\Rightarrow$ $Rank(\mathbf{\Xi})<N_g(N_g+1)/2$

For $N_g=2$

$$\hat{\mathbf{\Sigma}}=\{\sigma_{11},\sigma_{22},\sigma_{12}\}$$

$$\mathbf{\Xi}=\begin{pmatrix}\chi_1 & 0 & \chi_{12}\\ 0 & \chi_2 & \chi_{12}\\ \chi_{12} & \chi_{12} & \chi_1+\chi_2\end{pmatrix}$$

Non degenerate case : $Det[\Xi] = 0$ *and* $Rank[\Xi] = 2$

$$Det[\Xi] = (\chi_1 + \chi_2)(\chi_{12}^2 - \chi_1\chi_2) = 0$$

Consider the branch $\chi_1 = -\chi_2$, $\chi_{12}^2 \neq \chi_1\chi_2$

$$\sigma_{12} = -\frac{\chi_1\sigma_{11}}{\chi_{12}} \quad ; \quad \bar{\sigma}_{12} = -\frac{\chi_1\bar{\sigma}_{11}}{\chi_{12}}$$
$$\sigma_{22} = -\sigma_{11} \quad ; \quad \bar{\sigma}_{22} = -\bar{\sigma}_{11}$$

The p_{AB} eqns then imply

$$\bar{\sigma}_{11}\sigma_{11} = \frac{F_{11}\chi_{12}^2}{2(\chi_{12}^2 + \chi_1^2)}$$
$$F_{11} = F_{22} \quad ; \quad F_{12} = 0$$

$D_{B-L} = 0 \Leftarrow |\sigma_{AB}| = |\bar{\sigma}_{AB}|$ We assume $\sigma_{AB} = \bar{\sigma}_{AB}$. Then eqns for σ_{AB} soluble. Rest solved by combination of analytic and numerical methods.

Hidden Sector BM Superpotential

- Visible sector contribution to O(2) D term

$$Im[\frac{-4\chi_1^*|\sigma_{11}|^2}{\chi_{12}^*} + p_{12}^*(p_{11} - p_{22}) + 3a_{12}^*(a_{11} - a_{22}) + 6w_{12}^*(w_{11} - w_{22})]$$

- Bajc-Melfo (BM) superpotential with a pair of symmetric multiplets ϕ_{ab}, S_{ab} facilitates null family ($O(N_g)$) D-term[9].

$$W_H = TrS(\mu_B\phi + \sqrt{N_g}\lambda_B\phi^2)$$

- Local minimum ($< \phi_s >= -\frac{\mu_B}{2\lambda_B}$, $< S_s >$ undetermined) breaks SUSY.
- VEV of S_s is fixed by coupling to Supergravity.
- $O(N_g)$ charged components are fixed by minimizing potential from D-terms of family symmetry (and common Sugry scalar mass term).

[9] C. S. Aulakh, arXiv:hep-ph/1402.3979

A toy solution with 2 generations

Yukawa coupling eigenvalues and mixing angles in the quark and lepton sectors :

$$Y_u^{Diag} = \{0.068, 0.021\} \quad ; \quad Y_d^{Diag} = \{0.036, 0.005\}$$

$$Y_\nu^{Diag} = \{0.091, 0.003\} \quad ; \quad Y_l^{Diag} = \{0.015, 0.006\}$$

$$\theta_{CKM} = 7.41^\circ \quad ; \quad \theta_{PMNS} = 33.68^\circ$$

Values of YUMGUT couplings and vevs of Higgs fields :

$$\xi = 0.872 + 0.547i \quad ; \quad \eta = 0.4$$

$$\lambda = -0.038 + .005i \quad ; \quad \gamma = 0.32 \quad \bar{\gamma} = -1.6$$

$$W = \begin{pmatrix} 0.141 - 0.203i & 0.317 + 0.189i \\ 0.317 + 0.189i & -0.267 + 0.307i \end{pmatrix} \quad P = \begin{pmatrix} -0.236 - 0.200i & 0.179 - 0.028i \\ 0.179 - 0.028i & -0.230 + 0.120i \end{pmatrix}$$

$$A = \begin{pmatrix} -0.230 - 0.352i & 0.338 + 0.078i \\ 0.338 + 0.078i & -0.447 + 0.120i \end{pmatrix} \quad \Sigma = \overline{\Sigma} = \begin{pmatrix} 0.086 - 0.237i & 0.197 + 0.104i \\ 0.197 + 0.104i & -0.086 + 0.237i \end{pmatrix}$$

3 generations

- Rank 5 (least degen.):

$$Det[\sigma] \sim Det[\Xi] \Rightarrow Det[\sigma] = 0$$

It provides the existence of one or more light sterile neutrino

  - One needs to consider the most general superpotential having different couplings for singlet and irrep.
  - Lower the rank $\Rightarrow$ Rank 4 (found acceptable solutions)

- Use only symmetric irrep of O(3) (worked Rank 4)

$$Y_u = \begin{pmatrix} \hat{h}(\frac{\hat{V}_1}{\sqrt{2}} + \frac{\hat{V}_2}{\sqrt{6}}) + \hat{f}(\frac{\hat{V}_6}{\sqrt{2}} + \frac{\hat{V}_7}{\sqrt{6}}) & \hat{h}\frac{\hat{V}_3}{\sqrt{2}} + \hat{f}\frac{\hat{V}_8}{\sqrt{2}} & \hat{h}\frac{\hat{V}_4}{\sqrt{2}} + \hat{f}\frac{\hat{V}_9}{\sqrt{2}} \\ \hat{h}\frac{\hat{V}_3}{\sqrt{2}} + \hat{f}\frac{\hat{V}_8}{\sqrt{2}} & \hat{h}(-\frac{\hat{V}_1}{\sqrt{2}} + \frac{\hat{V}_2}{\sqrt{6}}) + \hat{f}(-\frac{\hat{V}_6}{\sqrt{2}} + \frac{\hat{V}_7}{\sqrt{6}}) & \hat{h}\frac{\hat{V}_5}{\sqrt{2}} + \hat{f}\frac{\hat{V}_{10}}{\sqrt{2}} \\ \hat{h}\frac{\hat{V}_4}{\sqrt{2}} + \hat{f}\frac{\hat{V}_9}{\sqrt{2}} & \hat{h}\frac{\hat{V}_5}{\sqrt{2}} + \hat{f}\frac{\hat{V}_{10}}{\sqrt{2}} & -2\hat{h}\frac{\hat{V}_2}{\sqrt{6}} - 2\hat{f}\frac{\hat{V}_7}{\sqrt{6}} \end{pmatrix}$$

$$Y_d = \begin{pmatrix} \hat{h}(\frac{\hat{W}_1}{\sqrt{2}} + \frac{\hat{W}_2}{\sqrt{6}}) + \hat{f}(\frac{\hat{W}_{11}}{\sqrt{2}} + \frac{\hat{W}_{12}}{\sqrt{6}}) & \hat{h}\frac{\hat{W}_3}{\sqrt{2}} + \hat{f}\frac{\hat{W}_{13}}{\sqrt{2}} & \hat{h}\frac{\hat{W}_4}{\sqrt{2}} + \hat{f}\frac{\hat{W}_{14}}{\sqrt{2}} \\ \hat{h}\frac{\hat{W}_3}{\sqrt{2}} + \hat{f}\frac{\hat{W}_{13}}{\sqrt{2}} & \hat{h}(-\frac{\hat{W}_1}{\sqrt{2}} + \frac{\hat{W}_2}{\sqrt{6}}) + \hat{f}(-\frac{\hat{W}_{11}}{\sqrt{2}} + \frac{\hat{W}_{12}}{\sqrt{6}}) & \hat{h}\frac{\hat{W}_5}{\sqrt{2}} + \hat{f}\frac{\hat{W}_{15}}{\sqrt{2}} \\ \hat{h}\frac{\hat{W}_4}{\sqrt{2}} + \hat{f}\frac{\hat{W}_{14}}{\sqrt{2}} & \hat{h}\frac{\hat{W}_5}{\sqrt{2}} + \hat{f}\frac{\hat{W}_{15}}{\sqrt{2}} & -2\hat{h}\frac{\hat{W}_2}{\sqrt{6}} - 2\hat{f}\frac{\hat{W}_{12}}{\sqrt{6}} \end{pmatrix}$$

$$Y_u^{Diag} = \{0.125, 0.055, 0.001\} \quad ; \quad Y_d^{Diag} = \{0.026, 0.011, 0.004\}$$

$$Y_\nu^{Diag} = \{0.275, 0.122, 0.003\} \quad ; \quad Y_l^{Diag} = \{0.038, 0.014, 0.01\}$$

$$\theta_{13}^Q = 3.48^\circ \quad ; \quad \theta_{12}^Q = 6.01^\circ \quad ; \quad \theta_{23}^Q = 8.67^\circ$$

$$\theta_{13}^L = 14.33^\circ \quad ; \quad \theta_{12}^L = 20.53^\circ \quad ; \quad \theta_{23}^L = 35.13^\circ$$

$$\xi = 7.68 + 0.16i \quad ; \quad \lambda = 0.18 - .03i \quad ; \quad \eta = 0.034$$

$$\gamma = -0.53 \quad ; \quad \bar{\gamma} = -2.60 \quad ; \quad h = .14 \quad ; \quad f = .23 + .04i$$

$$A' = \begin{pmatrix} -1.8304 - 0.7199i & 1.5213 - 0.445i & -0.0854 + 0.0056i \\ 1.5213 - 0.445i & 0.6854 + 0.7288i & -0.0906 - 0.0529i \\ -0.0854 + 0.0056i & -0.0906 - 0.0529i & 1.145 - 0.009i \end{pmatrix}$$

$$P' = \begin{pmatrix} -0.2976 + 0.0331i & 0.3609 - 0.0673i & -0.2904 + 0.0343i \\ 0.3609 - 0.0673i & 0.2394 + 0.4181i & -0.5147 - 0.2562i \\ -0.2904 + 0.0343i & -0.5147 - 0.2562i & 0.0581 - 0.4512i \end{pmatrix}$$

$$W' = \begin{pmatrix} 1.0622 + 0.3626i & -0.7826 + 0.3279i & -0.0567 + 0.0102i \\ -0.7826 + 0.3279i & -0.3667 - 0.2597i & -0.1406 - 0.0597i \\ -0.0567 + 0.0102i & -0.1406 - 0.0597i & -0.6955 - 0.1029i \end{pmatrix}$$

$$\Sigma' = \begin{pmatrix} 1.5506 - 5.4394i & 1.7398 + 4.1109i & 0.0011 - 0.0811i \\ 1.7398 + 4.1109i & -1.6868 + 2.0758i & 0.0202 + 0.042i \\ 0.0011 - 0.0811i & 0.0202 + 0.042i & 0.1361 + 3.3636i \end{pmatrix}$$

$$D'^a_X = 275.15\delta_3^a$$

Summary and Remarks

- We have shown that quark and lepton Yukawa couplings can be generated dynamically in renormalizable SO(10) GUT extended with $O(N_g)$ family group.
- A hidden sector with a pair of (S_{ab}, ϕ_{ab}) fields break supersymmetry and facilitates $D_{O(N_g)} = 0$. Susy breaking is communicated via Supergravity.
- Dynamical Yukawa generation reduces no. of parameters dramatically and produced acceptable solution for $N_g = 2$ and $N_g = 3$ without optimization(worked till now).
- Both $N_g = 2$ and $N_g = 3$ have light scalars with mass $\sim M_S$ along with light modes in BM superpotential.
- Complete demonstration require optimization of numerical code for $N_g = 3$ respecting fitting criterion of NMSGUT!!

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