

New Minimal $SO(10)$ GUT A Theory for All Epochs

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Plan

- SO(10) MSGUTs : Phylogeny and Structure
- Accurate fits : Susy spectra prediction : Higgs Discovery
- Susy Spectra: Relic density, Δa_μ , $\epsilon_{leptogenesis}$
- Higgs Portal into GUT : $d = 5$ suppression
- Susy Seesaw Inflection failure in NMSGUT
- BICEP enables NMSGUT Inflation
- Dynamical MSGUT Flavour Unification
- Outlook

Target for any GUT

- **Data for GUT To Explain** : Measured ν SM parameters : (3 + 18) :
- **Gauge couplings** $g_{1,2,3}(M_Z)$
- **(Yukawa Coupling determined)** fermion masses and mixing parameters

$$\begin{aligned} m_{q,l} &: 10^{-4} - 10^2 \text{ GeV} & ; & \quad \sin \theta_i^{CKM} \sim .003 - .22 \\ \delta^{CKM} &\sim \pi/3 & ; & \quad \Delta m_\nu^2 \sim (10 \text{ meV}^2) \\ \theta_{12,23}^{PMNS} &\sim \pi/4 & ; & \quad \theta_{13}^{PMNS} \sim 8^\circ \pm 4^\circ \end{aligned}$$

- **Still Awaited (4)** : M_ν δ^{PMNS} $\alpha_{1,2}^{PMNS}$

Exotic Process limits and anomalies

- Baryon violation : Proton Lifetime :

$$\tau_P > 10^{34} \text{ yrs}$$

- Muon $g-2$ anomaly :

$$\Delta a_\mu \sim 3 \times 10^{-9}$$

- Quark & Lepton Flavour violation (SUSY \geq SM ! :

$$B.R.(B_s \rightarrow \mu\gamma) \sim 3 \times 10^{-4} \dots$$

$$B.R.(\mu \rightarrow e\gamma) \leq 2.410^{-12}$$

- Leptogenetic CP violation :

$$\epsilon_{CP} \simeq -\frac{3M_1}{8\pi M_2} \frac{\text{Im}[(Y_\nu^\dagger Y_\nu)^2]_{12}}{(Y_\nu^\dagger Y_\nu)_{11}} \sim 10^{-7}$$

Hints of Unification

- MSSM Gauge Unification at $M_X^0 \sim 10^{16.25}$ GeV
- $y_t \simeq y_b \simeq \tau(M_X)$ for $\tan \beta > 40 - 60 \Rightarrow$ same GUT irrep !
- $10^{1-2} \text{meV} = M_{\nu_L} \sim \frac{m_{top}^2}{(10^{-3} M_X^0)} \Rightarrow \Rightarrow M_{\nu_L^c} \sim 10^{-3} M_X^0$ (Type I Seesaw)

New Hints of Unification :Higgs Discovery (2012)

- **2012** : ATLAS/CMS Higgs Mass $\simeq 126\text{GeV}$.

$$\Delta M_H^2 = \frac{3m_t^4}{2\pi^2 v^2 \sin^2 \beta} \left[\log \frac{M_S^2}{m_t^2} + \frac{X_t^2}{2M_S^2} \left(1 - \frac{X_t^2}{6M_S^2} \right) \right]$$

- $X_f = A_f - \mu(\tan \beta)^{-2T_{3L}}$, $M_S = \frac{m_{\tilde{t}_1} + m_{\tilde{t}_2}}{2}$
- $\Delta M_H^2 \simeq 36\text{GeV}$ requires large M_S, A_f, μ !
- We found $10M_S, |A_0|, \mu \sim 100\text{TeV}$ required by NMSGUT fermion fit in 2008 !

New Hints of Unification : BICEP Tensor Mode Discovery (2014)

- BICEP B mode Polarization of CMB measured

$$r = \frac{P_T}{P_S} = 0.2 \pm 0.06 \quad \Rightarrow$$

$$V_0^{\frac{1}{4}} \simeq 2 \times 10^{16} \text{ GeV} \Rightarrow \Rightarrow$$

$$\phi_0 \geq 10 M_P \quad (\text{LythBound})$$

VIRTUES OF $SO(10)$

- $\{(Q_L, L_L, u_L^c, d_L^c, l_L^c) \oplus \nu_L^c\} \equiv 16$: Tight and complete

- Simple Tri-band FM Higgs Channel Spectrum

$$16 \otimes 16 = 10 \oplus 120 \oplus 126 \Rightarrow (10 + 120 + \overline{126})_H$$
$$\overline{126} = (15, 2, 2) + \Delta_R(10, 1, 3) + \Delta_L(\overline{10}, 3, 1) + (6, 1, 1)$$

- Crucial MSSM R-parity

$$(-)^{3(B-L)} \equiv M_p \subset U(1)_{B-L} \subset G_{LR} \subset G_{PS} \subset SO(10)$$

VIRTUES OF SO(10)

- Only Even B-L vevs $\langle \Delta_{L,R} \rangle \Rightarrow R_p \sqrt{\nu} \Rightarrow$ **Stable LSP**

- NATURAL HOME TO BOTH SEESAWS :

$$\text{Type I : } M_{B-L} \sim \langle \vec{\Delta}_R \rangle_{SM=0} \Rightarrow M_{\nu^c} \Rightarrow M_{\nu}^I \sim \frac{v_W^2}{M_{B-L}}$$

$$\text{Type II : } \langle \vec{\Delta}_L \rangle_{Y=2, T_{3L}=-1} \Rightarrow M_{\nu}^{II} \sim \frac{v_W^2}{M_{\Delta_L}}$$

TWO SCHOOLS OF SO(10)

| | |
|---|----------------------------------|
| Renormalizable SO(10) | NON-REN GUTS |
| Renormalizable couplings | Non Renorm. couplings |
| No ad-hoc discrete symmetries | Ad-hoc discrete necessary |
| Large(126,210,..) few (AS) | Small (10,16,45,54) irreps (AF) |
| # Parameter minimal | Unlimited # parameters |
| No Higgs duplication | Duplicates Higgs |
| $M_p \subset SO(10)$ | "string motivated" Z_2 |
| Higgs-Matter distinct | Higgs-Matter mix |
| Only B-L even vevs $\Rightarrow R_p \sqrt{\sqrt{}}$ | R_p broken |
| UNSTRUNG !! | STRING INSPIRED !! |
| a) $210 \oplus 126 \oplus \overline{126}$ | $16_H^n \oplus 10^m \oplus 45^l$ |
| b) $54 \oplus 45 \oplus 126 \oplus \overline{126}$ | Plethora |

New Minimal Supersymmetric Grand Unified Theory

- $3 \times 16_F, 10_H, \overline{126}_H, 126_H, 210_H, 120_H, 45_V$ ^{1 2}
- **AM Higgs** : $\langle 210, \overline{126}, 126 \rangle \Rightarrow \text{Susy } SO(10) \rightarrow \text{MSSM}$
- **Superpotential**

$$\begin{aligned} W = & m 210^2 + \lambda 210^3 + M 126 \cdot \overline{126} + \eta 210 \cdot 126 \cdot \overline{126} \\ & + 10 \cdot 210(\gamma 126 + \bar{\gamma} \overline{126}) \\ & + M_H 10^2 + h_{AB} 16_A \cdot 16_B + f'_{AB} 16_A 16_B \end{aligned}$$

Superpotential Parameters : $((2 \times 7 - 4) + 3 + 2 \times 6 = 25)$
Minimal³

¹CSA, Mohapatra(1982), Clark, Kuo and Nakagawa (1983)

²CSA, Garg(2006)

³CSA, Bajc, Melfo, Senjanovic, Vissani(2003)

- **GUT scale VEVs** : $SO(10) \rightarrow MSSM$

$$\langle (15, 1, 1) \rangle_{210} : a \quad \langle (15, 1, 3) \rangle_{210} : \omega \quad \langle (1, 1, 1) \rangle_{210} : p$$

$$\langle (10, 1, 3) \rangle_{\overline{126}} : \bar{\sigma} \quad ; \quad \langle (\overline{10}, 1, 3) \rangle_{126} : \sigma$$

- D Terms, preserve SUSY :

$$|\sigma| = |\bar{\sigma}| \quad \Rightarrow \quad D_{B-L} \equiv 0$$

- F terms : 4 Independent equations for $a, p, w, \sigma = \bar{\sigma}$

NMSGUT-SSB

- SSB completely analyzable Units : $\frac{m}{\lambda}$

$$\tilde{a} = \frac{(x^2 + 2x - 1)}{(1 - x)} ; \quad \tilde{p} = \frac{x(5x^2 - 1)}{(1 - x)^2}$$
$$\tilde{\sigma}\tilde{\sigma} = \frac{2 \lambda x(1 - 3x)(1 + x^2)}{\eta (1 - x)^2}$$

- **EOM reduce to single Cubic in** $x = -\lambda\omega/m$ with a single parameter $\xi = \frac{\lambda M}{\eta m}$:

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1 - x)^2$$

- Chiral GUT scale spectra : 52 MSSM multiplet sets,
26 MSSM types : 18 unmixed , 8 mixed : 504 Fields

Opening the Higgs Portal

- 6 pairs of doublets from $\{\mathbf{10}, \overline{\mathbf{126}}, \mathbf{126}_H, \mathbf{210}_H, \mathbf{120}\}_H$ mix into MSSM doublets mix in 6×6 Higgs mass matrix \mathcal{H} of which just one and only one pair pair must be light
- MSSM Higgs is PORTAL into UV completion *through* Consistency Condition(a.k.a Fine tuning) :

$$\text{Det } \mathcal{H} = 0$$

Opening the Higgs Portal

- Bi-Unitary transformation $\Rightarrow \bar{U}^T \mathcal{H} U$ is diagonal. Left and right Null Eigenvectors of \mathcal{H} define the Higgs Fractions $\alpha_j, \bar{\alpha}_j$:

$$\begin{aligned} \alpha_j &= U_{j1} & ; & & \bar{\alpha}_j &= \bar{U}_{j1} \\ H &= \sum_i \alpha_j^* h_j & ; & & \bar{H} &= \sum_i \bar{\alpha}_j^* \bar{h}_j \\ L_{eff} &: h_j \rightarrow \alpha_j H & ; & & \bar{h}_j &\rightarrow \bar{\alpha}_j \bar{H} \end{aligned}$$

- $SO(10)$ Matter Yukawas $\oplus \{\alpha_j \bar{\alpha}_j\} \Rightarrow$ MSSM Yukawas $y_{AB}^{u,d,l,\nu}$ and Majorana masses $M_{\bar{\nu}^c}$:

$$\Psi_A \cdot (h_{AB} H + f_{AB} \Sigma + g_{AB} \Theta) \Psi_B \quad \Rightarrow \quad 3 + 12 + 6 = 21 \quad \text{parameters}$$

for the MSSM Yukawas

Achievements of MSGUTs

- Completely realistic fit of all fermion mass mixing data⁴
- Prediction of distinctive MSSM spectra(2008)
 - Normal s-hierarchy ($m_{\tilde{q}_3, \tilde{l}_3} \gg m_{\tilde{q}_{1,2}, \tilde{l}_{1,2}}$)
 - Heavy stop, sbottom, large A_0 now (2012) necessary for $M_H^{Susy} \gg M_Z$
 - Large A_0 and μ parameter
 - NMSGUT requires these to survive !
 - Light smuon (muon g-2 and CDM co-annihilation)
- Generic mechanism⁵ for suppression of fast SUSY $d = 5$ operator mediated proton decay $\tau_p \sim 10^{27}$ yrs upto the Susy Gauge mediated level $\tau_p \sim 10^{36}$ yrs

⁴C.S.A. and S. K. Garg, Nucl. Phys. **B857** (2012)101

⁵C. S.A., I. Garg and C. K. Khosa, **NPB 882** (2014)397

M_S Threshold

- NMSGUT success is Quantum found / *not tree level engineered* : Quantum corrections to Light-Heavy matching resolve difficulties of $b - t - \tau$ unification in NMSGUT.
- Fermion masses : $\overline{126}$ couplings suppressed to fit $M_\nu \Rightarrow \Rightarrow$
- $10 \oplus 120$ only fits charged fermion masses $y_t \simeq y_b \simeq y_\tau(M_X)$ and $\tan \beta \simeq 50$ IF, MSSM radiative corrections raise $Y_{d,s}^{GUT}$ by 3-4 times while Y_b^{GUT} lowered by 5%.

M_S Threshold

- Precisely at large $\tan \beta$ gluino and chargino loops modify down type quarks sufficiently *provided*
- Light gauginos : $\sim .1 - 1.5$ TeV
- $M_S > 10$ TeV
- $\mu, A_0 \sim 100$ TeV
- *Normal s-hierarchy* $m_{\tilde{3}} \gg m_{\tilde{1,2}}$

| Field | Mass(GeV) |
|-------------------|---|
| $M_{\tilde{G}}$ | 1000.14 |
| M_{χ^\pm} | 569.81, 125591.22 |
| M_{χ^0} | 210.10 _{LSP} , 569.81, 125591.20, 125591.20 |
| $M_{\tilde{\nu}}$ | 15308.069, 15258.322, 21320.059 |
| $M_{\tilde{e}}$ | 1761.89, 15308.29, 211.57 _{smuon} , 15258.60, 20674.72, 21419.56 |
| $M_{\tilde{u}}$ | 11271.80, 14446.76, 11270.63, 14445.80, 24607.51, 40275.87 |
| $M_{\tilde{d}}$ | 8402.99, 11272.10, 8401.48, 11270.95, 40269.19, 51845.93 |
| M_A | 377025.29 |
| M_{H^\pm} | 377025.30 |
| M_{H^0} | 377025.28 |
| M_{h^0} | 124.00 _{h^0} |

Table: Large $\mu, B, A_0 \Rightarrow \Rightarrow$ LSP $\simeq \tilde{B}, \tilde{\chi}^\pm \tilde{W}_\pm$). Light gauginos, Normal Shierarchy \Rightarrow Higgs h^0 as found, Light smuon ! Other sfermions multi-TeV : Decoupled & Mini-split, large μ, A_0

Baryon Stability on the Higgs Dissolution Edge

$$W^{\Delta B} = L_{ABCD} Q_A Q_B Q_C L_D + R_{ABCD} \bar{U}_A \bar{U}_B \bar{D}_C \bar{L}_D$$

$$(L, R)_{ABCD} \sim \frac{(h/f/g)_{AB}(h/f/g)_{CD}}{M_X}$$

- MSSM Higgs blend of 6 pairs from NMSGUT Higgs $\Rightarrow \Rightarrow$
 $\sim 10^3$ heavy fields renormalize light Higgs : *Generically* drive it to
 “Higgs dissolution edge” :

$$Z_{H, \bar{H}} \simeq 0$$

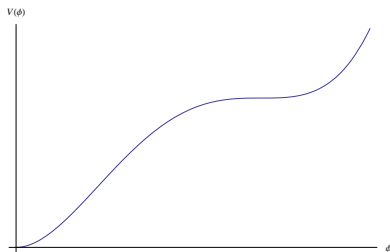
- $\Rightarrow \Rightarrow$

$$Y_{GUT} \sim \sqrt{Z_H} Y^{MSSM}(M_X) \ll Y^{MSSM}(M_X) < 1$$

- But $\mathcal{A}(\Delta B \neq 0, d = 5) \sim \frac{Y_{GUT}^2}{M_X} !!$ $\Rightarrow \Rightarrow$

- $\tau_p \gg 10^{27}$ yrs (generic) $\longrightarrow \longrightarrow$ $\tau_p > 10^{34}$ yrs !

Generic Renormalizable Inflection Point Inflation (GRIPi)



Generic Renormalizable Inflection Point Inflation

$$V = \frac{h^2}{12}\phi^4 - \frac{Ah}{6\sqrt{3}}\phi^3 + \frac{M^2}{2}\phi^2$$

Tuned for Inflection! $A = 4M\sqrt{1 - \Delta}$; $\phi_0 = \frac{\sqrt{3}M}{h}(1 - \Delta)$

$$V(\phi_0) = V_0 = \frac{M^4}{4h^2}(1 + 4\Delta)$$

GRIP parameters before BICEP

- GRIP parameters thought essential before BICEP :

$$N_{COBE} > 50 \Rightarrow \quad : \quad h^2 \sim 10^{-24.95 \pm 0.17} \left(\frac{M}{\text{GeV}} \right)$$
$$\Delta \sim 10^{-28.17 \pm .13} \left(\frac{M}{\text{GeV}} \right)^2 \quad : \quad V_0 \sim 10^{40} \left(\frac{M}{10^4 \text{ GeV}} \right)^3 \text{ GeV}^4$$

- Tensor scalar ratio r determines Vacuum energy scale

$$V_0^{1/4} = 2 \times 10^{16} \text{ GeV} \left(\frac{r}{0.1} \right)^{1/4}$$

- BICEP : $r \sim .2 \Rightarrow V_0^{1/4} \sim 2 \times 10^{16} \Rightarrow$ GUT scale controlled inflation
!!!!

Susy Chiral-Gauge systems and Inflationary Parameters

- Seek the Inflaton among fields of Particle Physics Models !!

- Generically GRIPI in Chiral-Gauge system has from V_D

$$h \sim g \sim 10^{-1} \gg h_{inflation}$$

- Composite Inflaton : e.g. LHN *flat direction* nullifies gauge coupling :

$$\tilde{N} = \tilde{\nu} = h_0 = \frac{\varphi}{\sqrt{3}} = \phi e^{i\theta}; \quad \phi \geq 0, \quad \theta \in [0, 2\pi)$$

MSSM Inflations and developments

- MSSM Inflation (Allahverdi, Mazumdar, Raidal et al 2002) MSSM flat direction inflaton e.g. LHN. suppresses gauge contributions to h ,
- $A, M \sim 1 - 10 \text{ TeV}$ from soft terms. Highly fine tuned : $\Delta \sim 10^{-20}!!$ and unstable.
- Supersymmetric Seesaw Inflation (CSA , Garg 2012) LHN flat direction,

$$M \simeq M_{\nu c} 10^8 - 10^{10} \text{ GeV} \quad \Rightarrow \quad V_0 \sim 10^{52} - 10^{58} \text{ GeV}$$

$$r \ll \ll 0 \cdot 1!!$$

- **AFTER BICEP :**

$$V_0 \simeq 10^{65} \text{ GeV} \quad \Rightarrow \quad M_{\text{Infl}} > 10^{12.5} \text{ GeV}$$

Susy Chiral-Gauge systems and Inflationary Parameters

- Superpotential couplings of inflaton components in MSSM inflation or SSI inflation φ should all be $\ll 10^{-8}$.
 - Trilinear coupling $A = 4M$.
 - Conceivable in factorized gauge model : impossible to Unify.
 - NMSGUT embedding of SSI failed !
-
- $M > 10^{13} \text{ GeV} \Rightarrow \Delta \sim 1$ Inflection breaks down, r large reanalyze !
-
- BICEP : $V_0^{1/4} \sim 10^{16.2} \Rightarrow M > 10^{13} \text{ GeV} : \Rightarrow \Delta \sim 1$ no Inflection
 $A <, \ll M$, acceptable !!

Post BICEP Renormalizable Inflation

- Slow roll still achievable for $10^{12.5} < M < 10^{14} \text{ GeV}$ without inflection.
- For $N_{\text{COBE}} > 50$ one requires

$$\omega = \frac{M}{hM_p} \geq 10$$

$$\begin{aligned} V_0 &\simeq 10^2 M^2 M_p^2 \simeq 10^{65} \text{ GeV} && \Rightarrow \\ M &\simeq 10^{13.5} \text{ GeV} && h \simeq 10^{-6} \end{aligned}$$

- **With heavy Higgs contribution** $\sim 10^{-3}$ to Inflaton mass $M \gg M_{\nu_1^c} \sim 10^7 \text{ GeV}$ easily achievable.

High Hopes on the Higgs Dissolution Edge



$$h \simeq \sqrt{h_{10}^\dagger h_{10} + f_{126}^\dagger f_{126} + g_{120}^\dagger g_{120}} \simeq 10^{-6}$$

Not easy to achieve simultaneously with accurate fits !.

- **However:** GUT threshold suppression of SO(10) Yukawas sufficient to lower *first generation* SO(10) Yukawas $\sim 10^{-6}$.
- After BICEP revolution in outlook NMSGUT may become first realistic unified matter theory to achieve Inflation and tie Inflaton to specific dynamics !!
- Current search (2 weeks) NMSGUT fits for > 18 target parameters has $\chi \sim 2$ for 18 target parameters *and* successful inflation with measured parameters ! But we hope for better fits.

OUTLOOK

- NMSGUT fits all fermion data
- Focus on Higgs portal highly rewarding : Distinctive Susy spectra predicted to be mini-split decoupled Susy with *normal* s -hierarchy
- A_0, μ, m_0 predicted to be large in 2008 as required by Higgs Discovery in 2012.
- $d = 5$ Baryon decay suppressed to tolerable levels near gauge dominated ones.

OUTLOOK

- Light smuon NLSP possible allows Susy resolution of muon $g-2$ anomaly and relic co-annihilation for correct Bino DM density.
- BICEP revolution may enable NMSGUT to become first realistic Unified theory with parameters compatible with Inflation and Inflaton made from GUT model Fields.
- Higgs mixing with UV completion and $\Delta\mathcal{H} = 0$ consistency condition again crucial : Proton Life time Inflated because universe lives on the Higgs dissolution edge which enable Inflation !
- Natural *renormalizable* MSGUT Yukawon-ification , identifying Horizontal and GUT scales emerges from MSGUT unification via exploitation of Higgs Portal consistency conditions. Predicts Light $\sim 10\text{GeV}$ viable CDM (DAMA/LIBRA).

THANK YOU