Generalized Parton Distributions for nucleon in the Light-front quark model

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Mohali

May 13-15, 2014

Quantum Chromodynamics (QCD)

- One of the challenging problem in high energy physics is to understand the hadron structure in terms of the confined quark and gluon quanta of QCD.
- It is essential to compute the detailed hadronic properties, such as, electromagnetic form factors, structure functions, distribution amplitudes, excitation dynamics of hadron resonances, etc., starting from the QCD Lagrangian.
- The strong coupling in the infrared region makes it has been difficult to find analytic solutions in terms of the wavefunctions.
- This gives impetus on the formulation of alternative approaches to calculate the hadronic properties and to make precise predictions in the nonperturbative regime.

Light-front holography (LFH)

- LFH is one of the important approach based on the AdS/CFT correspondence between the string theory on a higher-dimensional anti-de Sitter (AdS) space and conformal field theory (CFT) in physical space-time to study the hadronic properties.
- It is based on a mapping of string modes in the AdS fifth dimension to hadron light-front wave functions (LFWFs) in physical space-time. The AdS/CFT correspondence has led to a semiclassical approximation for strongly-coupled QFTs which provides physical insights into its nonperturbative dynamics.
- The models based upon AdS/QCD holography incorporate the essential features of QCD, such as, confinement and chiral symmetry breaking, and successfully explain the general properties of mesons, such as, mass spectra (Regge trajectories), electromagnetic and gravitational form factors, decay constants, and other physical quantities.

Generalized parton distributions (GPDs)

- The idea of matching the matrix elements of AdS modes to the light-front QCD has been successfully applied in the baryonic sector.
- Using the simple prescription, one can constrain the GPDs for valence quarks indirectly via the sum rules that connect them with form factors and one can also extract the LFWFs explicitly.
- GPDs have gained a lot of theoretical and experimental attention in the recent past. The first moments of GPDs are related to the electromagnetic form factors.

$$F_1^q(t) = \int dx\, H^q(x,t)\,, \qquad F_2^q(t) = \int dx\, E^q(x,t)\,.$$

- The GPDs contain much more information about the nucleon structure and spin compared to the ordinary parton distribution functions (PDFs) which are functions of x only. The GPDs reduce to the PDFs in the forward limit.
- These are off forward matrix elements and hence they do not have probabilistic interpretation.

- At zero skewness, the Fourier transform (FT) of the GPDs with respect to the momentum transfer in the transverse direction $(q_\perp)$ gives the impact parameter dependent GPDs.
- Impact parameter GPDs provide us information about partonic distributions in the impact parameter $(b_\perp)$ or the transverse position space for a given longitudinal momentum (x).
- The transverse impact parameter $(b_\perp)$ is a measure of the transverse distance between the struck parton and the center of momentum of the hadron.
- They obey certain positivity constraints and have probabilistic interpretation in terms of distribution functions.
- GPDs can be extracted from the exclusive processes like deeply virtual compton scattering and vector meson production.
- Recent experiments at DESY and JLab are planning to determine of GPDs in the valence quarks region, whereas measurements at COMPASS will explore the region of sea quarks, gluons, and transverse spin asymmetries.

Plan of work

- Recently, a new phenomenological light-front wave function (LFWF) for mesons has been proposed by matching the form factors in the AdS/QCD and Light-front QCD at an initial scale[1].
- A light-front quark model (LFQM) for nucleon has been formulated based on the proposed LFWF, which successfully explains the experimental data for the nucleon form factors.
- In this work, we extend the phenomenological LFQM to study the GPDs of up and down quark in nucleon. We compare our results with the soft wall model of AdS/QCD and a recent parameterization of parton densities using the gaussian ansatz and $t-$dependance.
- Further, we aim to discuss the behaviour of valence GPDs in the momentum space and impact parameter space and also compare our results with the phenomenological models discussed above.

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[1] T. Gutsche, V.E. Lyubovitskij, I. Schmidt, A. Vega, Phys. Rev. D **89**, 054033 (2014)

GPDs in the light front quark-scalar diquark model

- The Dirac and Pauli form factors $F_1(q^2)$ and $F_2(q^2)$ for the spin $\frac{1}{2}$ particles are defined by

$$\langle P'|J^\mu(0)|P\rangle = u(P')\big[\gamma^\mu F_1(q^2) + \frac{\iota\sigma^{\mu\alpha}q_\alpha}{2M_N}F_2(q^2)\big]u(P)\,,$$

  Dirac and Pauli form factors are normalized to electric charge and anomalous magnetic moment of the corresponding nucleon.

- In the light-front formalism the Dirac and Pauli form factors:

$$\begin{aligned} F_1^q(q^2) &= \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2 k_\perp}{16\pi^3}\Big[\psi^{*\uparrow}_{\frac{1}{2}}(x,k'_\perp)\,\psi^{\uparrow}_{\frac{1}{2}}(x,k_\perp) + \psi^{*\uparrow}_{-\frac{1}{2}}(x,k'_\perp)\,\psi^{\uparrow}_{-\frac{1}{2}}(x,k_\perp)\Big], \\ F_2^q(q^2) &= \frac{-2M}{q_1-\iota q_2}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2 k_\perp}{16\pi^3}\Big[\psi^{*\uparrow}_{\frac{1}{2}}(x,k'_\perp)\,\psi^{\downarrow}_{-\frac{1}{2}}(x,k_\perp) + \psi^{*\uparrow}_{\frac{1}{2}}(x,k'_\perp)\,\psi^{\downarrow}_{-\frac{1}{2}}(x,k_\perp)\Big], \end{aligned}$$

- where x is the fraction of the momentum carried by the active quark and $t = -q^2$ is the square of momentum transferred.

- The explicit form of LFWFs for the nucleon

$$
\begin{aligned}
\psi^{\uparrow}_{\frac{1}{2}}(x,k_\perp) &= \phi^1_q(x,\,k_\perp)\,,\\
\psi^{\uparrow}_{-\frac{1}{2}}(x,k_\perp) &= -\left(\frac{k^1+\imath k^2}{xM_n}\right)\phi^2_q(x,\,k_\perp)\,,\\
\psi^{\downarrow}_{\frac{1}{2}}(x,k_\perp) &= \left(\frac{k^1-\imath k^2}{xM_n}\right)\phi^2_q(x,\,k_\perp)\,,\\
\psi^{\downarrow}_{-\frac{1}{2}}(x,k_\perp) &= \phi^1_q(x,\,k_\perp)\,.
\end{aligned}
$$

- The $\phi^i_q(x,k_\perp)$ is the generalization of the twist $(\tau=2)$ LFWF derived from recent work on soft-wall holographic model

$$
\phi^i_q(x,\,k_\perp)=\frac{4\pi}{\kappa}\,N^i_q\sqrt{\frac{\log(1/x)}{1-x}}\,x^{a^i_q}(1-x)^{b^i_q}\,e^{\left(-\frac{k^2_\perp}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right)}\,.
$$

Here N^i_q is the normalization constant, a^i_q and b^i_q are the parameters to be fitted to the experimental data on electromagnetic form factors, magnetic moments, and charge radii.

- The expression for the GPDs for the up and down quark in nucleon in LFQM

$$H^q(x,t) = \frac{(N_q^1)^2\, x^{2a_q^1}(1-x)^{2b_q^1+1}}{I_1(0)}\left[1+\sigma^2\left(\frac{\kappa^2}{\log(1/x)}-\frac{q_\perp^2}{4}\right)\right]e^{\left(-\frac{\log(1/x)}{4\kappa^2}q_\perp^2\right)},$$

$$E^q(x,t) = \frac{2}{I_2(0)}\, N_q^1 N_q^2\, x^{a_{1q}+a_{2q}-1}(1-x)^{b_{1q}+b_{2q}+2}e^{\left(-\frac{\log(1/x)}{4\kappa^2}q_\perp^2\right)}.$$

- where

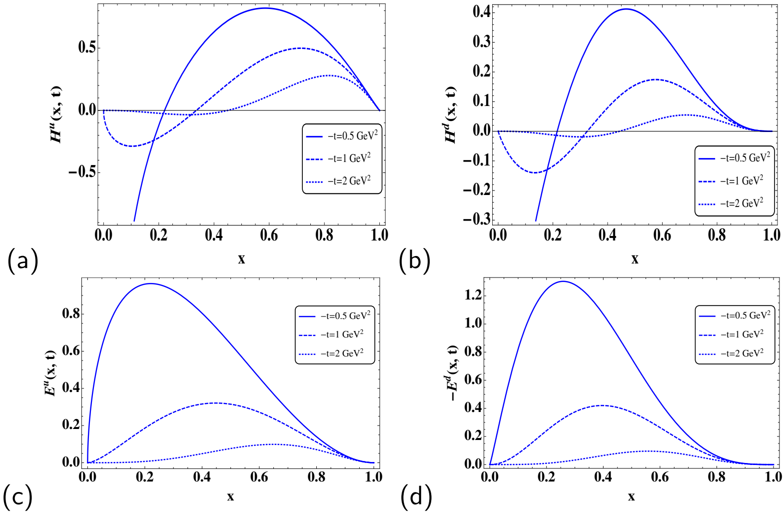
$$I_1^q(q^2) = \int_0^1 \mathrm{d}x\, (N_q^1)^2\, x^{2a_q^1}(1-x)^{2b_q^1+1}\left[1+\sigma^2\left(\frac{\kappa^2}{\log(1/x)}-\frac{q_\perp^2}{4}\right)\right]e^{\left(-\frac{\log(1/x)}{4\kappa^2}q_\perp^2\right)},$$

$$I_2^q(q^2) = 2\int_0^1 \mathrm{d}x\, N_q^1 N_q^2\, x^{a_{1q}+a_{2q}-1}(1-x)^{b_{1q}+b_{2q}+2}e^{\left(-\frac{\log(1/x)}{4\kappa^2}q_\perp^2\right)},$$

and $\sigma = \frac{N_q^2}{N_q^1}\, x^{(a_q^2-a_q^1)}(1-x)^{(b_q^2-b_q^1+1)}$.

- For the numerical calculation, we have used
$a_u^1 = 0.285, a_d^1 = 0.7, b_u^1 = 0.050, b_d^1 = 1,$
$a_u^2 = 0.244, a_d^2 = 0.445, b_u^2 = 0.109, b_d^2 = 0.336.$

Plots of GPDs $H^{u,d}(x,t)$ and $E^{u,d}(x,t)$ vs x for fixed values of $(t = -Q^2)$ for u and d quark



- We have presented the GPD $H^u(x,t)$ and $H^d(x,t)$ as a function of x for different values of t for the up and down quark.
- GPD $H(x,t)$ decreases with x and then increases, reaches a maximum and then falls to zero at $x \to 1$. One can see that the qualitative behaviour of the GPDs is same for up and down quarks, however the fall off behaviour is faster with the increasing values of x for the down quark.
- We have also present the GPD $E^{u/d}(x,t)$ as a function of x for the different values of t for the up and down quark.
- In this case the GPDs increases to a maximum value and then decreases, however, the fall off behaviour with x is same for both up and down quark.
- For all cases the peak of GPDs shifted towards a higher value of x as the value of t increases.

AdS/QCD Holography

- One of the important model in AdS/QCD is the soft-wall model in which the conformal invariance is broken by introducing a confining potential in the action of a Dirac field.
- Following the work of Brodsky and Teŕamond[2], the nucleon wavefunction in the soft wall model

$$\psi_+(z) = \frac{\sqrt{2}\kappa^2}{R^2} z^{7/2} e^{-\kappa^2 z^2/2},$$
$$\psi_-(z) = \frac{\kappa^3}{R^2} z^{9/2} e^{-\kappa^2 z^2/2}.$$

- The Dirac and Pauli form factors in terms of the LFWFs

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q^2, z) \psi_+^2(z),$$
$$F_1^n(Q^2) = -\frac{1}{3} R^4 \int \frac{dz}{z^4} V(Q^2, z)(\psi_+^2(z) - \psi_-^2(zf)),$$
$$F_2^{p/n}(Q^2) = \kappa_{p/n} R^4 \int \frac{dz}{z^4} V(Q^2, z) \psi_-^2(z).$$

[2] S.J. Brodsky, F.G. Cao, and G.F. de Teramond, Phys. Rev. D **84**, 075012 (2011); G.F. de Teramond S.J. Brodsky, arXiv:1203.4025.

- The bulk-to-boundary propagator for the soft-wall model is given as

$$V(Q^2, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

  where $U(a, b, z)$ is the Tricomi confluent hypergeometric function.
- The propagator can be written in an integral form:

$$V(Q^2, z) = \kappa^2 z^2 \int \frac{dx}{(1-x)^2} x^{Q^2/(4\kappa^2)} e^{-\kappa^2 z^2 x/(1-x)} .$$

- Using the nucleon wavefunctions and integral form of the bulk to the boundary propagator, one can calculate the form factors and GPDs. It has been proved that nucleon form factors matches very well with the experimental data for $\kappa = 0.4066$ GeV.

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[3] D. Chakrabarti and C. Mondal, Phys. Rev. D **88**, 073006 (2013); Eur. Phys. J. C **73**, 2671 (2013).

Parameterization Method (PM)

- There are various phenomenological parameterizations for the quark distribution functions based upon Gaussian ansatz, Regge parameterization, etc.. In a recent PM, the Gaussian ansatz is modified in order to incorporate the $t-$dependance of the GPDs in the form factors[4].
- The GPDs for the Dirac form factor

$$H^q(x,t) = q(x)\, \exp\,(a_+ \frac{(1-x)^2}{x^m} t)\,,$$

where $a_+ = 1.1$ and $m = 0.4$ is fixed by the low t experimental data.
- The function $q(x)$ is expressed as

$$\begin{aligned} u(x) &= 0.262 x^{-0.69}(1-x)^{3.50}(1+3.83x^{0.5}+37.65x)\,, \\ d(x) &= 0.061 x^{-0.65}(1-x)^{4.03}(1+49.05x^{0.5}+8.65x)\,. \end{aligned}$$

[4] O.V. Selyugin and O.V. Teryaev, Phys. Rev. D **79**, 033003 (2009).

- For the GPD $E^q(x,t)$, we use the widely used representation in the literature

$$E^q(x,t) = \mathcal{E}^q(x)\ \exp\left(a_+ \frac{(1-x)^2}{x^{0.4}} t\right),$$

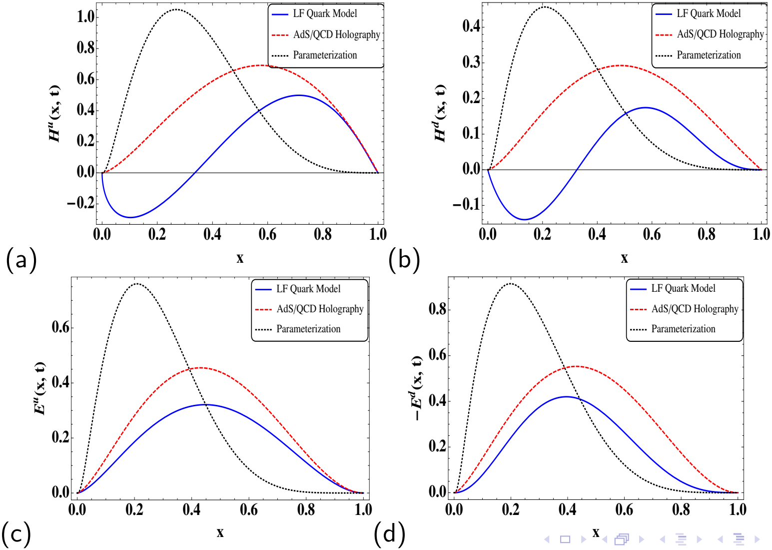
with

$$\mathcal{E}^u(x) = \frac{\kappa_u}{N_u}(1-x)^{\kappa_1}\, u(x)\,, \quad \mathcal{E}^d(x) = \frac{\kappa_d}{N_d}(1-x)^{\kappa_2}\, d(x)\,,$$

$k_1 = 1.53$, $k_2 = 0.31$, $\kappa_u = 1.673$, $\kappa_d = -2.033$, $N_u = 1.53$, $N_d = 0.946$.

- We compare our results in LFQM with AdS/QCD holography and PM. We plotted the GPDs $H^{u/d}(x,t)$ and $E^{u/d}(x,t)$ as a function of x at $t = -1$ GeV^2.
- The overall behaviour of GPDs is same for both up and down quarks. GPDs are sensitive to the small values of x in all the models.
- The results of LFQM and AdS/QCD holography are peaked at the almost same value, however, the results of PM are peaked at a lower value of x.

Comparison of GPDs $H^{u,d}(x,t)$ and $E^{u,d}(x,t)$ vs x at $t = -1$ GeV^2 for u and d quark.



GPDs in impact parameter space

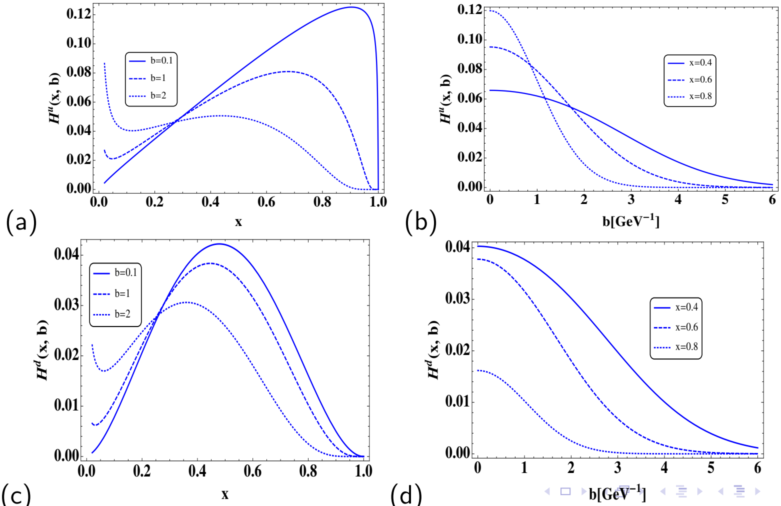
- GPDs in the momentum space are related to their impact parameter dependent parton distribution by the Fourier transform.

$$H(x,b) = \frac{1}{(2\pi)^2}\int \mathrm{d}^2 q_\perp e^{-b_\perp \cdot q_\perp} H(x,t)\,,$$

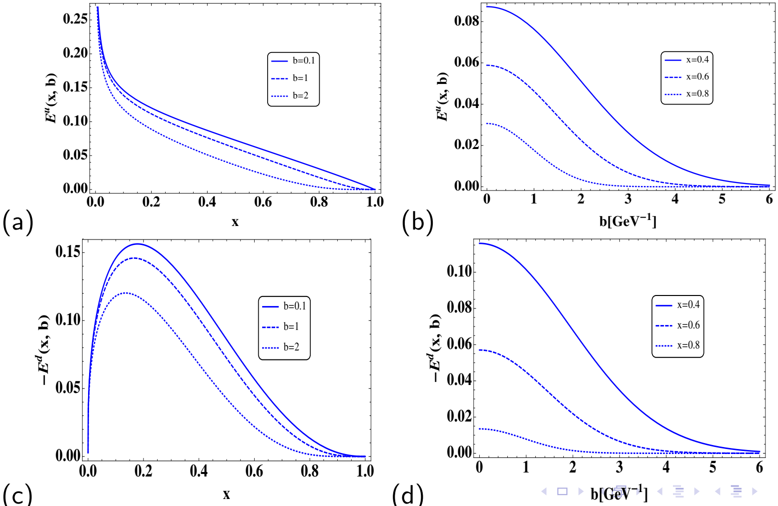
$$E(x,b) = \frac{1}{(2\pi)^2}\int \mathrm{d}^2 q_\perp e^{-b_\perp \cdot q_\perp} E(x,t)\,.$$

- The transverse impact parameter $b = |b_\perp|$ is a measure of the transverse distance between the struck parton and the center of momentum of the hadron.
- An estimate of the size of the bound state can be obtained from the relative distance between the struck parton and the center of momentum of the spectator system.

Plots of (a) $H^u(x,b)$ vs x for fixed values of b for u quark (b) $H^u(x,b)$ vs the impact parameter b for fixed value of $x = 0.4$ (c) and (b) same as in (a) and (b) but for d quark.



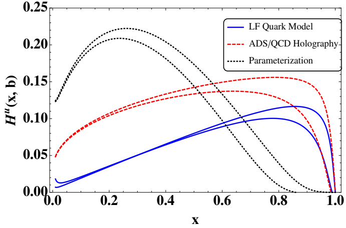
Plots of (a) $E^u(x,b)$ vs x for fixed values of b for u quark (b) $E^u(x,b)$ vs the impact parameter b for fixed value of $x = 0.4$ (c) and (b) same as in (a) and (b) but for d quark.



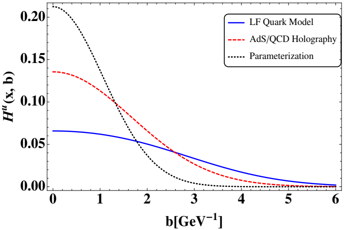
- We have plotted the behaviour of $H(x,b)$ with x for fixed values of $b = 0.1, 1, 2$ GeV^{-1}, and we have plotted the behaviour of same GPD with the impact parameter b for the fixed values of $x = 0.4, 0.6, 0.8$. We plot the same GPDs for the down quarks for the same set of parameters. Similar plots showing the behaviour of GPDs $E^{u/d}(x,b)$ are shown.
- In both cases, the maxima of GPDs shifted towards a lower value of x as b increases, therefore the transverse profile is peaked at $b = 0$ and falls off further.
- Also, for the small values of impact parameter b, the magnitude of GPD $H(x,b)$ is larger for up quark than down quark, whereas the magnitude of the GPD $E(x,b)$ is larger for down quark than up quark.

- It is also intriguing to compare the impact parameter dependent GPDs with AdS/QCD holography and parameterization method.
- We have compared the impact parameter GPDs $H^{u/d}(x,b)$ and $E^{u/d}(x,b)$ with the x and impact parameter b in the models mentioned above, for both up and down quarks.
- The qualitative behaviour of GPDs is almost same for both the up and down quarks in the impact parameter space, which is expected as the GPDs themselves behave in a similar manner.

Plots of (a) $H^u(x,b)$ vs x for fixed values of $b = 0.3, 0.6$ GeV^{-1}, (b) $H^u(x,b)$ vs the impact parameter b for $x = 0.4$ (c) and (d) same as in (a) and (b) but for d quark.



(a)



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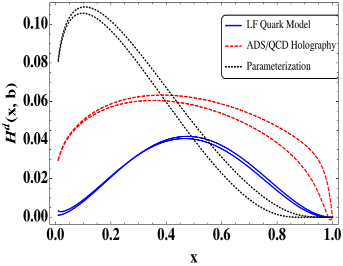
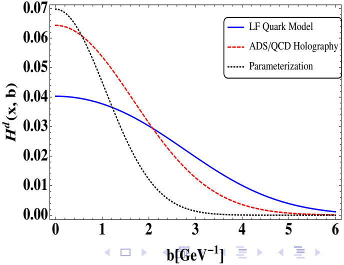
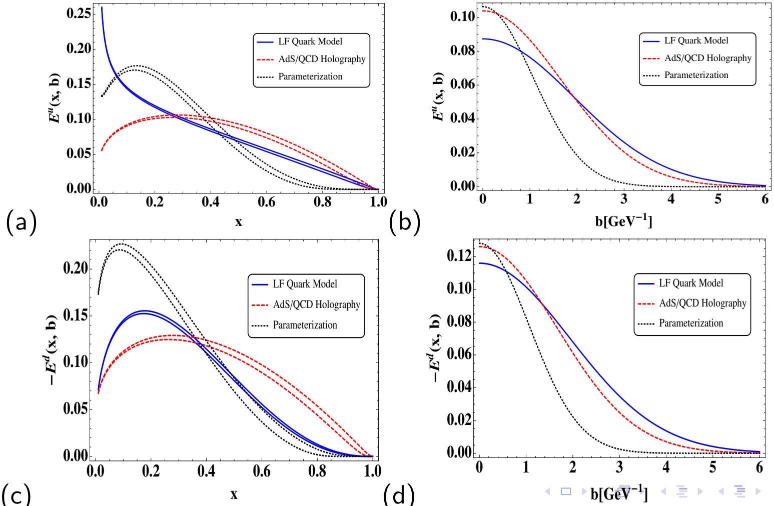


(c)



(d)

Plots of (a) $E^u(x,b)$ vs x for fixed values of $b = 0.3, 0.6$ GeV^{-1}, (b) $E^u(x,b)$ vs the impact parameter $b = |b_\perp|$ for $x = 0.4$ (c) and (d) same as in (a) and (b) but for d quark.



- For the variation of GPD $H^{u/d}(x,b)$ with x, the GPD is peaked at a higher value of x and falls off sharply in the LFQM and AdS/QCD, whereas in PM the same GPD is peaked at a much lower value of x. The GPD $E^{u/d}(x,b)$ show almost similar behaviour with x in different models, however for the small values of x, GPD has a large magnitude for the up quark in LFQM.
- The small difference with the behaviour of GPDs can be attributed to the fact that we restrict to the contribution of up and down quarks, while the contribution of the heavier strange and charm quarks has been ignored.
- For the variation of both the GPDs $H^{u/d}(x,b)$ and $E^{u/d}(x,b)$ with impact parameter b, the overall behaviour is same in different models and the transverse profile is peaked at $b=0$. It is also interesting to observe that for the small values of b, the magnitude of GPD $H(x,b)$ is larger for up quark than down quark, whereas the magnitude of the GPD $E(x,b)$ is marginally larger for down quark than up quark.

Summary and Conclusions

- We investigated the GPDs for up and down quark in nucleon using the effective light-front wavefunction obtained from matching the matrix elements in the AdS/QCD and light-front QCD at an initial scale.
- We have compared our results in the momentum space with the AdS/QCD holographic model and the phenomenological parameterization of the quark distribution functions.
- A detailed comparison of behaviour has been made in the transverse impact parameter space for the various models.
- Though we have considered only valence quarks contribution in the LFQM, it is interesting to observe that the qualitative behaviour of GPDs is same as the other phenomenological models in both momentum and impact parameter space.

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Thank you very much for your kind attention!

Advantages of the Diracs Front-Form for Hadron Physics

- Measurements are made at fixed $\tau = \frac{z}{c} + t$
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent – no boosts–No dependence on observers frame
- Dual to AdS/QCD
- LF Vacuum trivial – no condensates
- Implications for Cosmological Constant

- **Holography** The correspondence (or equivalence) between the AdS and CFT is said to be an example of holography, since it is similar to the way a 2 dimensional hologram encodes the information about a 3 dimensional object.
- **AdS/CFT Correspondence** AdS/CFT was originally motivated by heuristic arguments. Although the correspondence has not yet been proven rigorously, it has been tested extensively, and there is no reason to believe that it fails.
  One of the motivations for the duality comes from matching symmetries of the four dimensional conformal field theory with the symmetries of five dimensional AdS space. In the CFT side, we have fifteen generators. Ten of them come from the generators of the usual Lorentz transformation, one comes from the generator of scaling transformation, and the rest come from the so-called special conformal transformations. Overall, they satisfy an SO(4, 2) algebra, which is, also the isometry of the AdS_5.