

# *Why Supersymmetry? Physics Beyond the Standard Model*

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*Another name for this requirement is **Naturalness Principle**.*

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It is this reason that atomic physics described by the interactions of electrons and photons is not disturbed by the fact there are other heavier charged fermions in Nature:  $m_\mu \sim 200 m_e$ ,  $m_\tau \sim 3500 m_e$ , .....,  $m_{top} \sim 3.4 \times 10^{11} m_e$ . Effect of heavier fermions is decoupled.



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*No decoupling of the heavy mass scale from the low mass scale theory. Even in the limit  $m_L \rightarrow 0$ , this correction does not go away.*



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No enhancement of symmetry in the limit  $v \rightarrow 0$  at the quantum level.

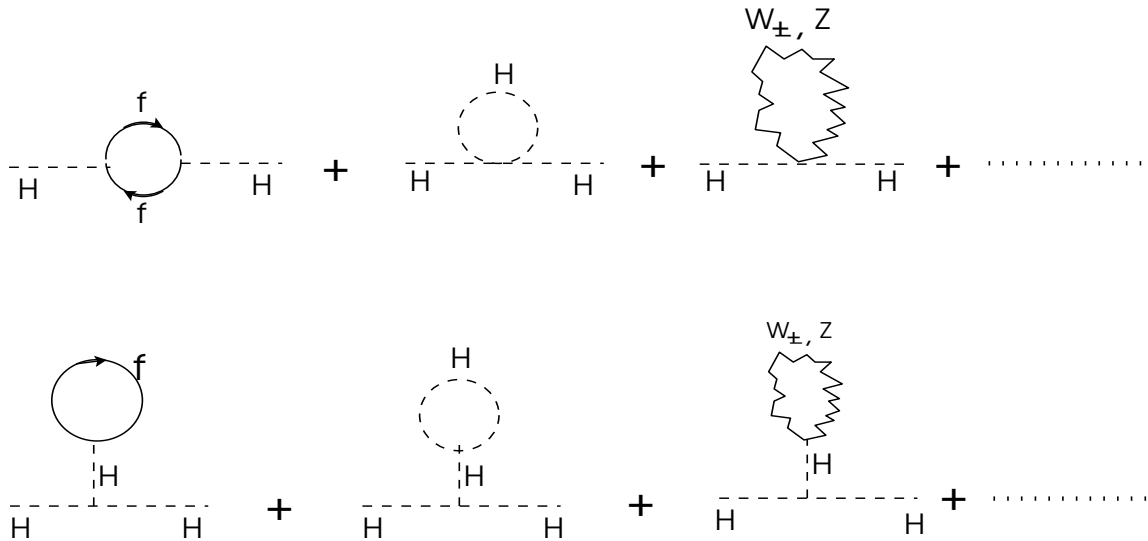
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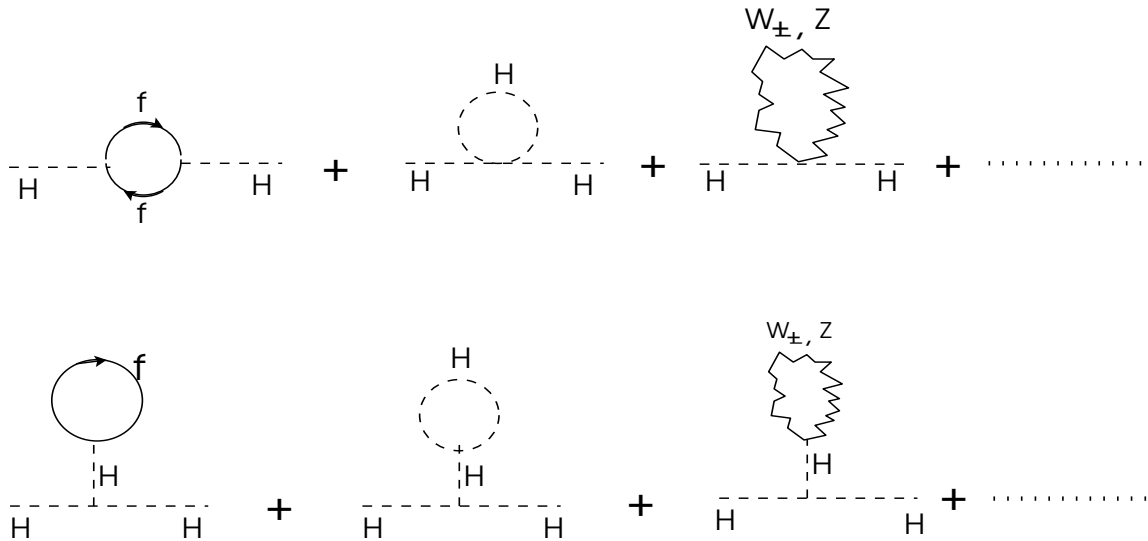
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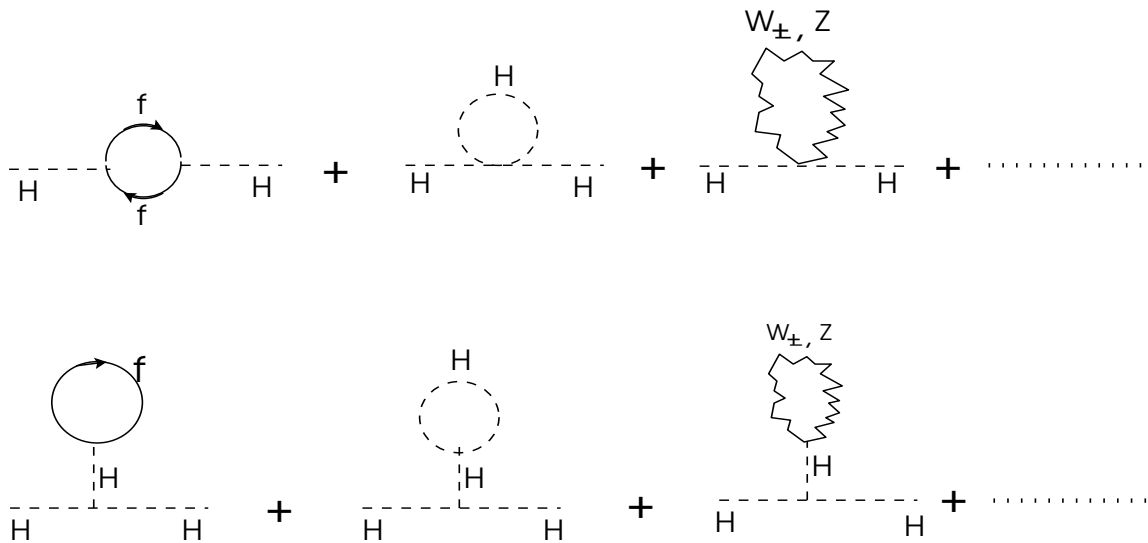
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But if there were a much heavier particle in Nature, such as in a GUT, the radiative corrections to Higgs mass would be controlled by this heavy scale and hence very large.

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This leads to the naturalness breakdown scale for EW theory to be:  $\Lambda_N \sim 1 \text{ TeV}$ .

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*Consider a gauge theory based on a gauge group  $G$  which is spontaneously broken at two stages:*

$$G \xrightarrow{F} G_1 \xrightarrow{f} G_2 ; \quad F \gg f$$

*This is achieved through v.e.v.'s of two scalar fields:  $\langle \Phi \rangle_{vac} = F \sim M_1$  and  $\langle \phi \rangle_{vac} = f \sim M_2$ .*

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*Quadratically divergent graphs for the two-point correlations of light scalar fields give large corrections ( $\sim F^2$ ) to its  $M_2^2$  ( $\sim f^2$ ).*

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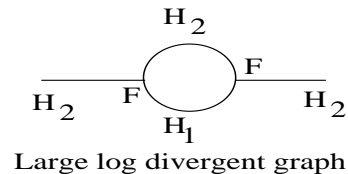
$$\begin{aligned} \mathcal{L}_{int} &\sim \kappa \Phi^2 \phi^2 = \kappa (H_1 + F)^2 (H_2 + f)^2 \\ &\sim \dots + c H_1^2 H_2 + d H_1 H_2^2 + \dots; \quad c \sim f, \quad d \sim F. \end{aligned}$$

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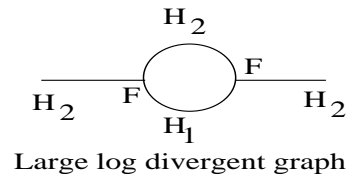


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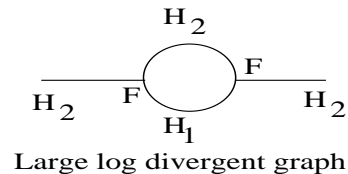
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*This is what supersymmetry does indeed provide.*

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(iii) RKK and P. Majumdar: CTS-TIFR preprint Sept 1981 (TIFR-TH-81-34), Nucl. Phys. (1982):

(a) In a SUSY theory with anomalous  $U(1)$  gauge invariance ( $\sum Q_{U(1)} \neq 0$ ), quadratic divergences are **not** absent; but in a theory which is anomaly free ( $\sum Q_{U(1)} = 0$ ), these are absent.

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(iv) RKK: CTS preprint October 1981 (Print-81-0867):

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(vi) Govindarajan, RKK, Vijayalakshmi: CTS-IIT Kanpur preprint Jan 1983, *J. Phys. G* (1983):

In SUSY theories with spontaneous broken  $U(1)$  gauge symmetry even when trace of  $U(1)$  charges is zero, the  $D$  term can get one loop corrections, but these are only logarithmically divergent.

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*Hopefully, more results from the LHC will provide this discriminating guidance that will finally result in the correct SUSY model.*

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*LI implies that we can make an arbitrary large boost transformation which would result in Lorentz contraction of lengths to arbitrary small values.*

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Thus  $\xi \neq 0$  would be a measure of low energy violation LI.

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However due to the possible Lorentz violations from the Planck scale physics, the free fermion propagator used here would get significantly modified at high scales.

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One way to do this is by introducing a Lorentz non-invariant cut-off by multiplying the free fermion propagator by a smooth function  $f(|\vec{k}|/\Lambda)$ :

$$\frac{-i(\gamma^\mu k_\mu - m_{low})}{(k^2 - m_{low}^2)} \rightarrow \frac{-i(\gamma^\mu k_\mu - m_{low})}{(k^2 - m_{low}^2)} f(|\vec{k}|/\Lambda)$$

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An example of such a cut-off function:  $f(|\vec{k}|/\Lambda) = \left(1 + (\vec{k}^2/\Lambda^2)\right)^{-1}$ .

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But at high energy (Planck scale),  $k^2 \sim \Lambda^2$ , the violation is large.

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[ Collins, Perez, Sudarsky, Urrutia, and Vucetich, PRL, 2004;

Polchinski, Class. Quantum Grav., 2012. ]



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[ Pankaj Jain and John P. Ralston, PLB 2005 ]

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Present day observational/exptal limits on violation of LI:

$$\frac{\Delta c}{c} (\sim \xi/4 + O(\xi^2)) < 10^{-20}.$$

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*Hopefully, experimental discovery of SUSY, though very likely not necessarily in the simplest version as represented by the **cMSSM**, but a more general **MSSM** framework, or even perhaps in a non-minimal form, may happen in near future.*



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*Thank You !*