UNDERSTANDING NEGATIVE INDEX METAMATERIALS

Project report

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Abstract

Negative index metamaterials (NIMS) are materials having negative refractive index because of simultaneous negative permittivity and permeability in a certain frequency range. Theoretical concept of NIMS was introduced by a Russian Physicist; Vaselago in 1967. But the natural incurrence of these materials hampered the advancement in this field for a long time. It was around 2000 that the design of NIMS became possible using structured materials (composite of split ring resonator and thin wire), whose inhomogeneities are on length scales much smaller than the wavelength of radiation. A variety of resonance can be performed to obtain these materials in the various frequency ranges of the electromagnetic spectrum. Up till now we are able to work in the microwave and terahertz frequencies. NIMS are found to show phenomena like reversed Snell's law, reversed Doppler Effect, obtuse angle Cherenkov radiation etc. The most striking application of NIMS is the perfect lens wherein sub wavelength image resolution is made possible using a single slab of NIM as a lens. The possibility of soliton formation in NIMs is also the current area of study. Experimental work is still on the pace to fabricate metamaterials which can work in higher frequencies like visible range. In this project, attempt is made to understand NIMs in detail.

1. *INTRODUCTION*

In optics, the refractive index (R.I.) of a material is conventionally taken to be a measure of optical density and is defined as

$$n = \frac{c}{v},$$

where c is the velocity of light in vacuum and v is the velocity of the electromagnetic plane wave in the medium.

From Maxwell's equation, we have

$$n^2 = \sqrt{\varepsilon\mu}$$

This implies $$n = \pm\sqrt{\varepsilon\mu},$$

where ε is the relative permittivity and μ is the relative permeability of the medium.

Naturally occurring materials has n= +ve value and they are termed as Positive Index Materials (PIM) or Right Handed Materials (RHM). But for the case where $\varepsilon<0$ and $\mu<0$, Vaselago in 1967 [1] proposed that

$$n = \sqrt{(-\varepsilon)(-\mu)}$$

$$= \sqrt{\varepsilon e^{i\Pi}}\sqrt{\mu e^{i\Pi}}$$

$$= -\sqrt{\varepsilon\mu}$$

Such materials with simultaneous negative values of both ε and μ so that we can have refractive index negative at overlapping frequencies are known as Negative Index Metamaterials (NIMs) also known as Negative Refractive Material (NRM) or Left Handed Materials (LHM). Since ε and μ are dispersive it is necessary to take into account that n depends on frequency otherwise the energy of the field given by

$$W = \varepsilon E^2 + \mu H^2,$$

will be negative when ε and μ are negative , which is impossible. When frequency dispersion exists the energy W must be given in a different manner:

$$W = \frac{\partial(\varepsilon\omega)}{\partial\omega} E^2 + \frac{\partial(\mu\omega)}{\partial\omega} H^2,$$

which is positive for a very broad class of dispersion equations for ε (ω) and μ(ω) [2].

All causal materials are dispersive which means ε and μ are complex functions of the frequency. They are negative below the plasma frequency,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2},$$

and

$$\mu(\omega) = 1 - \frac{\omega_m^2}{\omega^2},$$

where ω_p and ω_m are the electric and magnetic plasma frequency. However the approach towards absorptive resonances at lower frequencies increases the dissipation and hence their complex nature. So far there is no such material found in nature but its artificial fabrication is possible. Such materials are found to exhibit various strange phenomena. The interest in this field increases due to the possibility of superlens production using NIMs. It is also found that soliton can be formed when electromagnetic waves propagate through NIMs which will be a boon in the field of communication.

1.1. *CLASSIFICATION OF ELECTROMAGNETIC MATERIALS*

The electromagnetic (EM) response of a medium is determined by the values of ε and μ of that medium. Based on the relative signs of these two, the EM materials can be classified into four types in shown in figure (1) below.

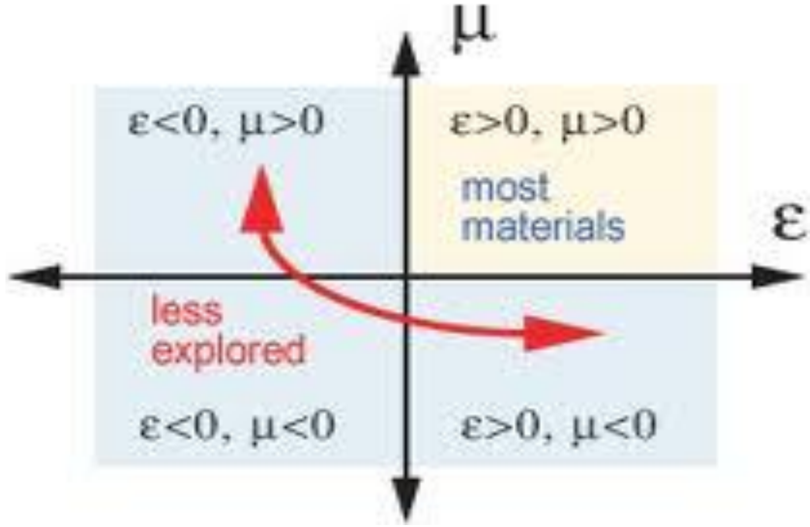

Fig (1): Re (ϵ) vs Re (μ) plane classifying em materials.

First quadrant- This corresponds to the normal material with ε>0 and μ>0. In such material we have propagating waves.

Second quadrant- ε <0 and μ>0 is the electric plasma material and here we get evanescent decaying waves. This material can support a host of resonant states localized at the surface known as surface plasmons.

Third quadrant- ε<0 and μ< 0 is the artificial NRM where we obtain propagating waves.

Fourth quadrant- ε >0 and μ <0 .This is the magnetic plasma material in which evanescent waves are obtained and can also support surface plasmons.

Our aim is to make a material of third quadrant using a composite of second and fourth quadrant materials over a common frequency range.

1.2 *PROPERTIES OF NIMS WHICH MAKE THEM DIFFERENT FROM NORMAL MATERIAL*

To study the electrodynamics of NIMs [1] which make them counter intuitive w.r.t the normal material let us consider the Maxwell's curl equations,

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{d\vec{B}}{dt} \qquad (1.2a)$$

$$\vec{\nabla} \times \vec{H} = \frac{1}{c}\frac{d\vec{D}}{dt} \qquad (1.2b)$$

(a) For a plane harmonic wave $e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}$ (1.2a) and (1.2b) reduce to

$$\vec{k} \times \vec{E} = \omega\mu\mu_0\vec{H}$$

$$\vec{k} \times \vec{H} = -\omega\epsilon\epsilon_0\vec{E}$$

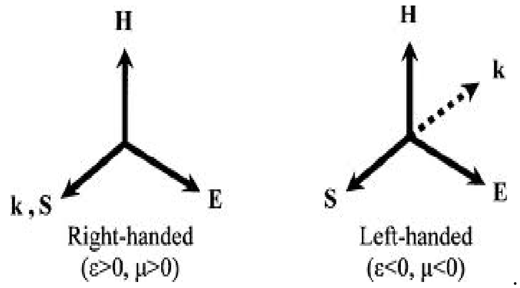

Fig (2): right handed and left handed triad.

For a medium with negative real parts of ϵ and μ with imaginary parts negligibly small $\overrightarrow{E}$, $\vec{k}$ and $\vec{H}$ form a left-handed triple of vectors whereas they form a right handed triple in normal materials.

(b) Poynting vector, $\vec{S} = \vec{E} \times \vec{H}$ and

$$\vec{k} = \frac{n\omega}{c}\hat{n} ,$$

where $\hat{n}$ is a unit vector along $\vec{E} \times \overrightarrow{H}$. This shows that $\overrightarrow{S}$ and $\overrightarrow{k}$ are parallel for n > 0 and antiparallel for n < 0. Thus in NIMs, waves propagate in a reverse phase. We also know that the phase velocity of the wave coincides with the direction of $\vec{k}$ and group velocity with $\vec{S}$. Therefore $\overrightarrow{S}$ is antiparallel to phase velocity in NIMs which means the phase wavefronts move backward.

(c) Group velocity, v_g is opposite to phase velocity, v_p in LHM.

Since

$$v_p = \frac{\omega}{k}\hat{k}$$

$$= \frac{\omega c}{n\omega}\hat{k}$$

Phase velocity in LHM is opposite to that in RHM. In linear, isotropic non dissipative media, group velocity is equal to the energy flow velocity associated with $\vec{S}$ which does not depend on material properties.

Hence for LHM phase velocity and group velocity are of opposite sign and wavefront travels towards the source.

1.3. *FABRICATION OF NIMs*

In nature we do not find any material exhibiting negative refraction at any frequency. But the theoretical implications suggest various useful applications for such material. So NIMs are artificially fabricated for the first time in the year 2000 and the fabrication consists of making a composite of an array of thin wires showing negative permittivity and split ring resonators (SRR) with negative permeability such that it has artificially designed arrays of LC oscillators mounted on electronic circuit plates capable to interact with em fields with frequency around 10 GHz. Graphing the general dispersive curve for SRRs, a region of propagation occurs from 0 up to a lower band edge followed by a gap and then an upper passband. When wires are symmetrically added between the splits rings a passband occurs within the forbidden gap.

Fig (3): composite of thin wire and SRRs

Most materials exhibiting a good electrical response can be found at almost any frequency from radiofrequency to ultraviolet frequencies but the magnetic response of most materials is limited to low microwave frequency as the magnetic polarization usually results from either unpaired electron spins or orbital electron currents. Therefore the collective excitations of these usually tend to occur at low frequency (microwave).

Let us study these components separately.

Thin wire medium: Mesh wire structures which consist of composites of randomly oriented long conducting fibers have been known to exhibit very high values of permittivity even at low concentration. Effective medium theories describe these systems when the wavelength of the incident radiation is much larger than the intrinsic length scales of the structure. However the radiation probes only the end surfaces of the metallic structures and hence it is hard to make it penetrate well into the bulk of the structure for the appearance of three dimensional effective medium to hold true in many cases. Pendry et al [3] and Sievenpiper [4] independently demonstrated that metallic wire-mesh structures have a low frequency stop band from zero frequency up to a cut off frequency which they attributed to the motion of electrons in the metal wires and therefore we can obtain a negative dielectric at low frequency.

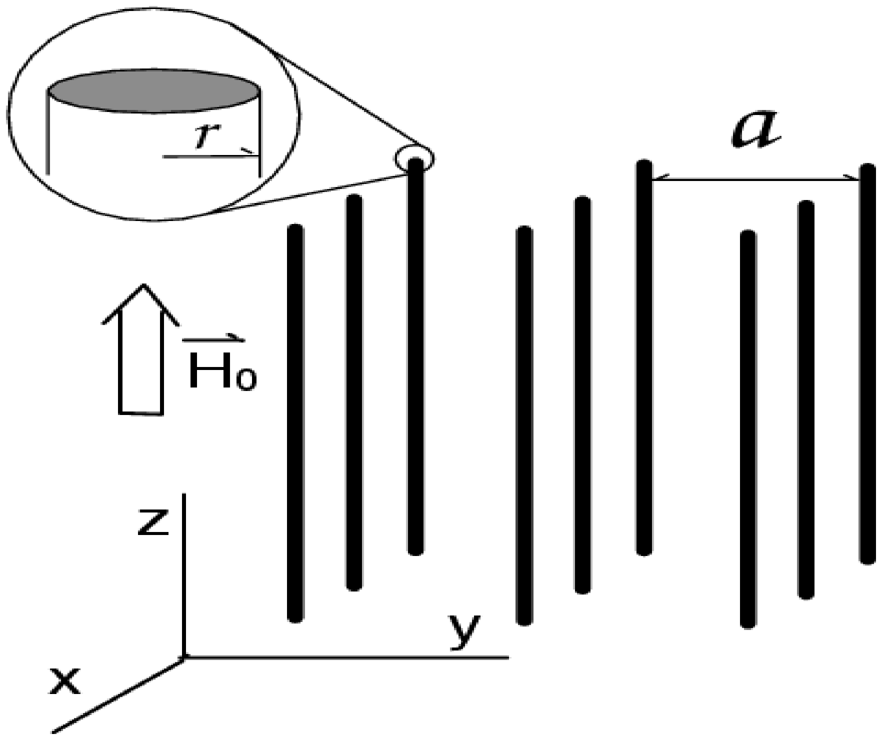

Fig (4): wire-mesh metallic structure as effective negative permittivity medium

By inherent property of thin wire medium, it has negative permittivity at frequencies below plasma frequency. Due to spatial confinement of the electrons to thin wires, the effective electron concentration in the volume of the structure is reduced which also decreases the plasma frequency. Thus an array of thin metallic wires by virtue of its macroscopic plasma like behaviour produces an effectively negative permittivity at microwave frequency.

For obtaining negative permittivity we exclude sphere and disc type media since the finite dimensions of these conducting inclusions transverse to applied field make the effective medium a diamagnetic response.

DISPERSION RELATION FOR PERMITIVITY ε (ω)

The dispersion relation is obtained using Drude-Lorentz model [5] as discussed below. The free electrons of conductors are considered to as negatively charged plasma. The long wavelength dielectric response ϵ (ω) of an electron gas is obtained from the equation of motion in an electric field.

$$m\ \ddot{x} = -e\,E$$

If x and E have time dependence e^{-iwt}, then

$$-\omega^2 m\,x = -e\,E$$

$$=> \quad x = \frac{eE}{m\omega^2}$$

The dipole moment of one electron is

$$-e\,x = -\frac{e^2}{m\omega^2}E\ .$$

The polarization defined as dipole moment per unit volume is

$$P = -\,ne\,x$$

$$= n\frac{e^2}{m\omega^2}E,$$

where n is electron concentration.

Since we know that, $\varepsilon\,(\omega) = \frac{D(\omega)}{E(\omega)} = 1{+}4\pi\,\frac{D(\omega)}{E(\omega)}$

$$\Rightarrow \varepsilon\,(\omega) = 1 + 4\pi\left(-\frac{ne^2}{m\omega^2}\right) \text{ (c.g.s)}$$

or

$$\varepsilon(\omega) = 1 - \frac{e^2}{\epsilon_0}\frac{n}{m\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \qquad (1)$$

If we consider the dissipation into account the relation is

$$\varepsilon\,(\omega) = 1 - \frac{e^2}{\epsilon_0}\frac{n}{\omega(\omega+i\gamma)}\ ;$$

γ is the damping or dissipation factor.

This is the dispersion relation for $\varepsilon(\omega)$ and it is negative for $\omega < \omega_p$ (plasma frequency).

Plasma frequency is defined by the relation

$$\omega_p^2 = ne^2/\varepsilon_0 m \quad \text{or} \quad \omega_p^2 = 4\pi ne^2/m$$

Split Ring Resonator (SRR): It consists of two rings with oppositely oriented splits. The splits in the rings are responsible for resonance at wavelengths much larger than the diameter of the rings [6]. The second split is oppositely oriented to generate a large capacitance at the small gap. With a single split a large electric dipole moment will be generated across the capacitive gap and this could well dominate over the weaker magnetic dipole moment generated in the ring. When there are two oppositely oriented splits, the dipole moment across opposite splits cancel each other and one only gets weak electric quadrupole moment whose effects can be dominated by the magnetic dipole moment. The periodic array of SRRs allows material to behave as a medium with effective μ at resonance since the incident wavelength cannot sense each individual unit. What we get is a response average over all the units. It works on the principle that a magnetic flux penetrating the metal rings will induce rotating currents in the rings which produce their own flux enhancing or opposing the incident field depending on the spin. This field pattern is dipolar.

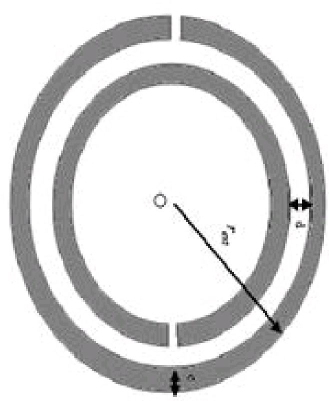

Fig (5): SRR

The magnetic flux produced can be understood as magnetic response. In other word, we can say that when an alternating magnetic field is applied perpendicular to the plane of split ring resonator, it behaves like a magnetic driven LC circuit exhibiting a resonance response at frequency Ω_m associated with the resonant circular currents in the SRR.

Where, $\Omega_m = \frac{1}{LC}$

This resonant circular currents give rise to a resonant magnetic dipole moment thereby we can recognize a SRRs system as a resonant effective permeability.

DISPERSION RELATION FOR PERMEABILITY μ (ω)

Consider the SRRs to be placed in a square lattice of lattice constant, a. In a SRR assuming the gap to be very small compared with the radius (r) and that the capacitance due to the large gaps in any single ring is negligible, we balance emf around the circuit with the ohmic drop in potential (Lenz Law).

By Lenz Law, $-\frac{d\phi}{dt} = -\frac{d}{dt}\int B.ds = \int E.dl - \frac{j}{i\omega c}$

.

$$\Rightarrow i\omega\mu_0\pi\, r^2(H_0 + j - \frac{\pi r^2}{a^2} j) = 2\pi\, r\, \rho\, j - \frac{j}{i\omega c},$$

Here we use, $B = \mu_0 H$ and $j = \sigma E$

$H = H_0 + j - \frac{\pi r^2}{a^2} j$, is the axial magnetic field inside the SRR

H_0 = applied magnetic field, j = induced current per unit length.

And third term is the depolarizing field due the induced current. ρ is the resistance per unit length.

$C = \frac{\epsilon_0}{3d}\varepsilon\pi r$ is the effective capacitance with ε as the relative dielectric permittivity of the material in the gap, d. Now for a homogeneous system of SRRs the effective magnetic field $H_{eff} = H_0 - \frac{\pi r^2}{a^2} j$.

Then $B_{eff} = \mu_0 H_0$

$$\mu_{eff} = \frac{B_{eff}}{\mu_{0H_{eff}}}$$

Solving we get,

$$\mu_{eff} = 1 - \frac{\pi r^2 / a^2}{1 - \left(3d / \mu_0 \epsilon_0 \epsilon \pi^2 r^3 \omega^2 \right) + i(2\rho / \mu_0 \omega\, r)}$$

$$= 1 + \frac{f\omega^2}{\omega_0^2 - \omega^2 - i\Gamma\omega}$$

Here $\omega_0 = \sqrt{\left(\frac{3d}{\mu_0 \epsilon_0 \epsilon \pi^2 r^3}\right)}$ is the resonant frequency and $f = \frac{\pi r^2}{a^2}$ is the filling fraction of the material. For frequencies larger than ω_0, the response is out of phase with the driving magnetic field and μ_{eff} is negative upto the magnetic plasma frequency given by

$$\omega_m = \sqrt{\frac{3d}{(1-f)\mu_0 \epsilon_0 \epsilon \pi^2 r^3}} \qquad (2)$$

TO SHOW NIMs WORK IN MICROWAVE REGION

Using the dispersion relation,

$$\epsilon = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

Where, γ = damping factor and ω_p = plasma frequency.

For small damping $\gamma = 0$, from (1) we get

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} \text{ and } \omega_p = \frac{ne^2}{m\epsilon_0}$$

For density of plasma (n) in the wires ~10^{17} $/m^3$

And putting the values of m= mass of electron, e = 1.6×10^{-19} C

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ N } m^2\, C^{-2}$$

We get $\omega_p \sim 10^{10}\ s^{-1}$ which corresponds to wavelength $\sim 10^{-2}$ m i.e in the microwave region.

For ϵ to be negative $\omega < \omega_p$ and it is possible in the microwave region.

Similarly in (2) if we put the values of r = 1.5 mm, a = 5 mm, d = 0.2 mm we have a resonant frequency in the microwave region where μ is negative.

1.4. *PHENOMENA EXHIBITED BY NIMs*

Some of the strange phenomena exhibited by NIMs are discussed as below.

(a) Reversed Snell's Law

According to Snell's Law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Where $n_1 and\, n_2$ are the refractive indices (R.I) of the rarer medium and denser medium and θ_1, θ_2 are the angles of incidence and refraction respectively. When both $n_1\, and\, n_2$ are positive refracted ray is on the opposite side of the normal while it is refracted on the same side of normal when $n_1 > 0$ but $n_2 < 0$ as shown in figure 6 below.

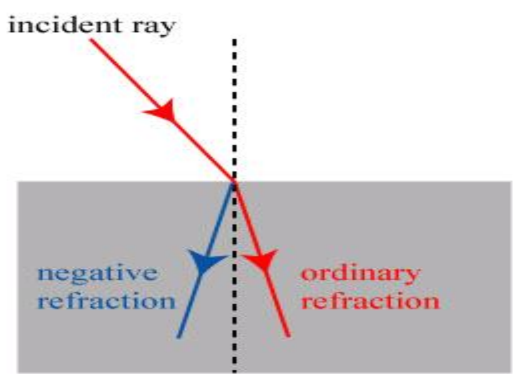

Fig (6): diagram shows positive and negative refraction

When $n_1 > 0$ but $n_2 < 0$ modified Snell's Law becomes

$$n_1 \sin\theta_1 = -\sin\theta_2$$

$$n_1 \sin\theta_1 = \sin(-\theta_2)\ ,$$

and hence the negative refraction.

(b) Reversed Doppler Effect

In Doppler Effect the frequency of a source increases or decreases when a detector is moving towards or away from it. But the thing is reversed in case of Reversed Doppler Effect.

Suppose if a source emits radiation at frequency ω and a detector is moving w.r.t source at velocity v, then the frequency received by the detector is given by

$$\omega' = \gamma\,(\omega + \vec{k}.\vec{v})$$

$$= \frac{1}{\sqrt{c^2-v^2}}\,\omega\,(1 + \frac{nv}{c}) \text{ where } |\vec{k}| = \frac{n\omega}{c}$$

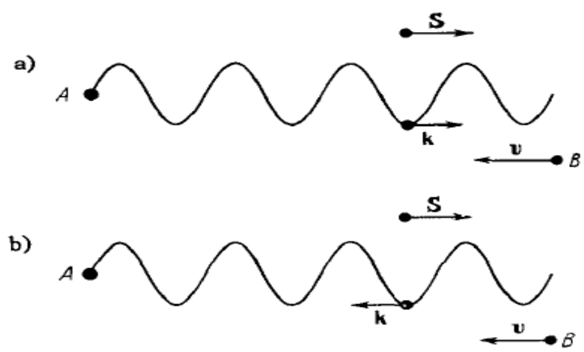

Fig (7): Doppler Effect in a right-handed substance; b) Doppler Effect in a left-handed substance. The letter A represents the source of the radiation, the letter B the receiver.

If n = +1,

$$\omega' = \omega\sqrt{\frac{c+v}{c-v}}\,.$$

But for NRM, $\vec{k}$ has negative sign since n (refractive index) is negative and

$$\omega' = \omega\sqrt{\frac{c-v}{c+v}}$$

Thus the frequency received by the detector will increase as the source is receding from it and vice versa.

(c) Obtuse angled Cherenkov's radiation

Cherenkov radiation is the cone of electromagnetic radiation when a charge particle such as electron passes through a dielectric medium at a speed greater than the phase velocity of light that medium. The charged particles polarize the molecules of the medium which then turn back rapidly to their ground state emitting radiation in the process.

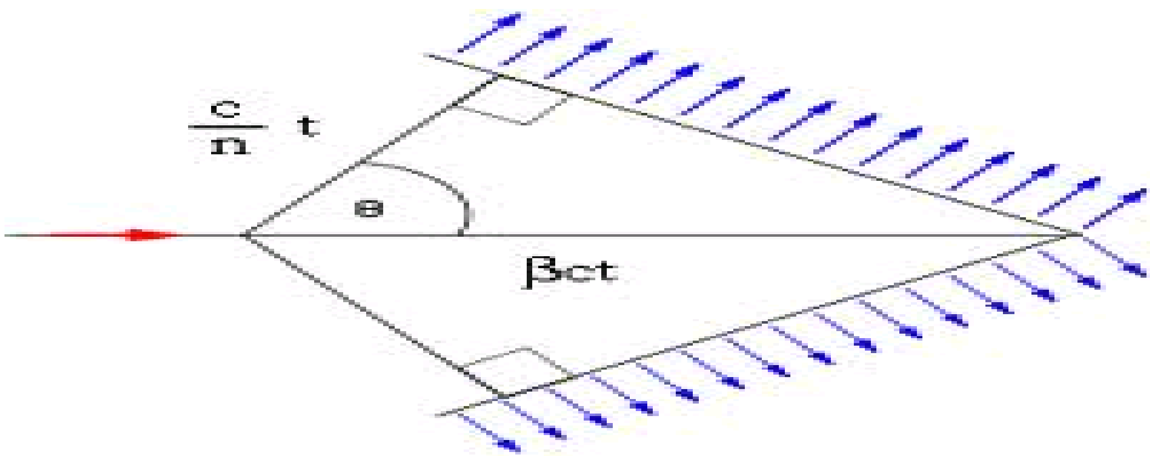

Fig (8): Cherenkov radiation.

Suppose at time t = 0, charge particle is situated at left hand corner of the diagram and traverses to the right corner with velocity phase velocity equal to βc in time t as shown in fig. 8 above. Distance traversed will be equal to βc t. If n is the R.I of the medium the cone will have traversed a distance $\frac{c}{n}$ t. Hence the acute angle of this cone is given by,

$$\cos\theta = \frac{c/n}{\beta c} = \frac{1}{\beta n}$$

For positive value of n, θ is acute while for the case of NRM n is negative and hence we will have radiation from a cone of obtuse angle.

1.5. *THEORY OF DIFRACTION LIMIT*

Consider an object and a lens placed along the z-axis so that the rays from the object are travelling along the z direction. The field emanating from the object can be written in terms of superposition of plane waves.

$$E(x, y, z, t) = \sum_{k_x k_y} A(k_x\,, k_y) e^{i(k_z z + k_y y + k_x x - \omega t)}$$

Where, $$k_z = \sqrt{\frac{\omega^2}{c^2} - (k_x^2 + k_y^2)}$$

Only positive square root is taken as the energy is going in the +z direction. All the components of the angular spectrum of the image for which k_z is real, are transmitted and refocused by an ordinary lens.

However if $k_x^2 + k_y^2 > \frac{\omega^2}{c^2}$ (higher resolution case), then k_z becomes imaginary and the wave is an evanescent wave whose amplitude decays as the wave propagate along the z- axis. The result is the loss of high frequency components of the wave which contain information about the high frequency features of the object being imaged. The highest resolution that can be obtained in a conventional lens is

$$k_{max} \simeq \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\therefore x_{min} \simeq \lambda$$

If the lens is placed at a distance larger than the operating wavelength, λ then k_z component will not be seen. In Pendry's Perfect lens, the transport of energy in the +z direction requires k_z to have opposite sign.

$$k_z = -\sqrt{\frac{\omega^2}{c^2} - (k_x^2 + k_y^2)}$$

For large angular frequencies, the evanescent wave grows so with proper lens thickness, all components of the angular spectrum can be transmitted through the lens undistorted. Thus the perfect lens is capable of capturing the near field components.

2. *APPLICATIONS*

2.1. *PERFECT LENS*

We consider a Vaselago's perfect lens [1] which consists of a slab of NRM with $\epsilon = -1$ and $\mu = -1$ capable of focusing both the propagating and evanescent waves emitted by an object.

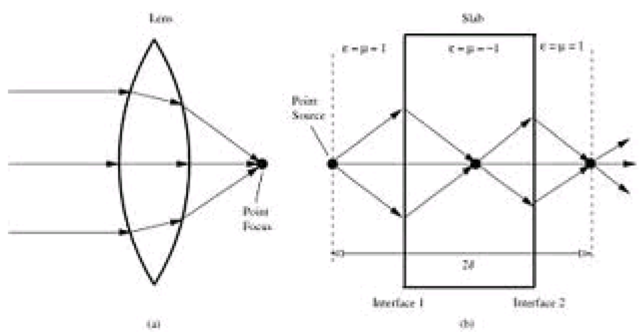

Fig 9 (a): Conventional lens   Fig 9 (b): perfect lens

When an object is placed in front of an material with $n = -1$, the waves are refracted so that they focus once inside the lens and once outside it. Such refraction allows for sub-wavelength resolution. Hence a perfect lens allows the near field rays to occur once within the lens and once outside enabling sub-wavelength imaging.

Inside the perfect lens, the amplification of the evanescent waves take place by producing excited states at the NRM surfaces. For this the surface current matches the evanescent waves from the object. We can mathematically show it using Pendry's Proposal.

Consider a Vaselago's lens consisting of a slab of thickness d with $\epsilon = -1$ and $\mu = -1$ surrounded by vacuum as shown in figure above. Source is at z = 0 (object plane). We are to calculate the fields at z = 2d (image plane).

The transmission and reflection coefficients at the interfaces are

$$T = \frac{t_{21} t_{32} e^{ik_{z2}d}}{1 - r_{12} r_{21} e^{2ik_{z2}d}}$$

$$R = \frac{r_{21} + r_{32} e^{2ik_{z2}d}}{1 - r_{12} r_{21} e^{2ik_{z2}d}}$$

When NRM has $\epsilon_{-} = -\epsilon_{+} = -1$, $\mu_{-} = \mu_{+} = -1$, we obtain trivially for propagating waves $k_{z2} = -k_{z1}$, $t_{jk} = 1$ and $r_{jk} = 0$ due to matched impedance.

$$\lim_{\substack{\epsilon_{-} \to -1 \\ \mu_{-} \to -1}} T = e^{-ik_{z1}d}$$

And

$$\lim_{\substack{\epsilon_{-} \to -1 \\ \mu_{-} \to -1}} R = 0$$

This clearly shows that the total phase change for propagation from the object plane to the image plane is zero.

For evanescent waves with

$$k_{z1} = i\sqrt{k_x^2 + k_y^2} - \epsilon_{+}\mu_{+} \frac{\omega^2}{c^2}$$

$$= ik_z$$

Then $k_{z2} = k_{z1}$ and the partial coefficients t_{jk} *and* r_{jk} diverge.

However the transmission and the reflection coefficients of the slab are still well defined in this limit.

$$\lim_{\substack{\epsilon_{-} \to -1 \\ \mu_{-} \to -1}} T = e^{k_z d}$$

$$\lim_{\substack{\epsilon_{-} \to -1 \\ \mu_{-} \to -1}} R = 0$$

i.e the slab actually increases exponentially the amplitude of the evanescent wave at the same rate by which it decays in free space.

Differences between conventional and perfect lens

Conventional lens	Perfect lens
(a) Its resolution is limited by the diffraction limit.	(a)Its resolution is not subjected to the diffraction limit.
(b) It focuses only the propagating wave of the electromagnetic radiation.	(b) It can focus both the propagating and evanescent waves of the electromagnetic radiation.
(c) A convex lens shows a converging nature and a concave lens a diverging one.	(c) A concave lens shows a converging nature and a convex lens a diverging one.

2.2. *CLOAKING*

The phenomenon of concealing an object from view is called cloaking. The principle of cloaking was first achieved in the microwave frequency on Oct 19, **2006**. An object is made invisible by covering it with a metamaterial cloak due to its ability to deflect the electromagnetic radiation. The radiation flows around the object as if nothing were there at all. We know that the bending of light is determined by refractive index. Metamaterials have a gradient in refractive index since it is inhomogeneous. The existence of this gradient in NIMs makes possible the creation of cloaking devices. Moreover the bending of light can be explained by Transformation Optics.

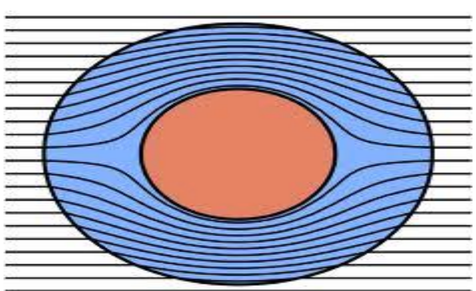

Fig 10 (a): diagram showing bending of light

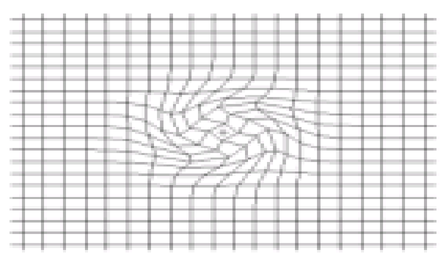

Fig 10 (b): twisting space coordinate

By transformation optics [7], if we twist the optical metamaterial it will affect its space into new coordinates. Maxwell's equations remain intact but change in the values of ε and μ by a common factor takes place and hence is the change in the value of refractive index. Then the light will travel in real space with a curved fashion in the twisted space.

Before conclusion we would like to point out an interesting aspect of EM propagation through NIMs. There is possibility of soliton formation in NIMs when EM waves propagate through it. So soliton formation in NIMs is an area of current research. Solitons [8] or solitary waves are the waves which are of localized shape and continue to travel with constant velocity for a long time without dissipating their energy. It is produced when non-linearity is cancelled out by dispersion. In a metamaterial, most of the electrostatic energy of the capacitor is located in the gap between the two rings which results in an enormously enhanced energy density making it a non-linear material and hence possibility for soliton formation. Such waves have been seen in many natural systems such as water waves [9], non-linear optics [10], BEC systems [11] and in quantum field theory (in 1, 2, 3 dimensions) [12]. Dispersive coefficient GHNLSE is a prototype equation which describes the propagation of EM waves in NIMs or non-linear optical fiber. Study of localized solutions in such media is current research interest.

3. *CONCLUSION*

In this project, the physics of the NIMs are studied with the values of ε and μ negative at overlapping frequencies when the wavelength of the incident radiation is very large compared to the inhomogeneities of the length scale. The most common method of fabrication is found to be the composite of thin wire medium and SRRs. The various properties of such materials have been discussed and the various strange phenomena exhibited by them are due to the fact that the propagation vector moves towards the source. It is also shown that the diffraction limit is being removed with perfect lens made of a slab of such material and the possibility to cloak an object in the microwave region. Soliton formation in NIMs will also be an effective tool for communication since it is it is non-dissipating. The future prospect is to make NIMs capable to work in the visible region which will be possible if we have the advanced technology to scale down the size of the unit structure.

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