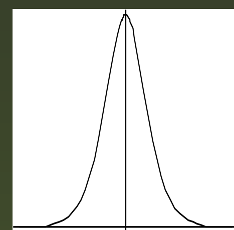
Solitons : An Introduction



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Contents

Waves

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-   

Wave Equation

- The simplest form of 1-dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0.$$

- The general form of solution

$$u(x,t) = f(x - ct) + g(x + ct).$$

$$u_t + cu_x = 0.$$

$$u(x,t) = Ae^{i(kx - \omega t)}.$$

- Dispersion relation $\omega = ck$

- Phase velocity $v_p = \frac{\omega}{k} = c$

Dispersive Wave Equation

- Simplest form of dispersive wave is

$$u_t + u_x + u_{xxx} = 0.$$

- Assume

$$u(x,t) = Ae^{i(kx-\omega t)}.$$
$$\omega = k - k^3.$$

- Phase velocity $\quad v_p = \frac{\omega}{k} = 1 - k^2$

  Group velocity $\quad v_g = \frac{d\omega}{dk} = 1 - 3k^2$

- Hence, $v_p > v_g$

  It means waves collide themselves and disperse.

Dissipative Wave Equation

- $$u_t + u_x - u_{xx} = 0.$$
- $$u(x,t) = Ae^{i(kx-\omega t)}.$$
  $$\omega = k - ik^2.$$

Hence, dispersion relation is imaginary.

$$u(x,t) = Ae^{i(kx-kt-ik^2t)}$$
$$= \; Ae^{-k^2t}e^{k(x-t)}$$

It means dissipation of amplitude with time.

Nonlinear Wave Equation

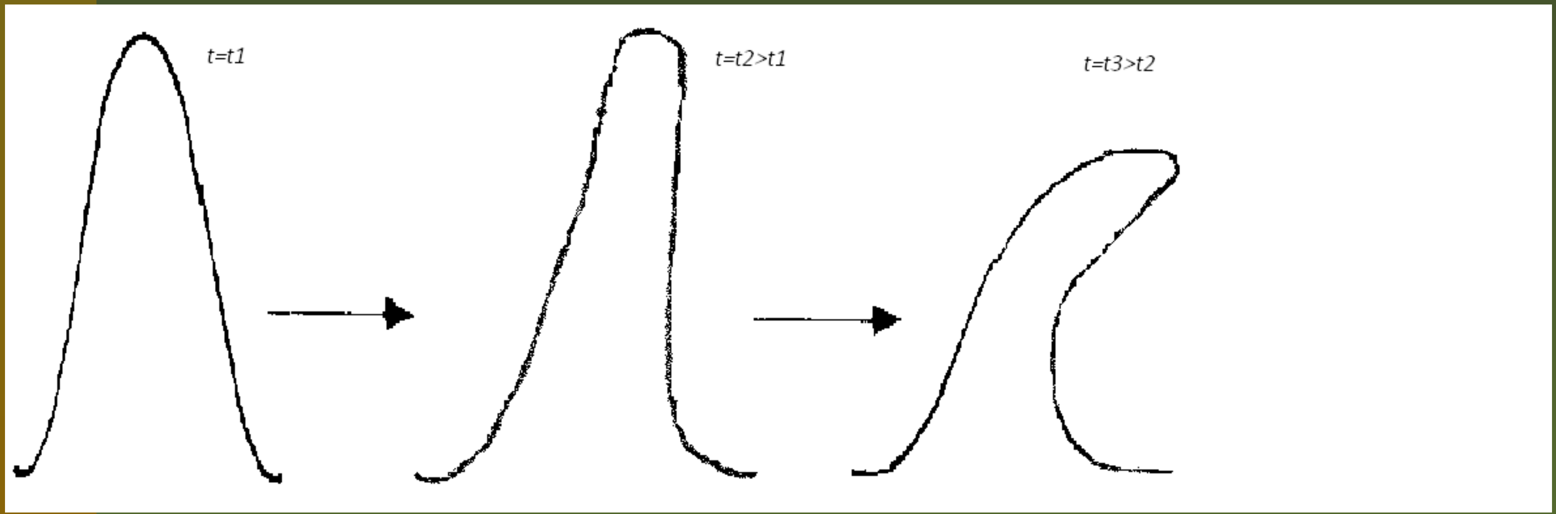
- Consider a simplest type of nonlinearity 'uu_x' to get

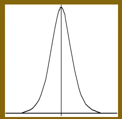
$$u_t + u_x + uu_x = 0.$$

- Compare it with $$u_t + cu_x = 0.$$

$$c = 1 + u$$

- $$u(x,t) = f(x - (1+u)t)$$





contd....

- Sometimes, we might obtain an equation which is both nonlinear and contains dispersive or dissipative terms (or both).

$$u_t + (1+u)u_x + u_{xxx} = 0.$$

$$u_t + (1+u)u_x - u_{xx} = 0.$$

contd....

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$$u_t + (1+u)u_x + u_{xxx} = 0.$$

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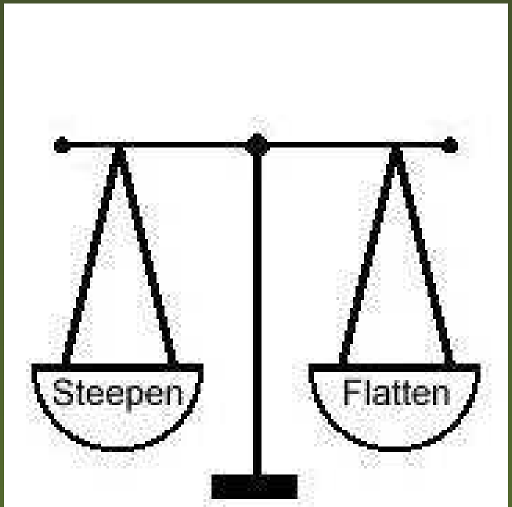
- Surprisingly, these two effects can actually neutralize each other to produce stable and localized solutions.



Solitary Waves

- A solitary wave is a non-singular and localized wave which propagates without change of its properties (shape, velocity etc.).

Solitary Waves

- A solitary wave is a non-singular and localized wave which propagates without change of its properties (shape, velocity etc.).
- Simple form of solitary wave type solutions are

$$u(x,t) = \mathrm{sech}(x-ct).$$

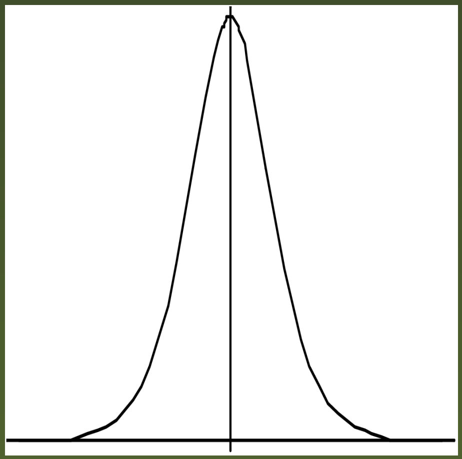
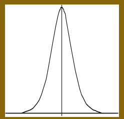
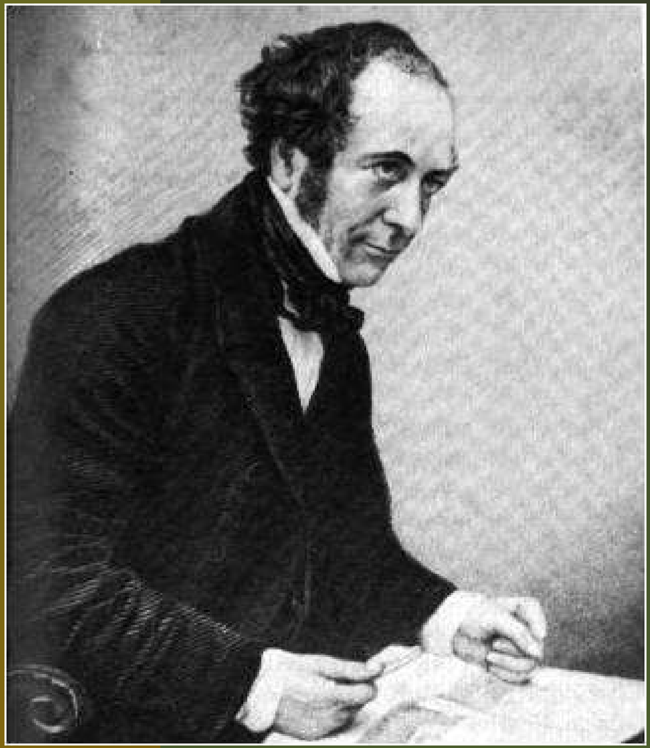


Figure 2: Plot of a solitary wave solution



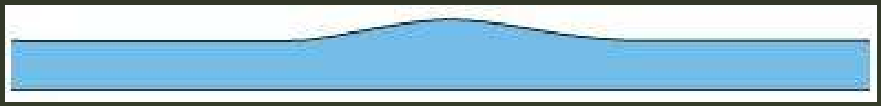
History

John Scott Russell (1808-1882)-Scotish Naval Architect



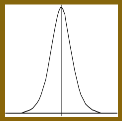
- Working on the shapes of ships
- Edinburgh-Glasgow canal in 1834
- "Great Wave of Translation"

Great Wave of Translation

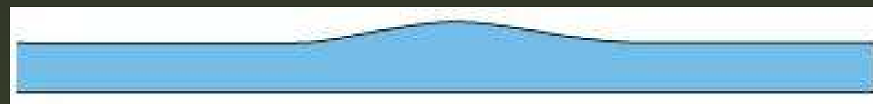


Russell's original description (1844) is still worth repeating:

"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind,rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed..."



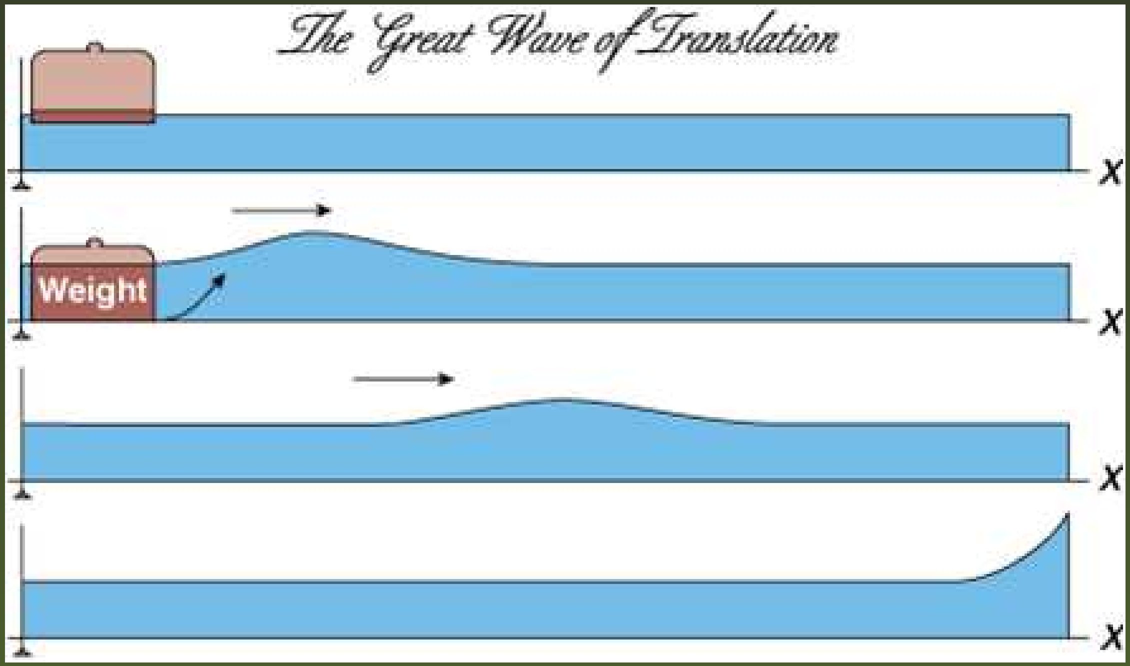
Great Wave of Translation



"...I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation."

Great Wave of Translation

Scott Russell also performed some laboratory experiments generating solitary waves.



$$c^2 = g(h+a)$$

Taller wave travel faster!

Recreation of the Wave of Translation (1995) in Glasgow

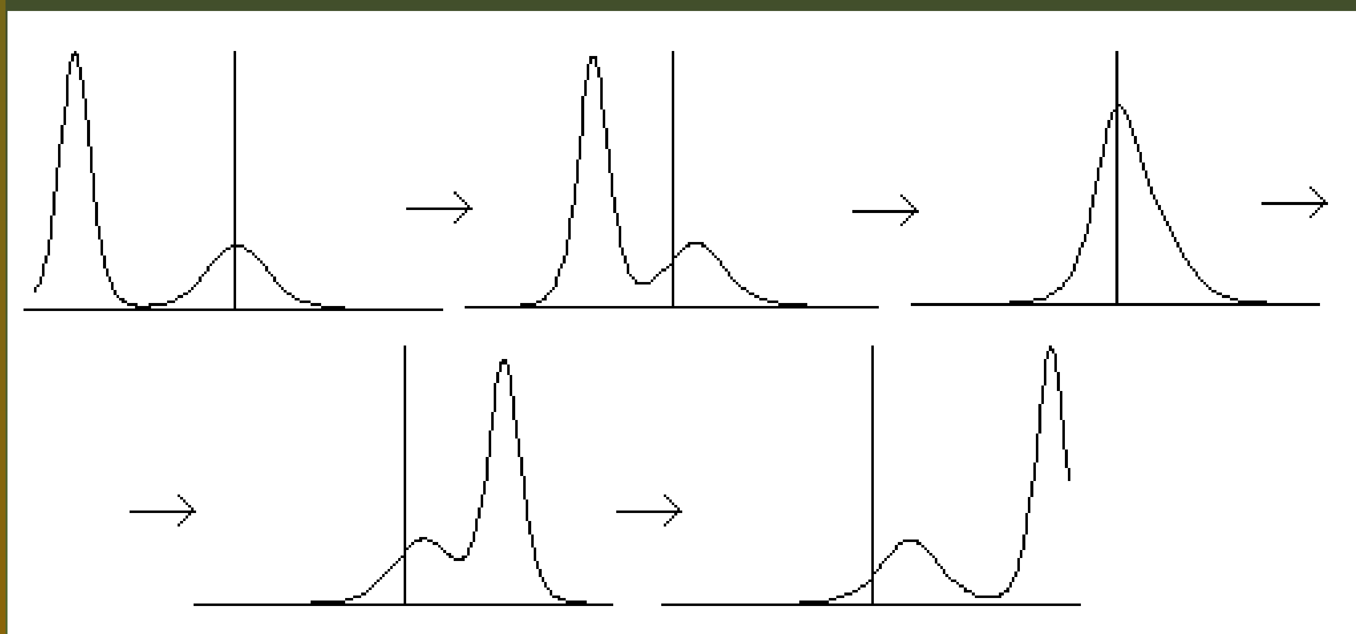
History contd...

- This phenomenon attracted attention of scientists.
- Two Dutch physicists, Korteweg and de Vries in 1898, presented their famous equation (known as KdV equation) for the evolution of long waves in a shallow one-dimensional water channel.
- In 1965, Zabusky and Kruskal solved the KdV equation numerically as a model for nonlinear lattice and found that solitary wave solutions interacted elastically with each other.
- Due to this particle-like property, they termed these solutions as solitons.

Soliton

A soliton is a self-reinforcing solitary wave solution of a NLEE which

- represents a wave of permanent form.
- is localized, so that it decays or approaches a constant value at infinity.
- is stable against mutual collisions with other solitons and retains its identity.



Examples of Nonlinear Equations

- KdV equation

$$u_t + \alpha u u_x + u_{xxx} = 0. \tag{1}$$

- Nonlinear Schrödinger equation (NLSE)

$$i u_t + \alpha u_{xx} + \beta |u|^2 u = 0. \tag{2}$$

- Sine-Gordon (SG) equation

$$u_{tt} - u_{xx} + \alpha \sin u = 0. \tag{3}$$

- Nonlinear reaction-diffusion-convection (NRDC) equation

$$u_t + v u^m u_x = D u_{xx} + \alpha u - \beta u^n. \tag{4}$$

Applications

- Soliton pulses can be used as the digital information carrying ‘bits’ in optical fibers.
- Solitary waves are very useful to study the fluid dynamical problems and in plasma studies.
- In Josephson junction, solitons are used to study the propagation of magnetic flux through the junction.
- Solitary waves arises in the study of nonlinear dynamics of DNA.

Nonlinear Equations for Real Systems

- The evolution equation for the propagation of weakly nonlinear waves in water of variable depth and width.
- In real optical fiber, the transmission of soliton is described by NLSE with the variable coefficients.
- To describe waves in an energetically open system with an external field acting on it which varies monotonically with time.
- In inhomogeneous media, the variable coefficient NRDC equations are studied due to variations of both convective and nonlinear effects.

Acknowledgement

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THANKS FOR
COMING !


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